



QUALITY IMPROVEMENT OF PETROLEUM PRODUCTS USING FUZZY CONTROL CHARTS

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ABSTRACT

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Quality improvement is one of the most important requirements to strengthen a competitive position in our markets today. So improving the quality, will lead to decrease variations, shrinkages and so production costs hence the customers acquire the appropriate products and services to use. Control charts have an effective usage field to keep the process under control. Control charts are illustrated as graphical analysis method which defines the products whether to stay in the acceptable limits or not and as a graphical analysis technique that specifies a signal in the state of product to be out of that limits. In this paper by detecting basic concept and essentials beyond the control charts usage and the improvement; so it combined with fuzzy approach to detect the optimal limits. Hence the application of the proposed fuzzy control chart is in Al-Dura Refinery to monitor variable quality characteristics. The proposed fuzzy control chart is under vague, imprecise, uncertain, and incomplete data and based on α -level fuzzy midrange for α – cut approach. As a result of the application, it's rational to say that constructing fuzzy control charts have a more flexible, a more convenient mathematical characterization concept and have more reasonable results than the traditional quality control chart techniques.

Contribution/ Originality: The paper's primary contribution is finding a way to improve the quality of petroleum products in Al- Dura refinery / Baghdad/ Iraq. This can be achieved by applying fuzzy control charts to monitor the petroleum products specification which is rarely mentioned in relative literature.

1. INTRODUCTION

Quality of manufacturing products and services is the primary key factor for the success and competitiveness of organizations and there are literally different definitions of quality. Based on the definition of Montgomery [1] quality is characterized as “inversely proportional to variability”. This definition of quality is rooted in the belief that an increase in the variability of key characteristics of a product or service results in a reduction in its quality. Hence, quality control techniques, especially Statistical Process Control, have absorbed significant amount of attention as an effective tool in reducing variability of processes and improving quality [2]. Quality control (QC) is a set of planned activities which include procedures and tests to achieve a specific specification of final products [3].

Quality Control focuses on the conversion of inputs into outputs. The purpose of quality control is to assure that processes are performing in an acceptable manner. This is accomplished by monitoring process output using statistical techniques [4]. Statistical process control is a dynamic monitoring method where product quality is actively measured and simultaneously charted while manufactured goods are being mass produced [5]. A major objective of statistical process control is to quickly detect the occurrence of assignable causes of process shifts so that investigation of the process and corrective action may be undertaken before many nonconforming units are manufactured [1]. The aim of this paper is to propose a fuzzy control chart to monitor the process of petroleum products because there are limited numbers of scientific papers about that. The proposed fuzzy control chart is validated by a case study applied in Al-Dura Refinery. This paper is organized as follows. In the second part it is mentioned a basic concepts about fuzzy numbers and fuzzy transformation techniques. In the third part, mentioned a related research of fuzzy control charts .in the fourth part mentioned the framework of the proposed methodology. In the fifth part, all mathematical models applied in this paper are listed. The next parts presents a case study based on real data collected from Al-Dura refinery to apply the proposed methodology. And then results of the proposed control charts with different approaches are compared with each other. In the last part, conclusions and findings have been interpreted.

2. FUZZY NUMBERS

Trapezoidal and triangular fuzzy numbers are the most famous shapes of fuzzy numbers. A real fuzzy number \tilde{A} is described as any fuzzy subset of the real line R with membership function f_A . A triangular fuzzy number (TriFN) as illustrated in Fig (1) is indeed a special case of the TraFN where $b=c$, and will be represented as (a, b, d) for the convenience to the TraFN. In this case, membership function of the TriFN becomes as follows [6]:

$$\mu(x) = \left\{ \begin{array}{ll} \frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1, & \text{for } x = b \\ \frac{d-x}{d-c}, & \text{for } b \leq x \leq d \\ 0, & \text{otherwise} \end{array} \right\} \quad (1)$$

Where a, b, c , and d are real numbers and $\mu(x)$ is: membership function of (x) .

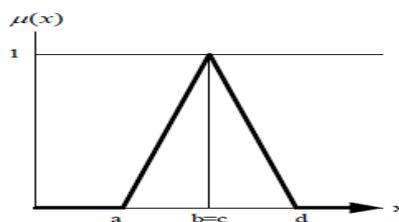


Fig-1. A Triangular Fuzzy Number (TRIFN).

- **Fuzzy Transformation Techniques:**

Fuzzy transformation techniques are used to transform the fuzzy numbers into crisp values. The four fuzzy measures of central tendency, fuzzy mode, α -level fuzzy midrange, fuzzy median and fuzzy average, which well-known in descriptive statistics, are given below:

- 1- **The fuzzy mode, f mode:** The fuzzy mode of a fuzzy set F is the value of the base variable where the membership function equals 1. This is stated as:

$$f_{mode} = \{x \mid \mu F(x) = 1\}, \forall x \in F. \quad \dots\dots\dots (2)$$

- 2- **The α -level fuzzy midrange, f_{α}^{mr} :** The average of the end points of an α -cut. An α cut, denoted by f_{α} , is a non-fuzzy subset of the base variable x containing all the values with membership function values greater than or equal to α . Thus

$$f_{\alpha} = \{x \mid \mu F(x) \geq \alpha\}, \text{ if } a_{\alpha} \text{ and } c_{\alpha} \text{ are end points of } \alpha\text{-cut } f_{\alpha}$$

Such that $a_{\alpha} = \text{Min} \{f_{\alpha}\}$ and $c_{\alpha} = \text{Max} \{f_{\alpha}\}$ then,

$$f_{\alpha}^{mr} = \frac{1}{2}(a_{\alpha} + c_{\alpha}) \dots\dots\dots (3)$$

- 3- **The fuzzy median, f_{med} :** This is the point that partitions the curve under the membership function of a fuzzy set into two equal regions satisfying the following equation:

$$\int_a^{f_{med}} \mu F(x) dx = \int_{f_{med}}^c \mu F(x) dx = \frac{1}{2} \int_a^c \mu F(x) dx \dots\dots\dots (4)$$

Where a and c are the end points in the base variable of the fuzzy set F Such that $a < c$

- 4- **The fuzzy average, f_{avg} :** Based on Zadeh, the fuzzy average is

$$f_{avg} = Av(x; f) = \frac{\int_{x=0}^1 x \mu F(x) dx}{\int_{x=0}^1 \mu F(x) dx} \dots\dots\dots (5)$$

It should be mentioned that there is no specific basis supporting any one precisely or the selection between them. Generally, the first two methods are easier to calculate than the last two when the membership function is nonlinear. Additionally, the fuzzy mode may lead to biased results when the membership function is extremely asymmetrical. The fuzzy midrange is more flexible because one can choose different levels of membership (α) of interest. If the area under the membership function is considered to be an appropriate measure of fuzziness, the fuzzy median is suitable [7].

3. LITERATURE SURVEY

In the literature there exist many papers about fuzzy control charts. Numerical examples using the data of real case studies are also given to highlight the practical usage of the proposed approaches as illustrated below:

Moraditadi [8] found out that fuzzy individual x and moving range (IX-MR) control chart in the uncertainty case for process parameters and data based on Fuzzy mode for triangular and trapezoidal fuzzy number coded by MATLAB software. It was found that the fuzzy control chart was more sensitive than the traditional control charts and more consistent with the actual situation. As well, fuzzy mode method was more sensitive for trapezoidal than triangular fuzzy numbers Moraditadi [8]. Chen and Yu [9] applied fuzzy zones instead of crisp ones to describe the monitored zones; and a fuzzy rule to construct the corresponding fuzzy zone control chart. It was found that the proposed fuzzy zone control chart could achieve better performance against the classical control charts the monitoring of process variation Chen and Yu [9]. Tadi and Darestani [10] Investigated the fuzzy IX-MR control chart when Data transformed into trapezoidal fuzzy number using fuzzy mode and Fuzzy Rules approaches. The results were grouped into four categories (in control, out of control, rather in control and rather out of control). It was found that fuzzy control chart is more sensitive than classical control chart Tadi and Darestani [10]. Sogandi, et al. [11] developed a new fuzzy control chart for monitoring attribute quality characteristics based on α -level fuzzy midrange approach. It was found the performance and comparative results of the proposed fuzzy control chart was measured in terms of average run length (ARL) by Mont Carlo simulation Sogandi, et al. [11]. Sorooshian

[12] tried an approach based on fuzzy set theory monitoring attribute quality characteristics which considers uncertainty and vagueness and for this purpose. It was found that the proposed approach has a better performance and detects abnormal shifts in the process, especially in small shifts and small sample size, faster than current related approaches [12].

4. THE PROPOSED METHODOLOGY

In this paper the proposed methodology as shown in Fig (4) has been focus on applying traditional control charts and fuzzy control charts. The main steps of the proposed methodology list as follows:

1. Problem Definition and Data Collection.
2. Selection of the control chart type.
3. Establish the control limits.
4. Analysis of the control chart
5. Decision making

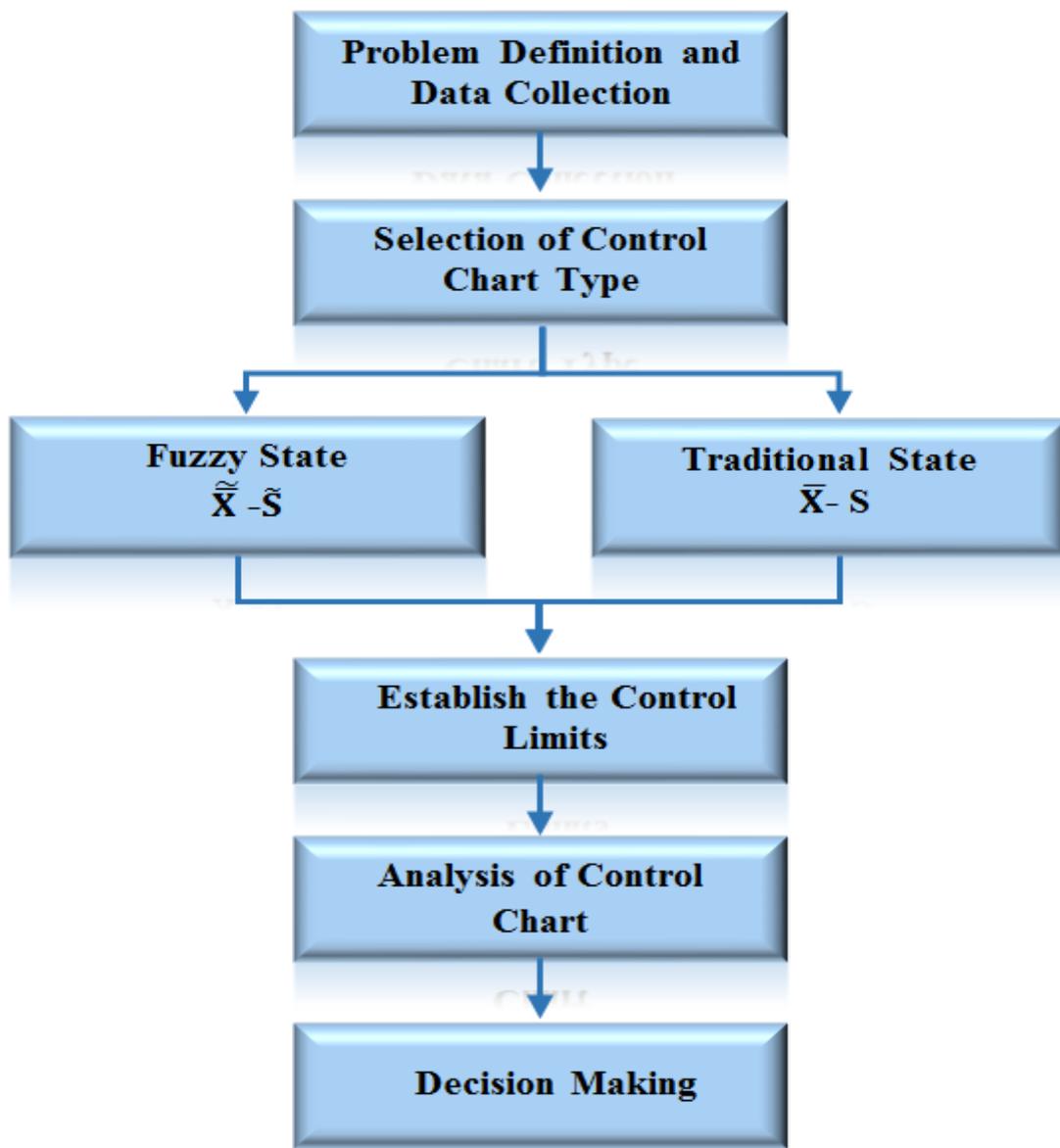


Fig-2.The Framework of Proposed Methodology.

5. MATHEMATICAL MODEL

In this section basic concepts of traditional control charts, fuzzy numbers and proposed fuzzy $\bar{X} - \bar{S}$ control charts.

A - Traditional \bar{X} and S control chart

For traditional \bar{X} and S control chart the control limits are calculated as follows:

\bar{X} Chart control limits

$$\left. \begin{aligned} \text{Center Line (CL)} &= \bar{\bar{X}} \\ \text{Lower Control Limit (LCL)} &= \bar{\bar{X}} - A_3 \bar{S} \\ \text{Upper Control Limit (UCL)} &= \bar{\bar{X}} + A_3 \bar{S} \end{aligned} \right\} \dots (6)$$

S chart control limits

$$\left. \begin{aligned} \text{Center Line (CL)} &= \bar{S} \\ \text{Lower Control Limit (LCL)} &= B_3 \bar{S} \\ \text{Upper Control Limit (UCL)} &= B_4 \bar{S} \end{aligned} \right\} \dots (7)$$

Where:

X_1, X_2, \dots, X_n is a sample of size n, then the average of this sample is:

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} \dots (8)$$

If $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_m$ is the average of each sample, the process average or the grand average is:

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_m}{m} \dots (9)$$

If X_1, X_2, \dots, X_n is a sample of size n, then the range of the sample is the difference between the largest and smallest observations; that is:

$$R = X_{\max} - X_{\min} \dots (10)$$

The sample standard deviation is defined as:

$$S = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}} \dots (11)$$

B. - Fuzzy \bar{X} - S control chart

The study lists all the formulas of establishing the control limits of the control charts for Fuzzy \bar{X} - S control charts, α - Cut Fuzzy \bar{X} - \bar{S} control charts and α - Level Fuzzy Midrange for α - Cut Fuzzy \bar{X} - \bar{R} Control Chart Control Limits as illustrated below:

Fuzzy \bar{X} Chart Control Limits

$$\begin{aligned} \bar{CL} &= (\bar{X}_a, \bar{X}_b, \bar{X}_c) \\ \bar{LCL} &= (\bar{X}_a - A_3 \bar{S}_c, \bar{X}_b - A_3 \bar{S}_b, \bar{X}_c - A_3 \bar{S}_a) \\ \bar{UCL} &= (\bar{X}_a + A_3 \bar{S}_a, \bar{X}_b + A_3 \bar{S}_b, \bar{X}_c + A_3 \bar{S}_c) \end{aligned} \quad \left. \vphantom{\begin{aligned} \bar{CL} \\ \bar{LCL} \\ \bar{UCL} \end{aligned}} \right\} \dots (12)$$

Fuzzy \bar{S} Chart Control Limits

$$\begin{aligned} \bar{CL} &= (\bar{S}_a, \bar{S}_b, \bar{S}_c) \\ \bar{LCL} &= (B_3 \bar{S}_a, B_3 \bar{S}_b, B_3 \bar{S}_c) \\ \bar{UCL} &= (B_4 \bar{S}_a, B_4 \bar{S}_b, B_4 \bar{S}_c) \end{aligned} \quad \left. \vphantom{\begin{aligned} \bar{CL} \\ \bar{LCL} \\ \bar{UCL} \end{aligned}} \right\} \dots (13)$$

Let quality characteristic of a sample with a size of n be represented as fuzzy triangular numbers by \bar{X}_i (X_{ija} , X_{ijb} , X_{ijc}), $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$. Using the fuzzy arithmetic the mean of the each subgroup and grand average of the samples can be calculated by Equations below:

$$\bar{X}_i = \left(\frac{\sum_{j=1}^n X_{ija}}{n}, \frac{\sum_{j=1}^n X_{ijb}}{n}, \frac{\sum_{j=1}^n X_{ijc}}{n} \right) = \bar{X}_{ia}, \bar{X}_{ib}, \bar{X}_{ic} \quad \dots (14)$$

$i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$

$$\bar{X} = \left(\frac{\sum_{i=1}^m X_{ia}}{m}, \frac{\sum_{i=1}^m X_{ib}}{m}, \frac{\sum_{i=1}^m X_{ic}}{m} \right) = (\bar{X}_a, \bar{X}_b, \bar{X}_c) \quad \dots (15)$$

Using the fuzzy arithmetic, the fuzzy standard deviation of the each subgroup and fuzzy average standard deviation of the samples can be derived by the Equations below:

$$(S_{ia}, S_{ib}, S_{ic}) = \sqrt{\frac{\sum_{j=1}^n [(X_{ija}, X_{ijb}, X_{ijc}) - (\bar{X}_{ia}, \bar{X}_{ib}, \bar{X}_{ic})]^2}{n-1}} \quad \dots (16)$$

$$(\bar{S}_a, \bar{S}_b, \bar{S}_c) = \left(\frac{1}{m} \sum_{i=1}^m S_{ia}, \frac{1}{m} \sum_{i=1}^m S_{ib}, \frac{1}{m} \sum_{i=1}^m S_{ic} \right) \quad \dots (17)$$

Control Limits for α – cut \tilde{X} control charts

The control limits for α - cut fuzzy \tilde{X} control charts based on standard deviation are obtained as follows:

$$\left. \begin{aligned} \overline{UCL}_{\tilde{X}}^{\alpha} &= (\overline{X}_a^{\alpha}, \overline{X}_b, \overline{X}_c^{\alpha}) + A_3(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha}) \\ \overline{CL}_{\tilde{X}}^{\alpha} &= (\overline{X}_a^{\alpha}, \overline{X}_b, \overline{X}_c^{\alpha}) \\ \overline{LCL}_{\tilde{X}}^{\alpha} &= (\overline{X}_a^{\alpha}, \overline{X}_b, \overline{X}_c^{\alpha}) - A_3(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha}) \end{aligned} \right\} \dots(18)$$

Where:

$$\overline{S}_a^{\alpha} = \overline{S}_a + \alpha (\overline{S}_b - \overline{S}_a) \dots (19)$$

$$\overline{S}_c^{\alpha} = \overline{S}_c - \alpha (\overline{S}_c - \overline{S}_b) \dots (20)$$

α - Level Fuzzy Midrange for α - Cut Fuzzy $\tilde{X} - \tilde{S}$ Control Chart:

The control limits and center line for α - Cut Fuzzy \tilde{X} control chart based on standard deviation using α - Level fuzzy midrange are:

$$\left. \begin{aligned} \overline{UCL}_{mr-\tilde{X}}^{\alpha} &= \overline{CL}_{mr-\tilde{X}}^{\alpha} + A_3\left(\frac{\overline{S}_a^{\alpha} + \overline{S}_c^{\alpha}}{2}\right) \\ \overline{CL}_{mr-\tilde{X}}^{\alpha} &= f_{mr-\tilde{X}}^{\alpha}(\overline{CL}) = \frac{\overline{X}_a^{\alpha} + \overline{X}_c^{\alpha}}{2} \\ \overline{LCL}_{mr-\tilde{X}}^{\alpha} &= \overline{CL}_{mr-\tilde{X}}^{\alpha} - A_3\left(\frac{\overline{S}_a^{\alpha} + \overline{S}_c^{\alpha}}{2}\right) \end{aligned} \right\} \dots (21)$$

The definition of α - level fuzzy midrange of sample j for fuzzy \tilde{X} control chart is:

$$S_{mr-\tilde{X}j}^{\alpha} = \frac{(\overline{X}_{aj} + \overline{X}_{cj}) + \alpha [(\overline{X}_{bj} - \overline{X}_{aj}) - (\overline{X}_{cj} - \overline{X}_{bj})]}{2} \dots (22)$$

Control Limits for α – cut fuzzy \tilde{S} control charts

The control limits and center line for α - Cut Fuzzy \tilde{S} control chart can be obtained as follows:

$$\left. \begin{aligned} \overline{UCL}_s^{\alpha} &= B_4(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha}) \\ \overline{CL}_s^{\alpha} &= (\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha}) \\ \overline{LCL}_s^{\alpha} &= B_3(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha}) \end{aligned} \right\} \dots(23)$$

α - Level Fuzzy Midrange For α - Cut Fuzzy \bar{S} Control Chart

The control limits for α - level fuzzy midrange for α Cut Fuzzy \bar{S} control chart based on triangular fuzzy numbers are obtained as follows:

$$UCL_{mr-s}^{\alpha} = B_4 f_{mr-s}^{\alpha}(\bar{CL})$$

$$CL_{mr-R}^{\alpha} = f_{mr-s}^{\alpha}(\bar{CL}) = \frac{\bar{S}_a^{\alpha} + \bar{S}_c^{\alpha}}{2} \dots (24)$$

$$LCL_{mr-s}^{\alpha} = B_3 f_{mr-s}^{\alpha}(\bar{CL})$$

The definition of α - level fuzzy midrange of sample j for fuzzy \bar{S} control chart can be calculated as follows:

$$S_{mr-s,j}^{\alpha} = \frac{(S_{aj} + S_{cj}) + \alpha [(S_{bj} - S_{aj}) - (S_{cj} - S_{bj})]}{2} \dots (25)$$

C- Determining the Process Condition

The process condition of the sample which identify the process whether in control or not as illustrated below:

α - Level Fuzzy Midrange for α - Cut Fuzzy \bar{X} Control Chart based on Standard Deviation

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \left\{ \begin{array}{l} \text{in - control for } LCL_{mr-\bar{X}}^{\alpha} \leq S_{mr-\bar{X},j}^{\alpha} \leq UCL_{mr-\bar{X}}^{\alpha} \\ \text{out - of control; otherwise} \end{array} \right\} \dots (26)$$

α - Level Fuzzy Midrange for α - Cut Fuzzy \bar{S} Control Chart

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \left\{ \begin{array}{l} \text{in - control for } LCL_{mr-s}^{\alpha} \leq S_{mr-s,j}^{\alpha} \leq UCL_{mr-s}^{\alpha} \\ \text{out - of control; otherwise} \end{array} \right\} \dots (27)$$

6. CASE STUDY I

The study selects the petroleum products to monitor the quality of it because of the importance of the oil industry. The study applied the traditional $\bar{X} - S$ control chart and Fuzzy $\bar{X} - \bar{S}$ control charts to monitor the Kerosene product the sulphur content specification as shown below:

A. The Constructing of Traditional $\bar{X} - S$ Control Chart

The study implements traditional control charts for Kerosene product the flash point specification by using MINITAB software 16.as illustrated in Fig(3).

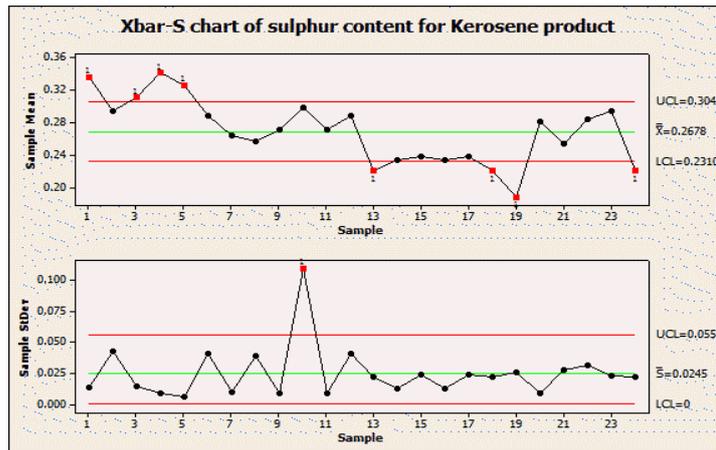


Fig-3. \bar{X} - S Control Chart of Sulphur Content for Kerosene Product

The results for \bar{X} - S control chart showed that the samples (1; 3; 4; 5; 13; 18;19;24) for \bar{X} chart and the sample (10) for S chart are failed using MINITAB software16 as shown in Fig (4).

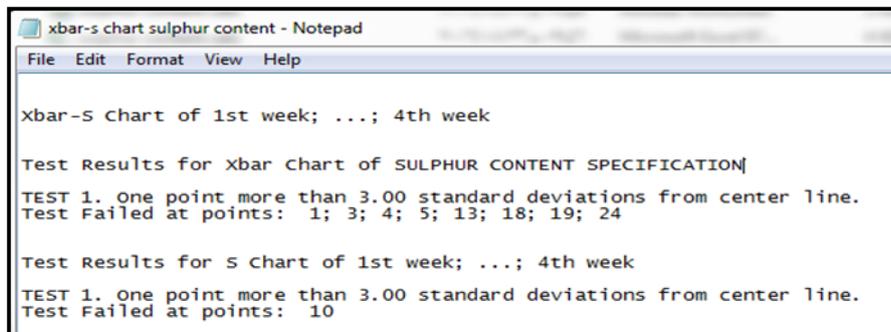


Fig-4. MINITAB Notepad Sheet for the Results of \bar{X} - S Control Chart.

B. The Construction of Fuzzy \bar{X} - \bar{S} Control Charts:

To construct fuzzy \bar{X} - \bar{S} control chart the data must be transformed to fuzzy triangular number (a, b, c) each observation must be transformed to fuzzy triangular number as shown in table (1) .Hence by applying the mathematical models and formulas mentioned above the Arithmetic Mean and Standard deviation for Fuzzy Data where calculated and by using Excel software13. The condition of process control for each sample can be identified by the condition rule mentioned above to detect the process control of the sample as shown in table (2).

Table-1. The Fuzzy Triangular Data

No	xa1	xb1	xc1	xa2	xb2	xc2	xa3	xb3	xc3	xa4	xb4	xc4
1	0.29	0.32	0.33	0.31	0.34	0.35	0.30	0.33	0.34	0.34	0.35	0.36
2	0.20	0.23	0.29	0.26	0.3	0.36	0.23	0.32	0.35	0.23	0.32	0.38
3	0.28	0.32	0.35	0.30	0.31	0.33	0.26	0.29	0.31	0.29	0.32	0.34
4	0.33	0.33	0.35	0.33	0.35	0.37	0.32	0.34	0.35	0.32	0.34	0.36
5	0.31	0.32	0.34	0.32	0.33	0.34	0.33	0.33	0.34	0.31	0.32	0.33
6	0.23	0.32	0.34	0.15	0.23	0.25	0.27	0.29	0.31	0.25	0.31	0.37
7	0.23	0.25	0.26	0.25	0.27	0.29	0.25	0.26	0.27	0.25	0.27	0.28
8	0.23	0.26	0.28	0.17	0.2	0.22	0.25	0.29	0.31	0.21	0.27	0.29
9	0.27	0.28	0.29	0.25	0.27	0.28	0.24	0.26	0.27	0.25	0.27	0.28
10	0.13	0.18	0.47	-0.04	0.23	0.35	0.23	0.4	0.69	0.15	0.38	0.55
11	0.28	0.28	0.29	0.25	0.27	0.28	0.24	0.26	0.27	0.25	0.27	0.28
12	0.30	0.32	0.43	0.15	0.23	0.27	0.18	0.29	0.40	0.22	0.31	0.42
13	0.21	0.22	0.25	0.16	0.2	0.22	0.16	0.21	0.26	0.20	0.25	0.30
14	0.20	0.23	0.23	0.21	0.22	0.25	0.22	0.25	0.28	0.20	0.23	0.25
15	0.23	0.27	0.31	0.18	0.22	0.23	0.21	0.24	0.25	0.18	0.22	0.23
16	0.21	0.23	0.25	0.20	0.22	0.24	0.22	0.25	0.26	0.22	0.23	0.24
17	0.26	0.27	0.28	0.19	0.22	0.25	0.23	0.24	0.26	0.18	0.22	0.24
18	0.17	0.22	0.27	0.16	0.2	0.25	0.16	0.21	0.22	0.24	0.25	0.30
19	0.13	0.18	0.20	0.20	0.22	0.27	0.11	0.16	0.21	0.14	0.19	0.24
20	0.27	0.27	0.29	0.26	0.28	0.30	0.26	0.28	0.28	0.27	0.29	0.31
21	0.18	0.22	0.23	0.23	0.24	0.28	0.23	0.27	0.29	0.27	0.28	0.32
22	0.17	0.24	0.31	0.22	0.28	0.35	0.24	0.31	0.32	0.29	0.3	0.37
23	0.27	0.32	0.38	0.26	0.28	0.33	0.22	0.27	0.29	0.25	0.3	0.36
24	0.17	0.22	0.27	0.16	0.2	0.25	0.17	0.21	0.26	0.22	0.25	0.30

Table-2. The Process Control Condition for α - Level Fuzzy Midrange For α - Cut (Fuzzy \bar{X} Control Chart Based on Standard deviation and Fuzzy S Control Chart)

No	$S_{ms-\bar{X}_j}^\alpha$	Process Control	Process Control	$S_{ms-s_j}^\alpha$
1	0.33	out of control	0.01	in control
2	0.29	in control	0.04	in control
3	0.31	in control	0.02	in control
4	0.34	out of control	0.01	in control
5	0.33	out of control	0.01	in control
6	0.28	in control	0.05	in control
7	0.26	in control	0.01	in control
8	0.25	in control	0.04	in control
9	0.27	in control	0.01	in control
10	0.31	in control	0.02	in control
11	0.27	in control	0.01	in control
12	0.29	in control	0.05	in control
13	0.22	in control	0.03	in control
14	0.23	in control	0.01	in control
15	0.23	in control	0.03	in control
16	0.23	in control	0.01	in control
17	0.24	in control	0.02	in control
18	0.22	in control	0.03	in control
19	0.19	out of control	0.03	in control
20	0.28	in control	0.01	in control
21	0.25	in control	0.03	in control
22	0.28	in control	0.03	in control
23	0.29	in control	0.03	in control
24	0.22	in control	0.02	in control

7. CASE STUDY II

The study applied the traditional $\bar{X}-S$ control chart and Fuzzy $\tilde{X}-\tilde{S}$ control charts to monitor the Benzene product the octane number specification as shown below:

A. The Constructing of Traditional $\bar{X}-S$ Control Chart.

The study implements traditional control charts for Benzene product the octane number specification by using MINITAB software 16.as illustrated in Fig(5).

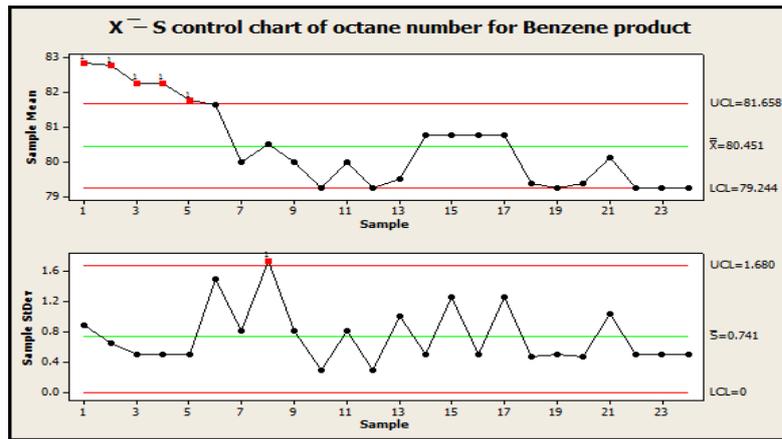


Fig-5. $\bar{X}-S$ Control Chart of Octane Number for Benzene Product

The results for $\bar{X}-S$ control chart showed that the samples (1; 2; 3; 4; 5)for \bar{X} chart and the samples (8) for S chart are failed using MINITAB software as shown in Fig (6)

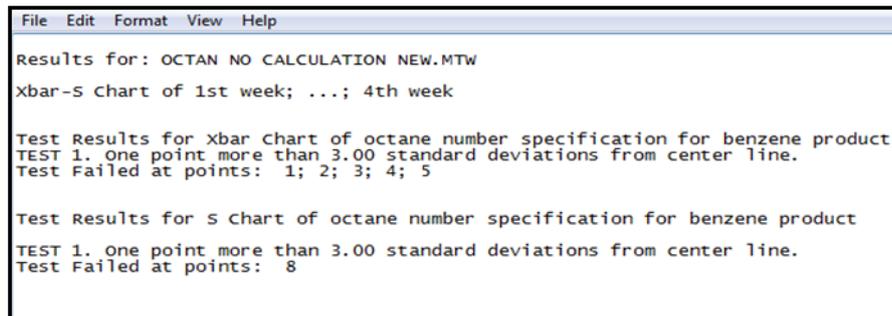


Fig-6. MINITAB Notepad Sheet for the Results of $\bar{X}-S$ Control Chart.

B. The Construction of Fuzzy $\tilde{X}-\tilde{S}$ Control Charts:

To construct fuzzy $\tilde{X}-\tilde{S}$ control chart the data must be transformed to fuzzy triangular number (a, b, c) each observation must be transformed to fuzzy triangular number as shown in table (3) .Hence by applying the mathematical models and formulas mentioned above the Arithmetic Mean and Standard deviation for Fuzzy Data where calculated and by using Excel software¹³. The condition of process control for each sample can be identified by the condition rule mentioned above to detect the process control of the sample as shown in table (4).

Table-3. The Fuzzy Triangular Data

No	xa1	xb1	xc1	xa2	xb2	xc2	xa3	xb3	xc3	xa4	xb4	xc4
1	81.09	83	83.74	80.54	82.3	82.72	80.09	82	82.74	83.58	84	84.42
2	83.04	83.5	84.48	81.72	83	83.98	81.11	82.5	82.96	80.61	82	83.39
3	80.69	82	83.31	81.76	82	82.76	81.93	83	83.76	80.93	82	83.07
4	82.76	83	84.07	81.01	82	83.31	80.93	82	82.76	80.93	82	82.99
5	80.93	82	83.31	80.75	82	82.63	81.84	82	83.07	79.93	81	81.76
6	78.80	82	82.71	79.05	82	82.71	82.29	83	83.71	77.23	79.5	81.77
7	78.25	80	81.24	78.39	80	81.75	79.76	81	82.24	77.25	79	79.68
8	77.76	79	79.82	78.76	80	80.82	81.34	83	83.82	77.37	80	80.82
9	79.76	81	82.24	78.39	80	80.68	77.25	79	79.68	78.25	80	81.24
10	79.36	79.5	80.26	78.28	79	79.32	78.56	79	79.76	78.88	79.5	80.26
11	80.61	81	82.24	78.39	80	80.90	77.25	79	79.90	78.25	80	80.68
12	79.36	79.5	80.26	78.43	79	79.32	78.24	79	79.76	78.88	79.5	80.26
13	80.53	81	81.83	77.02	79	79.96	76.85	79	80.52	76.85	79	80.52
14	79.69	81	81.16	80.76	81	82.07	78.69	80	81.07	79.69	81	81.76
15	80.09	82	83.91	79.09	81	81.59	79.40	81	81.59	77.09	79	79.59
16	80.24	81	81.76	80.01	81	81.76	78.93	80	80.24	80.76	81	81.24
17	81.41	82	82.59	79.40	81	82.38	80.41	81	81.90	77.09	79	80.20
18	78.47	79.5	80.53	78.05	79	80.03	77.97	79	79.16	79.77	80	81.03
19	77.93	79	79.42	79.58	80	81.07	77.93	79	80.07	77.93	79	80.07
20	79.34	79.5	80.53	78.05	79	80.03	77.97	79	79.23	78.97	80	81.26
21	77.44	79	79.49	79.01	79.5	81.06	79.44	81	81.74	80.51	81	82.56
22	77.93	79	80.07	78.01	79	80.07	77.93	79	79.24	79.76	80	81.07
23	77.93	79	80.31	78.45	79	80.07	77.93	79	79.42	78.93	80	81.31
24	77.93	79	80.07	78.01	79	79.42	78.01	79	80.07	79.45	80	81.07

Table-4. The Process Control Condition for α - Level Fuzzy Midrange For α - Cut (Fuzzy \bar{X} Control Chart Based on Standard deviation and Fuzzy S Control Chart)

No	$S_{ms-\bar{X}_j}^\alpha$	Process Control	Process Control	$S_{ms-s_j}^\alpha$
1	82.60	out of control	1.04	in control
2	82.71	out of control	0.75	in control
3	82.26	out of control	0.51	in control
4	82.30	out of control	0.62	in control
5	81.76	in control	0.62	in control
6	81.33	in control	1.48	in control
7	79.91	in control	0.94	in control
8	80.28	in control	1.75	in control
9	79.84	in control	0.93	in control
10	79.29	in control	0.37	in control
11	79.89	in control	1.01	in control
12	79.28	in control	0.38	in control
13	79.38	in control	1.15	in control
14	80.68	in control	0.58	in control
15	80.52	in control	1.39	in control
16	80.68	in control	0.62	in control
17	80.71	in control	1.36	in control
18	79.38	in control	0.65	in control
19	79.25	in control	0.63	in control
20	79.40	in control	0.62	in control
21	80.14	in control	1.16	in control
22	79.26	in control	0.66	in control
23	79.27	in control	0.57	in control
24	79.25	in control	0.60	in control

8. RESULTS AND DISCUSSION

The results of $\bar{X} - S$ control chart of the Kerosene product the Sulphur content specification in traditional state showed that there are (8) samples in \bar{X} chart are out of control and one sample in (S) chart, while by using fuzzy state the results show that only (4) samples in $\tilde{\bar{X}}$ chart are out of control and all samples are in control in (\tilde{S}) chart and the results of of $\bar{X} - S$ control chart of the Benzene product the octane number specification in traditional state showed that there are(5) samples in \bar{X} chart are out of control and one sample in (S) chart, while by using fuzzy state the results show that only (4) samples in $\tilde{\bar{X}}$ chart are out of control and all samples are in control in (\tilde{S}) as shown in table (5).

Table-5. The Results of Applying Traditional and Fuzzy Control Charts

Product Specification	State	Control Chart Type	Results
Kerosene	Traditional	\bar{X}	8 samples are out of control
		S	One samples is out of control
Flash point	Fuzzy	$\tilde{\bar{X}}$	4 samples are out of control
		\tilde{S}	All samples are in control
Benzene	Traditional	\bar{X}	5 samples are out of control
		S	One samples is out of control
Octane number	Fuzzy	$\tilde{\bar{X}}$	4 samples are out of control
		\tilde{S}	All samples are in control

9. CONCLUSION

1. The decreasing of the gap in the construction and interpretation of fuzzy control charts is achieved. α -cut of fuzzy sets is successfully applied so as to reflect the tightness of an inspection that can be subjectively set by the quality controller accordingly to the importance of an inspection. The proposed approach is efficient in detecting process shifts.
2. The fuzzy approach is convenient to detect the signals in the variable control charts, because its gives some flexibility to the control limits. Since the plotted values are close to the upper, lower and center line may cause false alarms with traditional control charts, fuzzy control limits can provide more flexibility for controlling a process.
3. In regard with the occurrence of a mean shift in a real process, hence the proposed fuzzy control chart can help researchers in the production management area to more accurately describe the implementation of fuzzy theory in a control chart. In practice, the quality control department of a factory can construct a specific fuzzy control chart according to its specific environment by following the steps illustrated in this

study and further more improve the performance of the control charts with the usage of computers, thus reducing the amount of unqualified products.

4. Constructing fuzzy control charts have more flexible than traditional control chart approach and give more meaning results than traditional quality chart technique, hence the proposed fuzzy control chart have better performance as compared with the traditional control chart. Finally for future work , the study can be extended by using multivariate fuzzy control charts to monitor more quality characteristic and it can be extended by monitoring the process capability.

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REFERENCES

- [1] D. C. Montgomery, *Introduction to statistical quality control*, 6th ed. USA: John Wiley & Sons, 2009.
- [2] G. Moghadam, G. Ali, and V. Amirzadeh, "New fuzzy EWMA control charts for monitoring phase II fuzzy profiles," *Decision Science Letters*, vol. 5, pp. 119-128, 2016. [View at Google Scholar](#)
- [3] N. Alwan, "Developing a database system for the laboratory tests," *Engineering & Technology Journal*, vol. 31, pp. 52-67, 2013. [View at Google Scholar](#)
- [4] I. Ertugrul and E. Aytac, "Construction of quality control charts by using probability and fuzzy approaches and an application in a textile company," *Journal of Intelligent Manufacturing*, vol. 20, pp. 139-149, 2009. [View at Google Scholar](#) | [View at Publisher](#)
- [5] M. H. Omar, "Statistical process control charts for measuring and monitoring temporal consistency of ratings," *Journal of Educational Measurement*, vol. 47, pp. 18-35, 2010. [View at Google Scholar](#) | [View at Publisher](#)
- [6] M. Gulbay, "Fuzzy process control and development of some models for fuzzy control charts," Ph.D Thesis, Dept. Industrial Eng, İstanbul Technical Univ, Turkey, 2006.
- [7] S. Senturk, "Fuzzy regression control chart based on α -cut approximation," *International Journal of Computational Intelligence Systems*, vol. 3, pp. 123-140, 2010. [View at Google Scholar](#) | [View at Publisher](#)
- [8] A. Moraditadi, "Development of fuzzy individual \bar{x} and moving range control chart," *International Journal of Productivity and Quality Management*, vol. 17, pp. 82-103, 2016. [View at Google Scholar](#)
- [9] L. Chen and H. C. Yu, "Research article design of a fuzzy zone control chart for improving the process variation monitoring capability," *Journal of Applied Sciences*, vol. 16, pp. 201-208, 2016. [View at Google Scholar](#)
- [10] A. M. Tadi and S. A. Darestani, "Development of fuzzy IX-MR control chart using fuzzy mode and fuzzy rules approaches," *International Peer-Reviewed Journal*, vol. 3, pp. 1-11, 2014.
- [11] F. Sogandi, S. M. Mousavi, and R. Ghanaatiyan, "An extension of fuzzy P-control chart based on α -level fuzzy midrange," *Advanced Computational Techniques in Electromagnetics*, vol. 2014, pp. 1-8, 2014. [View at Google Scholar](#) | [View at Publisher](#)
- [12] S. Sorooshian, "Fuzzy approach to statistical control charts," *Journal of Applied Mathematics*, vol. 2013, pp. 1-6, 2013. [View at Google Scholar](#) | [View at Publisher](#)

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