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RATE OF RETURN WOULD NOT INCREASE TO THE EXTENT OF ANNUITY SAVING DISCOUNT

Yin-Ching Jan¹ --- Yu-Chun Lin²

¹Department of Distribution Management, National Chin-Yi University of Technology, Taiwan

²Department of Distribution Management, National Chin-Yi University of Technology, Taiwan

ABSTRACT

This note demonstrates that when there is a discount on uniform cash flow, the rate of return would not increase to the extent of the discount. The extent to which the rate of return would increase depends on the investment horizon.

Keywords: Rate of return, Annuity, Discount, Investment horizon.

JEL Classification: G11, M21.

Contribution/ Originality

The relationship between annuity and rate of return had been well developed. The paper's primary contribution is to provide the effect of the annual saving discount on the rate of return.

1. INTRODUCTION

Many insurance companies provide discount on the annual insurance fee, and claim that the rate of return would increase to the extent of the discount. How to calculate the rate of return is well provided given annuity, future value, and horizon. See the textbook written by [Ross et al. \(2004\)](#) for example. Even taxation on annuity and future value are well discussed. For instance, [Caks \(1977\)](#) propose a method to compute the interest rate when coupon income is taxed differently than capital gains. [Reichenstein \(2008\)](#) discuss rate of return for after-tax value of assets held in taxable accounts and assets that earn tax-deferred returns. However, the effect of discount on annuity cash flow has not been discussed, as far as we know. As a result, this note demonstrates what the discount on saving annuity would affect the rate of return, and shows that the rate of return would not increase to the extent of the discount on the annual cash flow. The extent to which the rate of return would increase depends on the investment horizon.

2. THE ANALYSIS

If a uniform cash flow in the amount of A dollars is invested at the beginning of each period for T periods, and the future expected value, F , at the end of the T th period is obtained, then, the

rate of return, R , of this investment can be computed by using equivalence relation. That is, the R is i at which

$$F = A \times \frac{(1+i)^{T+1} - (1+i)}{i} \tag{1}$$

The method of solving Equation (1) normally involves trial-and-error calculations until the i is converged upon.

The discipline of calculating the rate of return has been well documented. However, if there is a discount, d , on the uniform cash flow A , how the rate of return, R , will be changed? Many investors intuitively think the new rate of return would be equal to $i+d$. We will show that the new rate of return is not equal to $i+d$, and depend on the investment period T .

When there is a discount d on the uniform cash flow A , then Equation (1) will become

$$F = A(1-d) \times \frac{(1+i)^{T+1} - (1+i)}{i} \tag{2}$$

If we take a total differential on Equation (2), we can get partial derivative of i with respect to d . i.e.,

$$\frac{\partial i}{\partial d} = \frac{(1+i)^{T+1} - (1+i)}{i[(T+1)(1+i)^T - 1] - [(1+i)^{T+1} - (1+i)]} \times \frac{i}{1-d} > 0, \text{ for } i > 0. \tag{3}$$

The derivation of Equation (3) can be found in Appendix. Equation (3) satisfies the common sense that the large the discount d is, the higher the rate of return is. Nevertheless, the investment horizon T appears in the equation of $\partial i / \partial d$, which implies that the extent of the increase on the rate of return depends on the investment horizon.

To explore the effect of the investment horizon T on the $\partial i / \partial d$, we take partial derivative of $\partial i / \partial d$ with respect to T . We get,

$$\frac{\partial^2 i}{\partial T \partial d} = \frac{i(1+i)^{T+1} [1 - (1+i)^T + T \ln(1+i)]}{\{i[(T+1)(1+i)^T - 1] - [(1+i)^{T+1} - (1+i)]\}^2} \times \frac{i}{1-d} < 0, \text{ for } i > 0, \tag{4}$$

where \ln denotes natural logarithm. The proof that the Equation (4) is smaller than zero can be seen in the Appendix. From Equation (4), we can see that the longer the investment period is, the smaller the effect of the discount d on the rate of return is. Therefore, if there is a opportunity to get a rate of return, R , when we invest a uniform cash flow A at the beginning of each period for T periods, the rate of return does not equal to $R+d$ if there is a discount d on the uniform cash flow. Moreover, the rate of return is drastically decreased with the investment horizon.

We show a numerical example in Figure 1. In Figure 1, we assume investors have the opportunity to get a 10% rate of return if they invest a uniform cash flow A at the beginning of each period for T periods. When there is a 5% discount on the uniform cash flow A , from Figure

1, we can see that the rate of return is negatively related to the investment horizon. If the investment horizon is only one period, then, the rate of return becomes 15.79%. When the investment period is extended to 2 periods, the rate of return becomes 13.76%. If the investment period is expanded to 10 or 20 periods, the rate of return drastically drops to 10.90% and 10.42%, respectively.

3. CONCLUSION

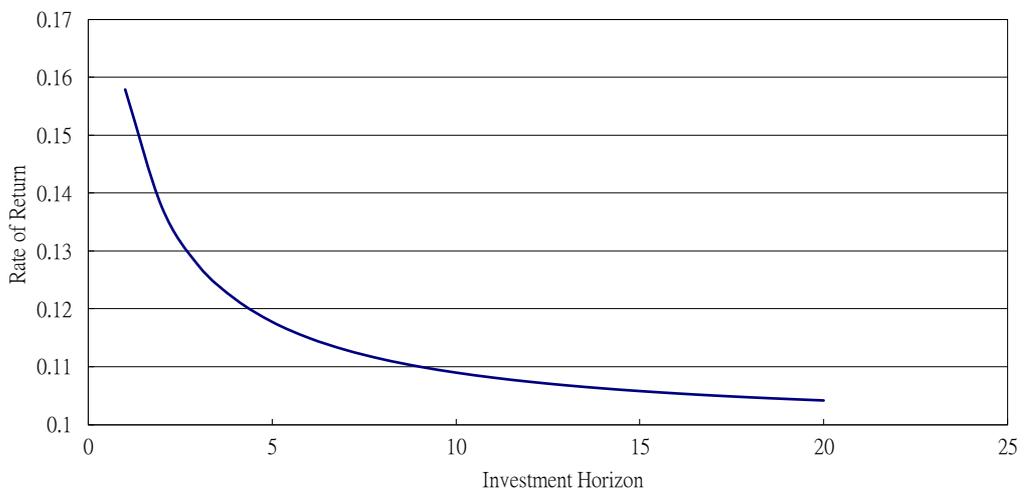
Investors always intuitively think that the rate of return would increase to the extent of the discount, when there is a discount on the saving annuity. In this note, we show that the rate of return would not increase to the extent of the discount. Moreover, the extent to which the rate of return would increase depends on the investment horizon. The new rate of return drops as investment horizon lengthens.

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Figure-1. An illustrated example

This figure shows how the investment horizon affects the rate of return when there is an annuity discount. In this example, we assume investors have the opportunity to get a 10% rate of return with a 5% discount on the uniform cash flow,



Appendix

To derivation of Equation (3), we take a total differential on Equation (2) as follows:

$$0 = \partial d \frac{(1+i)^{T+1} - (1+i)}{i} + (1-d) \frac{[(T+1)(1+i)^T \partial i - \partial i] i - [(1+i)^{T+1} - (1+i)] \partial i}{i^2}. \quad (A1)$$

Taking some algebra operations, we can get Equation (3).

To show why Equation (4) is smaller than zero, we need proof the bracket in the numerator is positive. We define $f(i, T)$ as follows:

$$f(i, T) = 1 - (1+i)^T + T \ln(1+i). \quad (A2)$$

The partial derivative of $f(i, T)$ with respect to i and T are

$$\begin{aligned} \frac{\partial f}{\partial i} &= -T(1+i)^{T-1} + \frac{T}{1+i} \\ &< 0, \text{ for } i > 0, \end{aligned} \quad (A3)$$

$$\begin{aligned} \frac{\partial f}{\partial T} &= -(1+i)^T \ln(1+i) + \ln(1+i) \\ &< 0, \text{ for } i > 0. \end{aligned} \quad (A4)$$

From Equation (A2) and (A3), we see that $f(i, T)$ is a strictly decreasing function with respect to i and T . When $T = 1$, $f(i, T)$ becomes

$$\begin{aligned} f(i, T = 1) &= -i + \ln(1+i) \\ &< 0, \text{ for } i > 0. \end{aligned} \quad (A5)$$

$f(i, T = 1)$ is smaller than zero under the condition of $i > 0$. Because $f(i, T)$ is a strictly decreasing function with respect to i and T , $f(i, T)$ is smaller than zero when $i > 0$ and $T > 1$.

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