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POSTBUCKLING ANALYSIS OF FUNCTIONALLY GRADED BEAMS USING HYPERBOLIC SHEAR DEFORMATION THEORY

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ABSTRACT

In the present study, post-buckling of a thick FGM rectangular beam is carried out using the hyperbolic shear deformation theory (HYSDT). The theory accounts for parabolic distribution of transverse shear stresses across the thickness satisfying the stress free boundary conditions at top and bottom surfaces of the beam. It is assumed that elasticity modulus is changing in the thickness direction and all other material properties are taken to be constant. Variation of elasticity modulus in the thickness direction, are described by a simple power law distribution in terms of the volume fractions of constituents. Governing equations of FGM beam for post-buckling problem were found by applying Hamilton principle and Navier type solution method was used to solve post-buckling problem. The results obtained for post-buckling analysis of functionally graded beams are compared with those obtained by other theories, to validate the accuracy of the presented theory.

Keywords: Functionally graded materials, Beams, Post-buckling, Hyperbolic function, Hamilton principle, Navier t solution.

Contribution/ Originality

This study originates new formula to investigate the postbuckling behaviors of simply supported FGM beams subjected to mechanical loads.

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1. INTRODUCTION

Functionally graded materials (FGMs), a type of composite materials formed of two or more constituent phases with a continuously variable composition, received increasingly growing research interests in recent years, because of their high performance in reducing the mismatch of material properties and improving the bonding strength [1]. To fit different requirements of the mechanical performance, the material properties along the gradient direction of FGMs are often designed into variety ways; therefore, the study of FGMs with arbitrary mechanical property is necessary. The investigations of dynamic fracture response in FGMs are also significant due to the fact that FGMs are usually subjected to dynamic load in engineering problems.

Miyamoto, et al. [2] discussed the methods of FGM fabrication and general information about FGMs including microstructure analysis of the graded materials.. The thermoelastic behaviour of functionally graded beams was also studied by Chakraborty, et al. [3]. A beam finite element based on Timoshenko's theory was developed, accounting for an exponential and a power law through-the-thickness variation of elastic and thermal properties. Zhao, et al. [4] studied the post-buckling of simply supported rod made of functionally graded materials under uniform thermal loading using the numerical shooting method. Li, et al. [5] studied the thermal post-buckling behaviour of a fixed-fixed beam based on the Timoshenko beam theory. They found the effect of shear on buckling of homogeneous beams and used the shooting method to analyze the post-buckling behaviour of FGM beams. Rastgo, et al. [6] discussed the buckling of functionally graded material curved beams under linear thermal loading. They studied both the in-plane and out of plane buckling of curved beams. Ke, et al. [7] presented the post-buckling of a cracked beam for hinged-hinged and clamped-hinged edge conditions based on the Timoshenko beam theory. Also, Ke, et al. [8] presented the free vibration and mechanical buckling of cracked beams using the first order shear deformation beam theory for three types of boundary conditions.

Recently, considerable interest has also been focused on investigating the performance of FGM plates. For example, Reddy [9] proposed an analytical formulation relied on a Navier's approach using the third-order shear deformation theory and the von Karman-type geometric non-linearity. Vel and Batra [10], Vel and Batra [11] introduced an exact formulation based on a power series for thermo-elastic deformations and vibration of rectangular FGM plates. Also, Bouazza and Adda-Bedia [12] expressed the mechanical buckling of plates under three types of mechanical loadings for simply supported plates in all edges. Bouazza, et al. [13] investigated the post-buckling behavior of a simply-supported FG beam by using Euler-Bernoulli beam theory, first-order shear deformation beam theory, parabolic shear deformation beam theory and exponential shear deformation beam theory.

In this present study on the nonlinear response of FGM beams using hyperbolic shear deformation theory. The material properties of the beams vary continuously in the thickness direction according to the power-law form. The formulations are developed by using HYSDT. Governing equations were found by applying Hamilton's principle. Navier type solution method was used to obtain critical buckling loads.

2. ANALYSIS

2.1. Material Properties

Consider a rectangular beam made of a mixture of metal and ceramic as shown in Fig. 1. The material in top surface and in bottom surface is ceramic and metal respectively. The modulus of elasticity E , and the Poisson's ratio ν are assumed as [12-14]

$$E(z) = E_c V_c + E_m (1 - V_c) \quad (1)$$

$$\nu(z) = \nu_0$$

where E_c and E_m denote values of the elasticity modulus at the top and bottom of the beam, respectively, and k is a variable parameter. According to this distribution, bottom surface ($z = -h/2$) of functionally graded beam is pure metal, whereas the top surface ($z = h/2$) is pure ceramics, and for different values of k one can obtain different volume fractions of ceramic. V_c denotes the volume fraction of the ceramic and is assumed as a power function as follows:

$$V_c = \left(\frac{z}{h} + \frac{1}{2} \right)^k \quad (2)$$

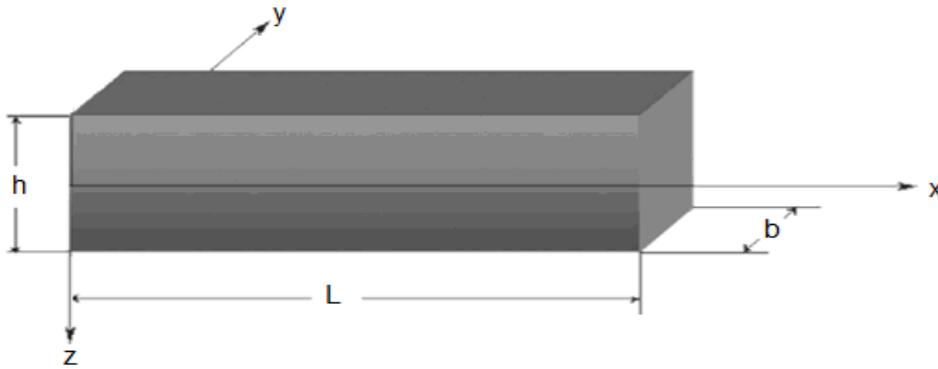


Fig-1. Co-ordinates and geometry of functionally graded beam.

Fig. 2 shows the variation of volume fractions of ceramic in the thickness direction of FGM beam. Here, volume fraction for ceramic increases from 0 at $z = -h/2$ to 1 at $z = h/2$.

The state of stress in the beam is given by the generalized Hooke's law as follows:

$$\sigma_x = Q_{11} \epsilon_x, \quad \tau_{xz} = Q_{55} \gamma_{xz} \quad (3)$$

where Q_{ij} are the transformed stiffness constants in the beam co-ordinate system and are defined as:

$$Q_{11} = \frac{E(z)}{1 - \nu^2}, \quad Q_{55} = \frac{E(z)}{2(1 + \nu)} \quad (4)$$

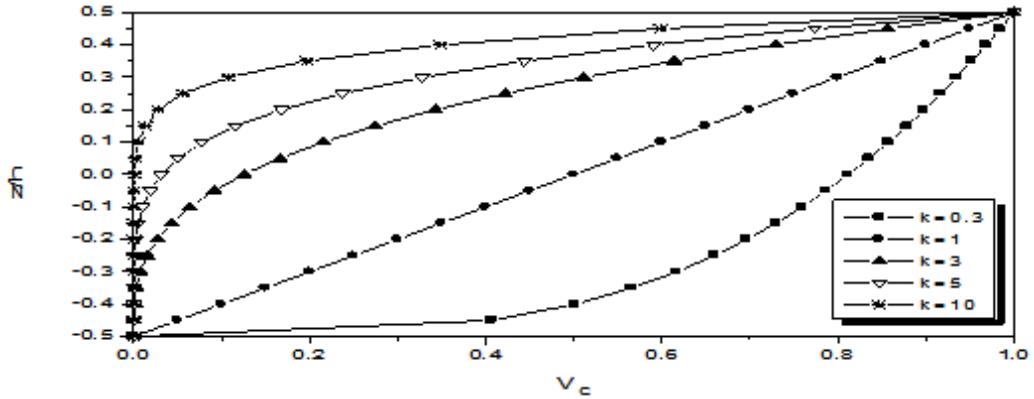


Fig-2. Volume fraction of ceramic along the thickness direction

2.2. Governing Equations

Based on the assumptions made in the previous section, hyperbolic shear deformation theory (HYSST) proposed by Soldatos [15], Soldatos [16] is used for the mathematical formulation. The displacement field of presented theory is as follows:

$$U(x, z, t) = u(x, t) - zw'(x, t) + \left[h \sinh\left(\frac{z}{h}\right) - z \cosh\left(\frac{1}{2}\right) \right] u_1(x, t) \quad (5)$$

$$V(x, z, t) = 0$$

$$W(x, z, t) = w(x, t)$$

Here, u and w represent middle surface displacement components along the x and z directions. The hyperbolic function in terms of thickness coordinate in both the displacement u is associated with the transverse shear stress distribution through the thickness of the beam and the function u_1 is the unknown function associated with the shear slopes.

According to the small-strain, moderate-rotation approximations, the nonvanishing strains are given as follows:

$$\varepsilon_x = \frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \quad (6)$$

$$\gamma_{xz} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}$$

where ε_x is the normal strain and γ_{xz} is the engineering shear strain.

Here the axial displacement u is assumed to be of order w^2 , which is based on the insignificant effect of the inplane inertia. Substituting Eq. (5) into Eq. (6) yields

$$\varepsilon_x = u' + \frac{1}{2}w'^2 - zw'' + \left[h \sinh\left(\frac{z}{h}\right) - z \cosh\left(\frac{1}{2}\right) \right] u_1' \quad (7)$$

$$\gamma_{xz} = \left[\cosh\left(\frac{z}{h}\right) - \cosh\left(\frac{1}{2}\right) \right] u_1$$

The following stress resultants are introduced

$$N = \int_A \sigma_x dA,$$

$$M = \int_A z \sigma_x dA, \quad (8)$$

$$M^s = \int_A \left[h \sinh\left(\frac{z}{h}\right) - z \cosh\left(\frac{1}{2}\right) \right] \sigma_x dA,$$

$$Q^s = \int_A \left[\cosh\left(\frac{z}{h}\right) - \cosh\left(\frac{1}{2}\right) \right] \tau_{xw} dA,$$

where N and M are the classical well-known force and moment stress resultants, Q^s and M^s are stress resultants associated with the shear deformation. Using Hook's law, the stress resultants are expressed in terms of the strains as follows:

$$\begin{Bmatrix} N \\ M \\ M^s \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} & E_{11} \\ B_{11} & D_{11} & F_{11} \\ E_{11} & F_{11} & H_{11} \end{bmatrix} \begin{Bmatrix} u' + \frac{1}{2}w'^2 \\ -w'' \\ u_1' \end{Bmatrix}, \quad (9a)$$

$$Q^s = A_{55}u_1 \quad (9b)$$

The extensional, coupling and bending rigidities appearing in Eq. (9a) are, respectively, defined as follows:

$$(A_{11}, B_{11}, D_{11}) = \int_A (1, z, z^2) Q_{11} dA,$$

$$E_{11} = \int_A \left[h \sinh\left(\frac{z}{h}\right) - z \cosh\left(\frac{1}{2}\right) \right] Q_{11} dA, \quad (10)$$

$$F_{11} = \int_A z \left[h \sinh\left(\frac{z}{h}\right) - z \cosh\left(\frac{1}{2}\right) \right] Q_{11} dA,$$

$$H_{11} = \int_A \left[h \sinh\left(\frac{z}{h}\right) - z \cosh\left(\frac{1}{2}\right) \right]^2 Q_{11} dA,$$

Moreover, the transverse shear rigidity appearing in Eq.(9b) is defined according to

$$A_{55} = \int_A \left[\cosh\left(\frac{z}{h}\right) - \cosh\left(\frac{1}{2}\right) \right]^2 \bar{Q}_{55} dA \quad (11)$$

It should be pointed out that the extensional A_{11} , coupling B_{11} and bending D_{11} rigidities are the ones usually appearing even in the classical beam theories. Among the additional rigidities in Eq. (9a), the one denoted as E_{11} is considered as additional coupling rigidity while the ones denoted as F_{11} and H_{11} are considered as additional bending rigidities.

The total potential energy can be expressed as follows:

$$V = \frac{1}{2} \int_v (\sigma_x \varepsilon_x + \tau_{xz} \gamma_{xz}) dv + \frac{1}{2} \bar{N} \int_0^L w^2 dx \quad (12)$$

Substituting Eq. (7) into Eq. (12) and noting the definition of the stress resultants, the potential energy can be expressed as follows:

$$V = \frac{1}{2} \int_0^L \left[N \left(u' + \frac{1}{2} w^2 \right) - M w'' + M^s u_1' + Q^s u_1 + \bar{N} w^2 \right] dx \quad (13)$$

The kinetic energy of the composite beam is given by

$$T = \frac{1}{2} \int_v \rho (\dot{U}^2 + \dot{w}^2) dv = \frac{1}{2} \int_v \rho \left[(\dot{u} - z\dot{w}' + f\dot{u}_1)^2 + \dot{w}^2 \right] dv \quad (14)$$

where ρ is the mass density per unit volume.

Hamilton's variational principle states that

$$\int_{z_1}^{z_2} \delta(T - V + W_{nc}) dt = 0 \quad (15)$$

where δ is the first variation and W_{nc} is the work done by non conservative forces. Applying this principle yields the following equations of motion:

$$-m_0 \ddot{u} - m_{01} \ddot{u}_1 - c_u \dot{u} + m_1 \ddot{w}' + N' + F_u = 0 \quad (16)$$

$$-m_0 \ddot{w} - m_1 \ddot{u}'' - m_{11} \ddot{u}_1' + m_2 \ddot{w}'' - c_w \dot{w} + M'' + (Nw')' - \bar{N} w'' + F_w = 0 \quad (17)$$

$$-m_{01} \ddot{u} - m_{02} \ddot{u}_1 + m_{11} \ddot{w}' + M'^s - Q^s = 0 \quad (18)$$

where

$$\begin{aligned} m_0 &= \rho(z), & m_1 &= \rho(z)z, & m_2 &= \rho(z)z^2, & m_{01} &= \rho(z)f(z), \\ m_{02} &= \rho(z)f^2(z), & m_{11} &= z\rho(z)f(z) \end{aligned} \quad (19)$$

The equations of motion can be expressed in terms of the displacements, u, w and u_1 . To this end, we substitute Eqs (9a) and (9b) into Eqs. (16)–(18) and obtain

$$-m_0\ddot{u} - m_{01}\ddot{u}_1 + m_1\ddot{w}' - c_u\dot{u} + A_{11}\left(u' + \frac{1}{2}w'^2\right)' + E_{11}u_1'' - B_{11}w''' + F_u = 0 \quad (20)$$

$$-m_0\ddot{w} - m_1\ddot{u}'' - m_{11}\ddot{u}_1' + m_2\ddot{w}'' - c_w\dot{w} + A_{11}\left(w'u' + \frac{1}{2}w^3\right)' + E_{11}(w'u_1') + B_{11}u''' + F_{11}u_1''' - D_{11}w'''' - \bar{N}w'' + F_w = 0 \quad (21)$$

$$-m_{01}\ddot{u} - m_{02}\ddot{u}_1 + m_{11}\ddot{w}' + E_{11}\left(u' + \frac{1}{2}w'^2\right)' + H_{11}u_1'' - F_{11}w''' - A_{55}u_1 = 0 \quad (22)$$

The boundary-value problem governing the static postbuckling response, expressed in terms of stress resultants, can be obtained from Eqs. (16)–(18) by setting all time-dependent terms equal to zero and disregarding the non conservative forces. The result is

$$\frac{dN}{dx} = 0 \quad (23)$$

$$\frac{dM^2}{dx^2} + \frac{d}{dx}\left(N\frac{dw}{dx}\right) - \bar{N}\frac{d^2w}{dx^2} = 0 \quad (24)$$

$$\frac{dM^s}{dx} - Q^s = 0 \quad (25)$$

As it is evident from Eq. (23), the stress resultant N , which is the total axial force exerted on the beam's cross section, is a constant. In the context of linear analysis, where the contribution of the midplane stretching is negligible, the induced axial force is simply equal to the externally applied axial load at the beam ends. As a matter of fact, the midplane stretching introduces a tension force on the beam's cross section. As a result, the total axial force N , which is a constant according to Eq. (23), will account for the applied axial force and the induced axial force due to midplane stretching. This means that for a compressive external axial force N , the stress resultant N will be less than the applied force by an amount that is equal to tension due to midplane stretching. Consequently, Eq. (24) that governs the transverse displacement w will be nonlinear. To this end, we express the equations governing the static response of the beam in terms of the displacements. Eqs. (23)–(25) can be expressed as follows:

$$A_{11} \left(u' + \frac{1}{2} w'^2 \right)' - B_{11} w''' + E_{11} u_1'' = 0 \quad (26)$$

$$A_{11} \left[\left(u' + \frac{1}{2} w'^2 \right) w' \right]' + E_{11} (w' u_1)' + B_{11} u''' + F_{11} u_1''' - D_{11} w'''' - \bar{N} w'' = 0 \quad (27)$$

$$E_{11} \left(u' + \frac{1}{2} w'^2 \right)' - F_{11} w'''' + H_{11} u_1'' - A_{55} u_1' = 0 \quad (28)$$

One notes that Eq. (26) may be solved for the axial displacement u , and hence it can be eliminated from the other two equations. This will lead to a flexural model that is given in terms of only the displacements unknowns w and u_1 . It is worth noting that this is applicable regardless of the symmetry property of the structural laminate. Integrating Eq. (26) with respect to the spatial coordinate x yields

$$A_{11} \left(u' + \frac{1}{2} w'^2 \right) - B_{11} w'' + E_{11} u_1' = c_1 \quad (29)$$

where c_1 is a constant that represents the induced axial tension force due to midplane stretching as it will be shown. Integrating Eq. (29) once more, we obtain

$$u(x) = -\frac{1}{2} \int_0^x w'^2 d\xi + \frac{B_{11}}{A_{11}} w' - \frac{E_{11}}{A_{11}} u_1 + \frac{c_1}{A_{11}} x + c_2 \quad (30)$$

For the midplane stretching to be significant, the beam ends must be restrained [17]. The boundary conditions for the axial displacement are assumed as follows:

$$u=0 \text{ at } x = 0; L$$

The constants c_1 and c_2 are now given by

$$c_2 = \frac{1}{A_{11}} [E_{11} u_1(0) - B_{11} w'(0)] \quad (31)$$

$$c_1 = \frac{A_{11}}{2L} \int_0^L w'^2 dx + \frac{E_{11}}{L} [u_1(L) - u_1(0)] - \frac{B_{11}}{L} [w'(L) - w'(0)]$$

Now, Eq. (29) can be rewritten as follows:

$$u' + \frac{1}{2} w'^2 = \frac{1}{2L} \int_0^L w'^2 dx - \frac{E_{11}}{A_{11}} u_1' + \frac{B_{11}}{A_{11}} w' + \frac{E_{11}}{LA_{11}} [u_1(L) - u_1(0)] - \frac{B_{11}}{LA_{11}} [w'(L) - w'(0)] \quad (32)$$

Eq. (26) and its first derivative can be expressed as follows:

$$\left(u' + \frac{1}{2}w'^2\right)' = -\frac{E_{11}}{A_{11}}u_1'' + \frac{B_{11}}{A_{11}}w'''' \tag{33}$$

and

$$u''' = -(w'w'') - \frac{E_{11}}{A_{11}}u_1''' + \frac{B_{11}}{A_{11}}w'''' \tag{34}$$

Substituting Eqs. (32)-(34) into Eqs. (27) and (28), we obtain

$$\left(\frac{B_{11}^2}{A_{11}} - D_{11}\right)w'''' - \left(\bar{N} - \frac{A_{11}}{2L} \int_0^L w'^2 dx\right)w'' + \left(F_{11} - \frac{B_{11}E_{11}}{A_{11}}\right)u_1''' + \beta w'' = 0 \tag{35}$$

$$\left(H_{11} - \frac{E_{11}^2}{A_{11}}\right)u_1'' + \left(\frac{B_{11}E_{11}}{A_{11}} - F_{11}\right)w'''' - A_{55}u_1' = 0 \tag{36}$$

Where β is a constant defined by (37)

$$\beta = \frac{1}{L} \left\{ E_{11} [u_1(L) - u_1(0)] - B_{11} [w'(L) - w'(0)] \right\}$$

In view of Eqs.(10a) and (10b), the stress resultants M and M^s are given by

$$M = -D_{11}w'' + F_{11}u_1' \tag{38}$$

$$M^s = -F_{11}w'' + H_{11}u_1'$$

These two equations can be solved for w and u_i at the boundaries and obtain

$$\left(F_{11}^2 - H_{11}D_{11}\right)w''(\xi) = 0 \tag{39}$$

$$\left(F_{11}^2 - H_{11}D_{11}\right)u_1'(\xi) = 0, \quad \text{with } \xi = 0, L \tag{40}$$

Since F_{11}, H_{11} , and D_{11} do not vanish, the boundary conditions in terms of the displacements can be expressed as follows:

$$w = 0 \text{ and } u_1' = 0 \text{ at } x = 0, \tag{41}$$

The first buckling mode was proofed to be the only stable equilibrium position [18]. For simply supported boundary conditions outlined above, the following displacement field is assumed:

$$w(x) = a \sin \pi \frac{x}{L} \tag{42}$$

$$u_1(x) = b \cos \pi \frac{x}{L} \tag{43}$$

where a and b are unknowns to be determined. Substituting Eqs.(42) and (43) into Eqs. (35) and (36), yields three solutions: the first is the trivial solution, $a = 0$, that corresponds to the equilibrium position in the pre-buckling state and the other two solutions, $a \neq 0$, correspond to the stable equilibrium positions in the post-buckling state. As it is well-known, the pre-buckling equilibrium position becomes unstable beyond the state of buckling. The post-buckling response can be obtained as follows:

$$a = \pm \frac{2}{\pi \sqrt{A_{11}}} \sqrt{\overline{NL}^2 - \pi^2 D_{11} + \frac{\pi^4 F_{11}^2}{L^2 A_{55} + \pi^2 H_{11}}} \tag{44}$$

We note that the buckling amplitude a corresponds to the maximum buckling level that occurs at the midspan of the beam where $x = L/2$.

On the other hand, the critical buckling load, \overline{N}_{cr} , can be obtained by solving the linear counterpart of Eq. (36). The result is

$$\overline{N}_{cr} = \frac{\pi^2}{L^2} \left(D_{11} - \frac{\pi^2 F_{11}^2}{L^2 A_{55} + \pi^2 H_{11}} \right) \tag{45}$$

3. RESULTS AND DISCUSSION

The constituent material properties of the FGM beam were chosen as follows [13, 14]:

Al: $E_m = 70GPa; \nu_m = 0.3;$

Ceramic: $E_c = 380GPa; \nu_c = 0.3;$

The nondimensional critical buckling load, \overline{P}_{cr} , is defined as follows:

$$\overline{P}_{cr} = \frac{L^2}{bh^3 E_m} \overline{N}_{cr} \tag{46}$$

Non-dimensional first critical buckling load were given in Tables 1-5 for $L/h = 5, 10, 20, 50$ and 100, respectively, for different theories and for different material distributions. It is seen from the tables that critical buckling load is decreasing with increasing k and increasing with

increasing L/h ratios. Difference between the critical buckling load predicted by classical beam theory and shear deformation theories is decreasing with increasing L/h ratio.

Table-1. Comparison of nondimensional first critical buckling load with different theories for different material distribution (L/h = 5, a=0).

Theory	Ceramic	k = 0.3	k = 1	k = 3	k = 5	k = 10	Metal
Classical beam theory [13]	4.906	3.812	2.905	2.305	2.047	1.688	0.904
First-order shear deformation beam theory [13]	4.485	3.498	2.655	2.069	1.824	1.4999	0.826
Parabolic shear deformation beam theory [13]	4.4097	3.447	2.611	1.996	1.741	1.433	0.812
Exponential shear deformation beam theory [13]	4.413	3.721	2.613	1.995	1.737	1.433	0.813
HYSDT	4,4097	3,447	2,611	1,997	1,741	0,983	0.812

Table-2. Comparison of nondimensional first critical buckling load with different theories for different material distribution (L/h = 10, a=0).

Theory	Ceramic	k = 0.3	k = 1	k = 3	k = 5	k = 10	Metal
Classical beam theory [13]	4.906	3.812	2.905	2.305	2.047	1.688	0.904
first-order shear deformation beam theory [13]	4.794	3.728	2.838	2.241	1.987	1.636	0.883
parabolic shear deformation beam theory [13]	4.772	3.714	2.825	2.219	1.961	1.616	0.879
Exponential shear deformation beam theory [13]	4.773	3.788	2.826	2.219	1.9598	1.616	0.879
HYSDT	4,772	3,714	2,825	2,219	1,961	1,688	0.879

Table-3. Comparison of nondimensional first critical buckling load with different theories for different material distribution (L/h = 20, a=0).

Theory	Ceramic	k = 0.3	k = 1	k = 3	k = 5	k = 10	Metal
Classical beam theory [13]	4.906	3.812	2.905	2.305	2.047	1.688	0.904
First-order shear deformation beam theory [13]	4.878	3.791	2.888	2.288	2.032	1.674	0.899
Parabolic shear deformation beam theory [13]	4.872	3.787	2.885	2.283	2.025	1.669	0.897
Exponential shear deformation beam theory [13]	4.872	3.806	2.885	2.282	2.025	1.669	0.898
HYSDT	4,872	3,787	2,885	2,283	2,025	1,432	0.897

Table-4. Comparison of nondimensional first critical buckling load with different theories for different material distribution ($L/h = 50, a=0$).

Theory	Ceramic	k = 0.3	k = 1	k = 3	k = 5	k = 10	Metal
Classical beam theory [13]	4.906	3.812	2.905	2.305	2.047	1.688	0.904
first-order shear deformation beam theory [13]	4.902	3.809	2.902	2.302	2.045	1.685	0.903
parabolic shear deformation beam theory [13]	4.901	3.808	2.902	2.301	2.044	1.685	0.903
Exponential shear deformation beam theory [13]	4.901	3.811	2.902	2.301	2.044	1.685	0.903
HYSDT	4,901	3,808	2,902	2,301	2,044	0,863	0.903

Table-5. Comparison of nondimensional first critical buckling load with different theories for different material distribution ($L/h = 100, a=0$).

Theory	Ceramic	k = 0.3	k = 1	k = 3	k = 5	k = 10	Metal
Classical beam theory [13]	4.906	3.812	2.905	2.305	2.047	1.688	0.904
First-order shear deformation beam theory [13]	4.905	3.811	2.904	2.304	2.047	1.687	0.904
Parabolic shear deformation beam theory [13]	4.905	3.811	2.904	2.304	2.047	1.687	0.904
Exponential shear deformation beam theory [13]	4.905	3.812	2.904	2.304	2.046	1.6867	0.904
HYSDT	4,905	3,811	2,904	2,304	2,047	1,688	0.904

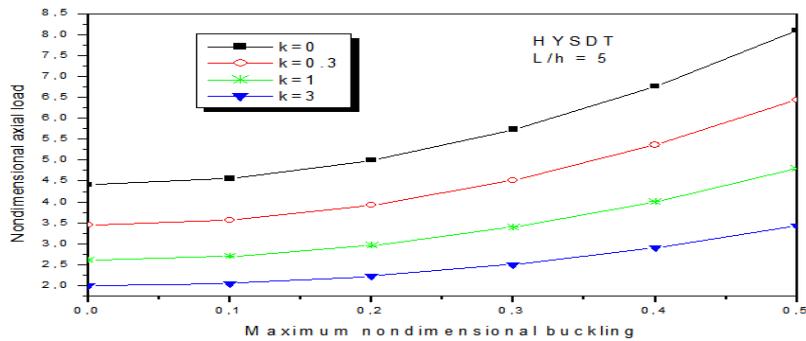


Fig-3. Variation of the maximum buckling with the applied axial load for $L/h = 5$.

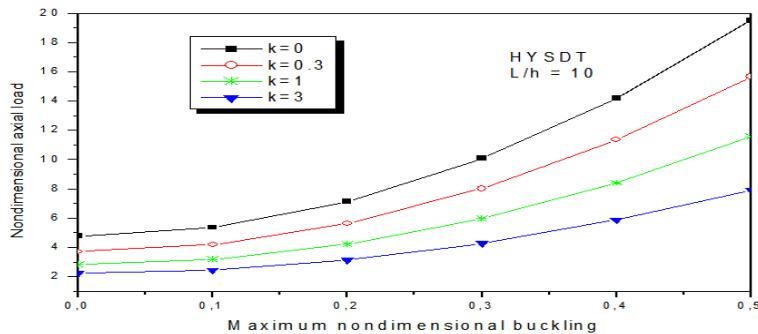


Fig-4. Variation of the maximum buckling with the applied axial load for $L/h = 10$.

It is worth investigating the significance of shear deformation not only on the critical buckling load but also on the resulting post-buckling response, which is considered to be the contribution of this study. The post-buckling response of simply supported FGM beams using the hyperbolic shear deformation theory (HYSDT) is presented.

Fig.3-4 shows the critical buckling load $\left(\overline{P}_{cr}\right)$ vs the nondimensional amplitude for different values of volume fraction exponent k for $L/h = 5$ and 10 , respectively. It is seen that the nondimensional axial load increases monotonically as the nondimensional amplitude increases. On the other hand, the critical buckling load decreases, when the material gradient index k is increased. Comparing Fig. 3 with Fig. 4, the responses are very similar, however, the critical buckling load of slenderness ratio $L/h=10$ is higher than that critical buckling load of slenderness ratio $L/h=5$.

4. CONCLUSION

In the present paper, equilibrium and stability equations for a simply supported rectangular functionally graded beam are obtained using the using the hyperbolic shear deformation theory (HYSDT), with the assumption of power law composition for the constituent materials. Closed form solutions for the critical buckling load and static post-buckling response of beams are presented. Post-buckling responses solutions for simply supported FGM rectangular beams are developed using the Navier procedure. From the numerical results and discussion, it is observed that, in case of isotropic beam, the critical buckling loads obtained by the presented theory are in excellent agreement with those of other refined theories. On the other hand, based on the post-buckling response, one can conclude that the effect of the shear deformation has considerable effect on the critical buckling load of functionally graded beam, especially for a thick beam.

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