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IMPROVED ANALYTICAL APPROACH OF VIBRATION BEHAVIOR DESTINED TO BALL BEARINGS IN THE CASE OF ANGULAR CONTACT

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ABSTRACT

In recent years many developments have been made in the area of the vibration behavior of the supported by bearings or journal bearings rotating shafts. The results, if they are generally acceptable for common-use in terms of operating conditions, are far from meeting the aircraft industry, where the machining at high speeds is an undeniable need. This research therefore is aimed to provide an analytical simulation based on theoretical developments of the vibration response of a ball bearing. The generation of vibration by a point in a rolling element bearing is modeled as a function of the rotation of the bearing, the distribution of the load in the bearing, the transfer function between the bearing, transducer and elasticity of the bearing structure. The essence of this work is to present an analytical model of a specific type single of row effect balls (SKF 6004). This study is a first step dedicated to define the geometric characteristics, and in second time to determine the equations governing the distribution of responsibilities within the ball bearing.

Keywords: Ball bearing, Damage, Vibration, Rolling element, Row balls, Effect balls.

1. INTRODUCTION

Even if the geometry of a ball bearing is perfect, it will still produce vibrations. The vibrations are caused by the rotation of a finite number of loaded rolling contacts between the balls and the guiding rings. Because these contacts are elastic, the bearing stiffness becomes explicitly dependent on time. In general, a time varying stiffness causes vibrations, even in the absence of external loads. Since the stiffness can be regarded as a system parameter, the variable stiffness leads to a so-called parametric excitation. It is one of the major sources of vibration in ball bearings. The first systematic research on this subject was conducted by Perret [1] and Meldau [2].

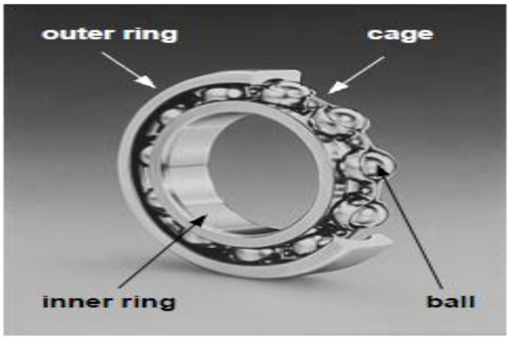
Dowson and Higginson [3] have proposed the first analytical solutions to EHD in case of cylinder/plane contact. Then, in the middle of 1970s, digital solutions contact ellipsoid/ maps have been proposed by Hamrock and Dowson [4]. From the viewpoint of contact fatigue in the presence of indentation, [5] exhibit a work based on the introduction of a ceramic rolling element (Si_3N_4), from the rolling ones in steel. This study demonstrates a "smoothing effect" created by the passage of ceramic rolling element which generates strong plastic deformation of bead. This "smoothing effect" makes it possible to reduce the height of beads and, at the same time, local overpressure. This suggests that the addition of ceramic allows increasing number of cycles to chipping adjacent indents. In addition, the work presented by Jacq [6] shows a smaller effect of the load on damage by chipping in vicinity with respect of sliding.

2. BEARING MODELING

2.1. General Model

The bearing is a mechanical body to allow rotation between two shafts, or between a shaft and housing in good condition, guide with little energy loss. The rotation is permitted by rolling bodies, separated by a cage, which roll on the inner ring and outer. The load applied to the bearing can be axial or radial, and of many geometries rings and rolling bodies exist depending on the nature of forces to be transmitted (Figure 1).

Figure-1. Overview of ball bearings components

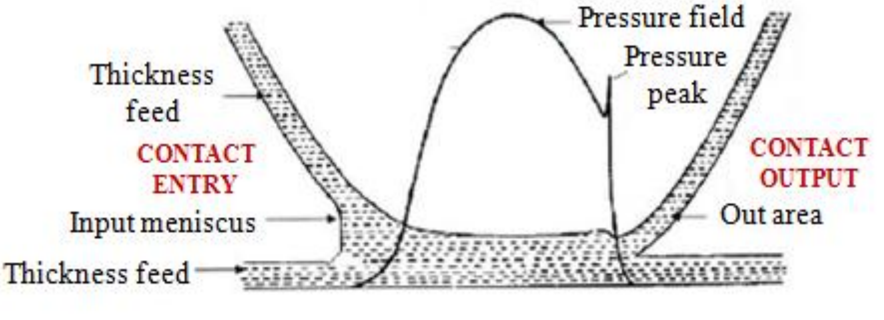


2.2. Bearing Lubrication

It is therefore advantageous to reduce the coefficient of friction by inserting a fluid in contact through lubrication. Indeed, as the Egyptians used water or animal fat to reduce sliding friction, current bearings are lubricated with special industrial oils.

Figure 2 shows the characteristics of contact EHD regime.

Figure-2. The EHD Contacts of field pressure and film thickness fluid



We thus distinguish three zones to describe an EHD contact. An inlet zone, high pressure zone and outlet zone of contact.

- Zone entrance in touch: it is an area of hydrodynamic lubrication resulting in increased contact pressure due to the presence of a convergent formed by elements.
- Pressure Zone: this is the elastohydrodynamic zone characterized by the presence of a high pressure that temporarily causes the transition to the glassy state of the lubricant and the elastic deformation of the contact surfaces.
- Zone output: the lubricant is ejected from the contact formed by the divergent parts. This implies a drop in oil thickness at contact exit due to flow conservation film.

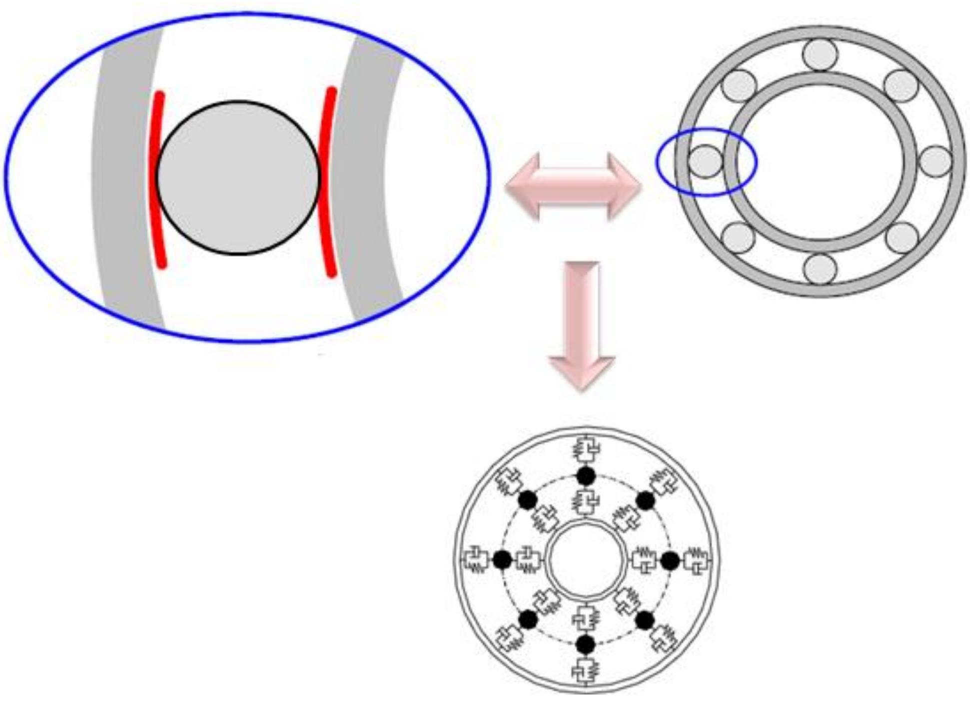
3. MATHEMATICAL DEVELOPMENT OF A SINGLE BEARING

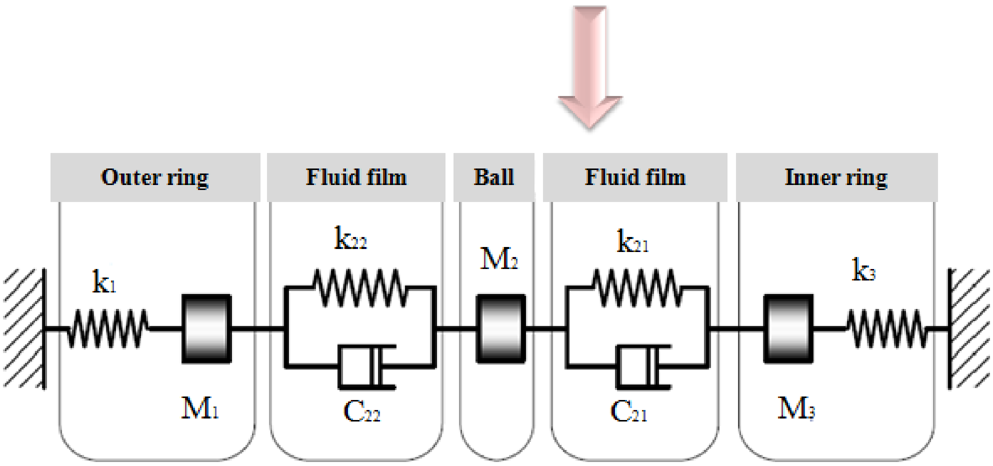
3.1. Initial Model

In this part we consider a rotor, bearing rings and support. The model becomes complex with a large number of degrees of freedom (ddl). In our model, we applied three degree of freedom system as described in Figure 3, in order to focus the study on the vibration response of the balls [7] and Tanaka and Mura [8].

The dynamic behavior of rigid bearing support and the contribution of the support is also considered. Thus, this simplified model that will be adjusted with the calculation of vibration response.

Figure-3. Model of bearing system





With:

M1: Mass of outer ring [kg]

M3: Mass of inner ring [Kg]

M2: Mass of ball [kg]

K1: stiffness of outer ring [N/m],

K3: stiffness of inner ring [N / m]

K21: stiff ball [N / m]

C22: damping ratio Film external fluid [N.s / m]

C21: damping coefficient of internal fluid film [N.s / m]

The different parameters governing the behavior of this model were developed using the following analytical elements:

3.2. The Rings

We considered a mass-spring system with neglected structural damping, the rigidities of the two rings is determined by the following equations:

Rigidity of the inner ring $k_3 = M_3\omega_3^2$ (1)

Rigidity of the outer ring $k_1 = M_1\omega_1^2$ (2)

Each of inner and outer rings will be represented by a mass-spring combination which corresponds to a single degree of freedom system. The internal damping has been neglected. The natural frequency of the flexural vibration mode number "n" is given by the following expression:

$$\omega_n = \frac{n[n^2-1]}{\sqrt{1+n^2}}\sqrt{\frac{EI}{\mu R^4}} \quad (3)$$

where:

ω_n angular frequency of mode [rad/s]

n order mode

E modulus longitudinal elasticity [N/m²]

I moment of inertia cross section of the ring [m⁴]

μ linear density [kg/m]

R radius of ring [m].

Modes 0 and 1 are considered as rigid modes. Therefore the second mode is considered as the first bending vibration mode. The angular frequency of the system to a single degree of freedom will be taken as:

$$\omega_n = 2.68\sqrt{\frac{EI}{\mu R^4}} \tag{4}$$

3.3. The Balls

Rigidity ball has the order of 8.3 10^9 N / m. Compared to other rigidities, it is considered infinitely rigid. Therefore modeling the ball was deemed to be a mass element.

3.4. The Fluid Film

The development of elastohydrodynamic theory of lubrication (EHD) showed that films with a few micrometers thick are occurs in rolling contact. Therefore, the characterization of lubrication system depends on the value of lubricant film thickness. The theory developed by Hamrock and Dowson [4], expressed in terms of stiffness and damping coefficients dimension film of fluid is given by the following expressions:

- Fluid film stiffness:

$$k = \frac{4}{W_r \lambda_k^2}\left[\frac{\varepsilon_0}{(1-\varepsilon_0^2)^2}\sin^2\phi_0 + \frac{3\pi\varepsilon_0^2}{4(1-\varepsilon_0^2)^{5/2}}\sin\phi_0\cos\phi_0 + \frac{2\varepsilon_0(1+\varepsilon_0^2)}{(1-\varepsilon_0^2)^3}\cos^2\phi_0\right] \tag{5}$$

- Amortization of fluid film :

$$C = \frac{4}{W_r \lambda_k^2}\left[\frac{\pi}{2(1-\varepsilon_0^2)^{3/2}}\sin^2\phi_0 + \frac{4\varepsilon_0}{(1-\varepsilon_0^2)^2}\sin\phi_0\cos\phi_0 + \frac{\pi(1+2\varepsilon_0^2)}{2(1-\varepsilon_0^2)^{5/2}}\cos^2\phi_0\right] \tag{6}$$

With :

$$\frac{4}{W_r \lambda_k^2} = \frac{(1-\varepsilon_0^2)^2}{\varepsilon_0[16\varepsilon_0^2 + \pi^2(1-\varepsilon_0^2)]^{1/2}}\cos^2\phi_0$$

$$\tan\phi_0 = \frac{\pi(1-\varepsilon_0^2)^{1/2}}{4\varepsilon_0}$$

$$\varepsilon_0 = 1 - \frac{h}{c}$$

And:

λ_k defines the ratio of width length of contact area
φ_0 is the maximal angle, described by the distribution charge
W_r is the resultant force applied to fluid film

ε_0 is an eccentricity coefficient

c is the diametrical bearing clearance
h is the thickness of the fluid film

4. RESULTS AND DISCUSSIONS

The general equation to solve a system of three degrees of freedom subjected to three forces, depending on the mass matrices, damping and stiffness being have a general form:

$$[A].\{\ddot{y}\}+[B].\{\dot{y}\}+[C].\{y\}=\{F\}$$

A, B and C represent respectively the masses of balls, damping and stiffness system.

$$A=\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \qquad B=\begin{bmatrix} C_{22} & -C_{22} & 0 \\ -C_{22} & C_{22}+C_{21} & -C_{21} \\ 0 & -C_{21} & C_{21} \end{bmatrix}$$

$$C=\begin{bmatrix} k_1+k_{22} & -k_{22} & 0 \\ -k_{12} & k_{22}+k_{21} & -k_{21} \\ 0 & -k_{21} & k_3+k_{21} \end{bmatrix}$$

The motion vector [5] and the force vector [5]. The vector F represents the impact forces generated by the fault of passage balls.

$$\{y\}=\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} \qquad \{F\}=\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

The equation of motion, governing this system model is the following:

$$\begin{cases} M_1\ddot{y}_1+C_{22}\dot{y}_1-C_{22}\dot{y}_2+(k_1+k_{22})y_1-k_{22}y_2=F_1 \\ M_2\ddot{y}_2-C_{22}\dot{y}_2+(C_{22}+C_{21})\dot{y}_2-C_{21}\dot{y}_3-k_{22}y_1+(k_{22}+k_{21})y_2-k_{21}y_3=F_2 \\ M_3\ddot{y}_3+C_{21}\dot{y}_3-C_{21}\dot{y}_2+(k_3+k_{21})y_3-k_{21}y_2=F_3 \end{cases} \quad (7)$$

This can be written in matrix form:

$$\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{Bmatrix} + \begin{bmatrix} C_{22} & -C_{22} & 0 \\ -C_{22} & C_{22}+C_{21} & -C_{21} \\ 0 & -C_{21} & C_{21} \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_{22} & -k_{22} & 0 \\ -k_{22} & k_{22}+k_{21} & -k_{21} \\ 0 & -C_{21} & k_3+k_{21} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

The trivial solution of the equation can be written as:

$$AY'' + BY' + CY = F$$

It becomes, after multiplication by A^{-1}:

$$Y'' + A^{-1}BY' + A^{-1}CY = A^{-1}F$$

We work in the case without default so F1 = F2 = F3 = 0,

Therefore $Y'' + A^{-1}BY' + A^{-1}CY = A^{-1}F$

A^{-1} is calculated by the shape:

$$A^{-1} = \frac{1}{\det A} coA^t$$

With:

$$\det(A) = M_1 M_2 M_3$$

And $$A^{-1} = \begin{bmatrix} \frac{1}{M_1} & 0 & 0 \\ 0 & \frac{1}{M_2} & 0 \\ 0 & 0 & \frac{1}{M_3} \end{bmatrix}$$

By replacing matrices by actual experimental values of the masses (M_1, M_2, M_3), damping (k_1, k_3, k_{21}, k_{23}) and stiffness (C_{21}, C_{23}).

k_1, k_2 calculated from (5)

C_{21}, C_{23}: calculated from (6)

$M_1 = 25*10^{-3}$ kg, $M_2 = 2*10^{-3}$ kg, $M_3 = 23*10^{-3}$ Kg

$k_1 = 10^7$ N/m, $k_3 = 10^7$ N/m, $k_{21} = 0, 9925$ N/m, $k_{23} = 0, 9925$ N/m

$C_{21} = 9, 9072$, $C_{23} = 9, 9072$

The mass matrix becomes:

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 9,9 & -9,9 \\ 0 & -9,9 & 9,9 \end{pmatrix}$$

With:

$$A^{-1}B = \begin{pmatrix} 40 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 43,5 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 9,9 & -9,9 \\ 0 & -9,9 & 9,9 \end{pmatrix}$$

And:

$$A^{-1}B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4950 & -4950 \\ 0 & -413,25 & 413,25 \end{pmatrix}$$

The damping matrix:

$$C = \begin{pmatrix} 10^7 & 0 & 0 \\ 0 & 0,99 & -0,99 \\ 0 & -0,99 & 10^8 \end{pmatrix}$$

With:

$$A^{-1}C = \begin{pmatrix} 40 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 43,5 \end{pmatrix} \begin{pmatrix} 10^7 & 0 & 0 \\ 0 & 0,99 & -0,99 \\ 0 & -0,99 & 10^8 \end{pmatrix}$$

And:

$$A^{-1}C = \begin{pmatrix} 40.10^7 & 0 & 0 \\ 0 & 495 & -495 \\ 0 & -43 & 435.10^7 \end{pmatrix}$$

The matrix L which explicitly interpolating numerical application:

$$L = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 40.10^7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 495 & -495 & 0 & 4950 & -4950 \\ 0 & -43 & 435.10^7 & 0 & -413,25 & 413,25 \end{pmatrix}$$

In seeking solutions of X:

$$A - I_6 X = \begin{pmatrix} -X & 0 & 0 & 1 & 0 & 0 \\ 0 & -X & 0 & 0 & 1 & 0 \\ 0 & 0 & -X & 0 & 0 & 1 \\ 40.10^7 & 0 & 0 & -X & 0 & 0 \\ 0 & 495 & -495 & 0 & 4950 - X & -4950 \\ 0 & -43 & 435.10^7 & 0 & -413,25 & 413,25 - X \end{pmatrix}$$

The polynomial to be solved is given by:

$$P(X) = X^6 - 5,363.X^5 - 4,75.10^9.X^4 + 2368.10^{13}.X^3 + 1,74.10^{18}.X^2 - 8,613.10^{21}.X - 8,61$$

The own values are:

$$V_1 = 2.10^4$$

$$V_2 = -2.10^4$$

$$V_3 = -6,5763.10^4$$

$$V_4 = 6,6178.10^4$$

$$V_5 = 4,948.10^3$$

$$V_6 = -0,1$$

The vectors associated with each own values are:

$$\overrightarrow{V_1} = 5.10^{-5}\overrightarrow{e_1} + \overrightarrow{e_4}$$

$$\overrightarrow{V_2} = 5.10^{-5}\overrightarrow{e_1} - \overrightarrow{e_4}$$

$$\overrightarrow{V_3} = 1,0618.10^{-6}\overrightarrow{e_2} + 1,517.10^{-5}\overrightarrow{e_3} - 6,98.10^{-2}\overrightarrow{e_5} - 9,97.10^{-1}\overrightarrow{e_6}$$

$$\overrightarrow{V_4} = -1,22.10^{-6}\overrightarrow{e_2} - 1,5.10^{-5}\overrightarrow{e_3} + 8,06.10^{-2}\overrightarrow{e_5} - 9,97.10^{-1}\overrightarrow{e_6}$$

$$\overrightarrow{V_5} = 2,02.10^{-4}\overrightarrow{e_2} + 9,55.10^{-8}\overrightarrow{e_3} + \overrightarrow{e_5} + 4,72.10^{-4}\overrightarrow{e_6}$$

$$\overrightarrow{V_6} = -9,95.10^{-5}\overrightarrow{e_2} + 3,83.10^{-10}\overrightarrow{e_3} - 9,95.10^{-2}\overrightarrow{e_5} - 3,83.10^{-11}\overrightarrow{e_6}$$

The explicit matrix vectors calculated:

$$P=\begin{pmatrix} 5.10^{-5} & 5.10^{-5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1,0618.10^{-6} & 1,22.10^{-6} & 2,02.10^{-4} & 9,95.10^{-1} \\ 0 & 0 & 1,517.10^{-5} & -1,5.10^{-5} & 9,55.10^{-8} & 3,83.10^{-10} \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6,98.10^{-2} & 8,06.10^{-2} & 1 & -9,95.10^{-2} \\ 0 & 0 & -9,97.10^{-1} & -9,97.10^{-1} & 4,72.10^{-4} & -3,83.10^{-11} \end{pmatrix}$$

With:

P: matrix that contains the own vector

Furthermore P^{-1} results in the following matrix:

$$P^{-1}=\begin{pmatrix} 10^{4} & 0 & 0 & 85.10^{-1} & 0 & 0 \\ 10^{4} & 0 & 0 & -5.10^{-1} & 0 & 0 \\ 0 & -3,05.10^{-4} & 3,31.10^{4} & 0 & -2,93.10^{-3} & -0,5 \\ 0 & -3,53.10^{-4} & -3,31.10^{4} & 0 & 3,4.10^{-3} & -0,5 \\ 0 & 9,994.10^{-2} & 4,98.10^{3} & 0 & 1 & 5,83.10^{-3} \\ 0 & 1 & -1 & 0 & -2,02.10^{-4} & -3,47.10^{-8} \end{pmatrix}$$

And:

$$P^{-1}AP=\begin{pmatrix} 2.10^{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.10^{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & -6,5763.10^{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 6,6178.10^{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 4,948.10^{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & -0,1 \end{pmatrix}$$

And we consider :

$$\begin{pmatrix} y' \\ -y'' \end{pmatrix}=\begin{pmatrix} y_1^{'} \\ y_2^{'} \\ y_3^{'} \\ -y_4^{'} \\ -y_5^{'} \\ -y_6^{'} \end{pmatrix}$$

This vector can be written as follows:

$$\begin{pmatrix} y_1' & 0 & 0 & 0 & 0 & 0 \\ 0 & y_2' & 0 & 0 & 0 & 0 \\ 0 & 0 & y_3' & 0 & 0 & 0 \\ 0 & 0 & 0 & -y_1'' & 0 & 0 \\ 0 & 0 & 0 & 0 & -y_2'' & 0 \\ 0 & 0 & 0 & 0 & 0 & -y_3'' \end{pmatrix} = e^{Vt} + K$$

With the vector V can be written as:

$$V = \begin{pmatrix} 0 & I \\ A^{-1}C & A^{-1}B \end{pmatrix}$$

The previous system is reflected in the resolution of the matrix:

$$\begin{pmatrix} y_1' & 0 & 0 & 0 & 0 & 0 \\ 0 & y_2' & 0 & 0 & 0 & 0 \\ 0 & 0 & y_3' & 0 & 0 & 0 \\ 0 & 0 & 0 & -y_1'' & 0 & 0 \\ 0 & 0 & 0 & 0 & -y_2'' & 0 \\ 0 & 0 & 0 & 0 & 0 & -y_3'' \end{pmatrix} = \begin{pmatrix} e^{2.10^4 t}+1 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-2.10^4 t}+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-6,5763.10^4 t}+1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{6,6178.10^4 t}+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{4,948.10^3 t}+1 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-0,1t}+1 \end{pmatrix}$$

Hence the withdraws:

$$y_1' = e^{2.10^4 t} + 1$$

Incorporating includes:

$$y_1 = \frac{e^{2.10^4 t}}{2.10^4} + t + A$$

With the initial conditions imposed on the constant A to the value:

At t = 0 then $y_1=0$ $\quad A = -\frac{1}{2.10^4}$

Hence the solution y_1:

$$y_1 = \frac{e^{2.10^4 t}}{2.10^4} + t - \frac{1}{2.10^4} \qquad (10)$$

Similarly for y1 we find the general expression of y_2:

$$y_2' = e^{-2.10^4 t} + 1$$

$$y_2 = -\frac{e^{-2.10^4 t}}{2.10^4} + t + B$$

With the initial conditions imposed there is the value of the constant B:

At t = 0 y2 =0 $B = \frac{1}{2.10^4}$

Where:

$$y_2 = -\frac{e^{-2.10^4 t}}{2.10^4} + t + \frac{1}{2.10^4} \tag{11}$$

Integrating is the solution of the vector y_3:

$$y_3' = e^{-6,5763.10^4 t} + 1$$

$$y_3 = -\frac{e^{-6,5763.10^4 t}}{6,5763.10^4} + t + C$$

With the initial conditions imposed there is the value of the constant C:

At t = 0 $y_3 = 0$ $C = -\frac{1}{6,5763.10^4}$

$$y_3 = -\frac{e^{-6,5763.10^4 t}}{6,5763.10^4} + t - \frac{1}{6,5763.10^4}$$

5. CONCLUSION

Vibration analysis, for conditional preventive maintenance, proves a wonderful tool for several decades for the industry. It serves three levels of analysis: monitoring, diagnosis and damage of equipment state.

This work helped to develop an analytical model that simulates the behavior of a vibratory ball bearing with three degrees of freedom.

The problem addressed in this paper is the development of an analytical simulator vibration bearing. Theoretical research is detailed enough to provide a simple model that lends itself easily to programming, but powerful enough to incorporate the effect of a maximum of influential parameters on the vibration of bearings.

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