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AN ESTIMATE OF THE TENSILE STRENGTH BASED ON P-WAVE VELOCITY AND SCHMIDT HARDNESS REBOUND NUMBER

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ABSTRACT

Tunnels, roads, bridges, dams, power plants, etc., are constructed in rock formation. Various rock parameters are important for the design of these structures such as compressive and tensile strengths of the embedded rocks. Preparation of samples and performing uniaxial and Brazilian tensile tests require a lot of time and money; and they are not operable in all samples. Therefore, much easier and less costly methods are used to determine tensile strength. Schmidt hammer and sound velocity tests are among these simple and inexpensive tests. Another advantage of these tests is that they are non-destructive and operable in the deserts. The main objective of this study was to obtain the relationship between the sound velocity, the Schmidt hammer rebound number and the tensile strength. This study was conducted on samples of sandstone, tuff, marble, limestone, red sandstone and marl. Simple and multivariate regression models were used to obtain this relationship. Root Mean Square Error (RMSE) and value account for (VAF) were determined to control the performance of provided equations. These two indices were equal to 2.20 and 46.5 in the relation between the sound velocity and the tensile strength; and they were equal to 0.09 and 81.49 in the relation between the Schmidt hardness rebound number and the tensile strength when the simple regression analysis was used, while they were equal to 0.24 and 80.91 when multivariate regression analysis.

Keywords: Tensile strength, P waves, Schmidt hammer, Brazilian test, Velocity, Schmidt hardness.

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Contribution/ Originality

This study is one of very few studies which have investigated the relationship between the sound velocity, the Schmidt hammer rebound number and the tensile strength. This study documents that tensile strength can be estimated with a high level of accuracy by using the proposed equations.

1. INTRODUCTION

Tunnels, roads, bridges, dams, power plants, etc., are constructed in rock formation. Various rock parameters are important for the design of these structures such as compressive and tensile strengths of the embedded rocks (Hosseini *et al.*, 2012). Tensile strength is widely used in determining the properties of intact rock, choosing a proper rock failure criterion, and suggesting the initial designs of rock structure. This feature is determined using both direct (uniaxial tensile) and indirect (Brazilian) tests. These tests may not be accurately performed on all rocks and require careful sample preparation and the expensive and precision equipment. In addition, the results are highly dependent on the sample size, method of loading, human errors, external factors, etc. (Arjomand Pour and Hosseini Tudeshaki, 2013). To solve the above problems, determination of an index to show the tensile strength of rocks, simple tests such as dry density, saturated density, Schmidt hammer, the sound velocity and porosity are performed.

Studies conducted on the estimation of the tensile strength are very limited, and most studies have been conducted on the estimation of the uniaxial compressive strength. A number of these studies are given in the following paragraphs.

In a study conducted in 2012, Hosseini et al., estimated the marl engineering features using punch tests and obtained several relations (equations) which among them, the relation between tensile strength and point load index (with a correlation coefficient of 0.96) and the relation between uniaxial compressive strength and the tensile strength (with a correlation coefficient of 0.81), can be mentioned. Both relations are linear (Hosseini *et al.*, 2012).

Arjomand Pour and Hosseini Tudeshaki evaluated the relationship between tensile strength and dry density, saturated unit-weight and porosity in 2013. The results of simple regression showed better performance of the porosity index compared with other properties of rock in the estimation of tensile strength. Also, a more accurate estimate of the tensile strength can be obtained for each variable using multivariate regression analysis (Arjomand Pour and Hosseini Tudeshaki, 2013).

Gurocak *et al.* (2012)examined the relationship between tensile strength and point load index, Schmidt rebound number, and the specific gravity in 2012. Relations proposed to estimate the tensile strength using the simple and multivariate regression models were linear. The relation obtained by multivariate regression model has the highest correlation coefficient (0.83) compared with the relations obtained using simple regression model (Gurocak *et al.*, 2012).

While considering the principle indicating that the compressive force applied to the sample in a particular direction induce a tensile force perpendicular to it, Atapoor estimated the tensile strength based on the uniaxial compressive strength and presented some relations (equations)(Atapour, 2012).

By conducting experiments on the Ilam Formation limestone in 2012, Ghorbani Saber and Ghobadi (2011) offered relations with high correlation coefficients to estimate the tensile strength and uniaxial compressive strength based on the point load index (Ghorbani Saber and Ghobadi, 2011).

Kahraman et al. conducted a research on compressive and tensile strength of different type of rocks. Based on their results, they proposed a linear correlation between UCS and BT. However, the coefficient of determination, R2, of their study was almost 0.5 which is not reliable enough (Kahraman *et al.*, 2012).

The purpose of research conducted by Kohno and Maeda is to investigate on relationship between point load strength index and uniaxial compressive strength of hydrothermally altered soft rocks (Kohno and Maeda, 2012).

Khandelwal estimated the tensile strength using the P-wave velocity (Khandelwal, 2013).

The research conducted by Minaeian and Ahangari (2011) on an estimate of uniaxial compressive strength based on the P-wave velocity and Schmidt rebound number yielded good results. Relations obtained by the simple linear regression analysis had correlation coefficients between 0.93 and 0.94 and relation obtained by the multivariate linear regression analysis had a correlation coefficient of 0.92 (Minaeian and Ahangari, 2011).

With the assessment of concrete compressive strength by ultrasonic testing in 2012, Hadian Fard and Jafari (2012) indicated the relationship between these two parameters by an exponential function (Hadian Fard and Jafari, 2012).

Abbasi estimated the uniaxial compressive strength using Schmidt hammer and point load tests in 2011 and presented some relations (Abbasi, 2011).

Heydari et al. presented relations for estimating the fragility of granitoid rocks district in Samen district in 2011using non-destructive ultrasound velocity and Schmidt hammer tests (Heidari *et al.*, 2011).

By conducting two non-destructive testing of ultrasound and Schmitt hammer on 50 cubic concrete samples with different grades of cement in 2012, Noferesti and Heydari (2012) presented relations to estimate the uniaxial compressive strength with an acceptable correlation coefficient (Noferesti and Heydari, 2012).

In this study, it was tried to evaluate the relationship of wave velocity and Schmidt hardness rebound number with tensile strength.

2. CHARACTERISTICS OF TESTED SAMPLES

All of the used samples have been cored from 7 blocks including sandstone, tuff, marble, sandstone, red sandstone, limestone, and marl.

12 Cores were derived from these 7 blocks, the characteristics of them are found in Table 1 after cutting and polishing, for doing sound velocity and Schmidt hammer tests.

Number associated with each Figure, from left to right, indicates the block number, core number and disc number (in the disc samples).

After performing the sound velocity and Schmidt hammer tests, sample are cut in the form of disc for Brazilian testing. Their characteristics are shown in Table 2.

3. CONDUCTED EXPERIMENTS

To obtain the parameters needed to estimate the tensile strength, three following experiments were performed, respectively:

A. Test of Sound Velocity

By this test, the time used for the passage of pressure wave through the sample and the pressure wave velocity can be calculated according to relation 1 (Fahimifar and Soroush, 2001).

$$V_P = \frac{L}{t} \times 100 \qquad (1)$$

V_P: Pressure wave velocity (m.s)

T: the transition time of pressure wave (μs)

L: Length of sample (mm)

B. Schmidt Hammer Test

Ernest Schmidt, a Swiss engineer made a Rebound Hammer in 1948 to measure the concrete surface hardness (Luke and Snell, 2012).

Based on the suggestion of ISRM, it is better to place hammer on one of the 3 directions of vertical upward, horizontal, and vertical downward on the sample. If it is not possible to perform the test in any of the above-mentioned directions, the test can be performed in a desired position; and then the results are corrected for horizontal or vertical position (Fahimifar and Soroush, 2001). In Table 3, correction of results in non-horizontal position is shown.

At least 20 separate tests are conducted on each rock sample, and the mean values are obtained after correcting the read numbers.

C. Brazilian Test

Brazilian test was on trial for the first time during the period between 1930 and 1960, at the National Institute of Brazil (INT) in Rio de Janeiro. In 1940, when Brazil joined World War II, this test contributed significantly to the determination of the tensile strength of concrete (RILEM Publications SARL, 2002). In this test, the tensile strength, based on the load at the moment of failure, was calculated by relation 2 (Hosseini, 2006).

$$\sigma_t = .0636 \frac{P}{D\, t} \qquad (2)$$

P: load at the moment of failure (kN)

D: Diameter of Sample (mm)

t: Thickness of the sample (mm)

σ_t: Tensile strength (MPa)

The results of these tests are given in Table 4.

4. ANALYSIS OF THE RESULTS

In this section, the relations between the numbers obtained from the sound velocity, Brazilian and Schmidt hammer tests were evaluated; and other relations are presented to estimate the tensile strength for the other two components, in a way that two separate relations, one between the tensile strength (TS) and the sound velocity (V_P) and the other between the tensile strength and Schmidt hammer rebound number (SHS) were obtained using the simple regression analysis. Then, with the help of multivariate regression analysis, the relation between these three components was obtained. In these relations, V_P and SCH were considered as independent variables; and TS was considered as the dependent variable. For the analysis of the results, charting and presentation of the relations (equations) software such as Excel, SPSS and Table curve ^{3D} were used.

A. The Relation between TS and VP

All functions were evaluated using SPSS to examine the relation between these two parameters and the function with the highest correlation coefficient was selected. Table 5 includes all functions together with the correlation coefficient and parameters of the equation. The diagrams of these functions can be seen in Figure 1.

Given the values of R Square and Sig of functions, three-order function (Cubic) with the highest correlation coefficient (0.467) and the lowest Sig (0.059) was selected. Its diagram can be seen in Figure 2.

The relation obtained from this function is given in Equation 3.

$$TS = -41.594 + 0.019\, V_P - 2.762 \times 10^{-10} V_P^3 \qquad R^2 = 0.467 \qquad (3)$$

B. The Relation between TS and SCH

This section is like the previous section.

Table 6 contains all the functions and their parameters and Figure 3 shows their diagrams. Given the values of R Square and Sig in Table 6, the exponential function with the highest correlation coefficient (0.884) and a Sig value of zero was selected (Of course, Growth and Logistic functions had the same conditions, as well; but with regard to the simple form of exponential function, two other functions could not be selected). Their diagram is shown in Figure 4.The relation obtained from this function is given in Equation 4.

$$(4) \qquad TS = 0.297 e^{0.084 SCH} \quad R^2 = 0.884$$

C. The Relation between VP, SCH and TS

The relation between these three variables was evaluated by multivariate linear regression using SPSS, and non-linear multivariate regression using Table curve $^{3D.}$ Then the best equation was selected according to the correlation coefficients. Tables 7 and 8 show the details of the linear multivariate regression models. Three-dimensional diagram and relation associated with the non-linear multivariate regression have been shown in Figure 5.

Relations 5 and 6 show those equations obtained from linear and nonlinear models of multivariate regression. Given the correlation coefficient, its nonlinear model is more acceptable and usually used in the estimation of the tensile strength (Equation 6).

$$TS = -21.796 + 0.735\,SCH + 0.001V_P \qquad R^2=0.803 \qquad (5)$$

$$ln\,TS = 1.669 + 1.416 \times 10^{-5}SCH^3 - \frac{5482336.9}{V_P^2} \qquad R^2=0.853 \qquad (6)$$

D. Validation of the Proposed Relations

Accuracy of relations is discussed now. This means that the values obtained from relations and Brazilian test are evaluated in the diagram and the accuracy of relations is achieved by considering the slope of the line passing through these points. The more the slope of these curves is closer to 1, the more the relationship is acceptable.

Also, in order to control the performance of the proposed equations, the root mean square error (RMSE) and value account for (VAF) are determined using relations 7 and 8. y and y' are the measured and estimated tensile strengths, respectively (Minaeian and Ahangari, 2011).

In the best situation, RMSE is equal to zero and VAF is equal to 100. After these steps, the best equation to estimate the tensile strength can be proposed.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - y_i')^2} \qquad (7)$$

$$VAF = \left[1 - \frac{var(y-y')}{var(y)}\right] \times 100 \qquad (8)$$

Table 9 shows the values obtained by Brazilian test and each of relations provided, where σ_t is the tensile strength obtained from the Brazilian test, TS_{VP} is the tensile strength estimated by relation (equation) 3, TS_{SCH} is tensile strength estimated by relation 4, and $TS_{VP,\ SCH}$ is the tensile strength estimated by relation 6.

Table 10 shows the values related to R^2, RMSE and VAF for each relation related to the estimation of tensile strength.

6. CONCLUSION

This study was conducted on samples of sandstone, tuff, marble, limestone, red sandstone and marl.

The main objective of this study was to obtain the relation between the sound velocity, Schmidt hammer rebound number and the tensile strength.

Schmidt hammer and sound velocity tests are among simple and inexpensive tests. Another advantage of these tests is that they are non-destructive and operable in the desert.

Two separate relations, one between the tensile strength (TS) and the sound velocity (VP) and the other between the tensile strength and the Schmidt hammer rebound number (SCH) were first obtained using simple regression analysis. Then, with the help of multivariate regression analysis, a relation between these three components was obtained. In these relations, VP and SCH TS were considered as independent variables and TS was considered as the dependent variable. For the analysis of results, charting and presentation of equations, Excel, SPSS and Table curve 3D were used.

The results indicate that:

A. To choose the best relations between the velocity of longitudinal waves and tensile strength, with respect to the values of R Square and Sig of functions, a three-order function (Cubic) with the highest correlation coefficient (0.467) and the lowest Sig (0.059) is selected.

B. An exponential function with the highest correlation coefficient (0.884) and zero Sig was selected to determine the relationship between Schmidt hammer rebound number and the tensile strength based on the values of R Square and Sig in Table 4-3 (Of course, both Growth and Logistic functions had the same condition, but because of the simple form of the exponential function, two other functions could not be selected).

C. The relation between the three variables of Schmidt hammer rebound number, velocity of longitudinal waves and the tensile strength were examined through linear multivariate regression model using SPSS; and non-linear multivariate regression model using Table curve 3D.

The relation obtained by the linear multivariate regression had a R-Square equal to 0.803 and the non-linear multivariate had a R-Square equal to 0.853.

The root mean square error (RMSE) and VAF were determined to control the performance of provided equations. Values of these two indices were equal to 2.20 and 46.05, respectively when the simple regression was used to determine the relationship between the sound velocity and tensile strength; and they were equal to 0.09 and 81.49, respectively when the simple regression was used to determine the relationship between the Schmidt hardness rebound number and the tensile strength, while these two indices were calculated to be equal to 0.24 and 80.91 when multivariate regression model was used.

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Competing Interests: The authors declare that they have no competing interests.

Contributors/Acknowledgement: All authors contributed equally to the conception and design of the study.

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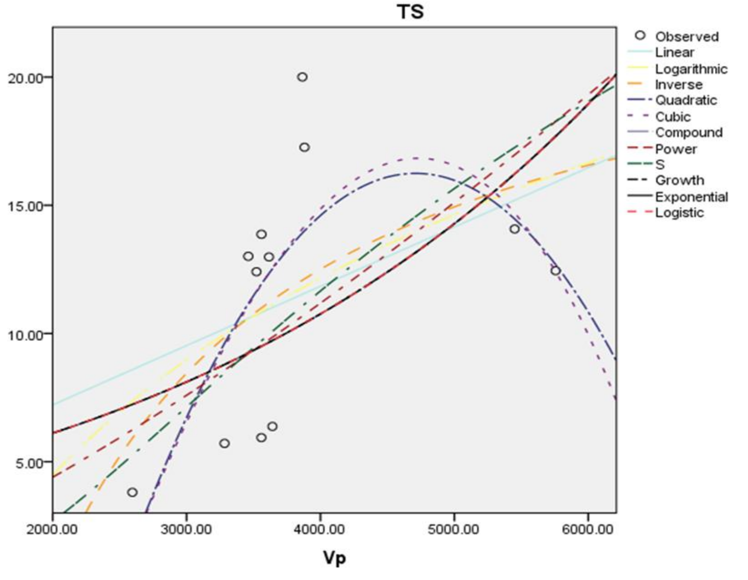

Figure- 1. Relations between TS and VP for rock using various functions

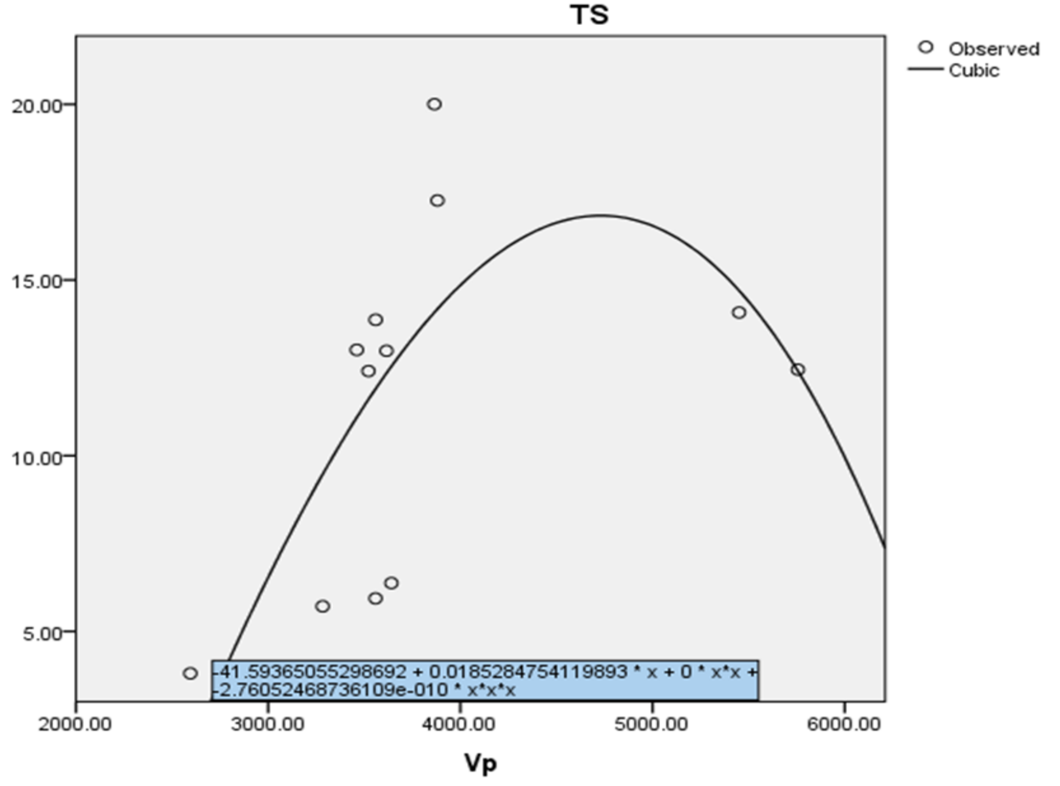

Figure-2. Third order function (Cubic) used to show the relation between TS and VP for a typical rock type

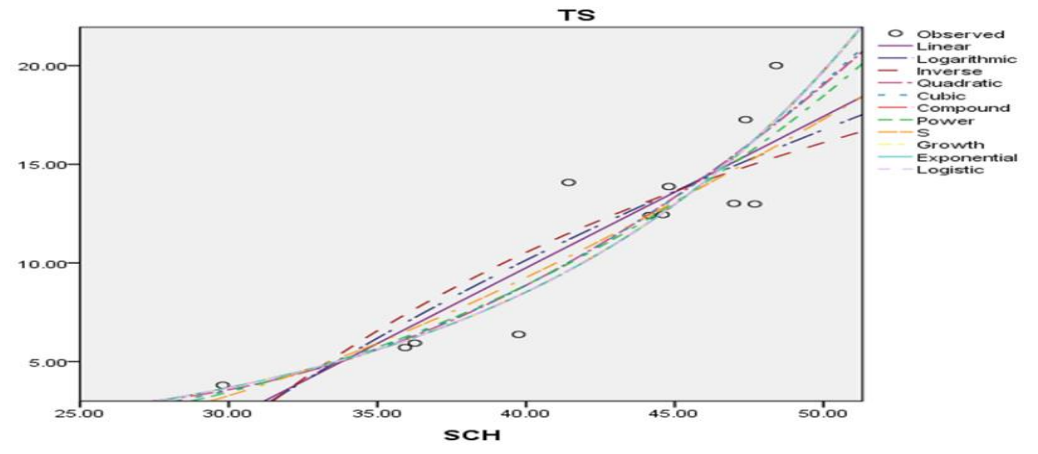

Figure-3. Relations between TS and SCH for rock using various functions

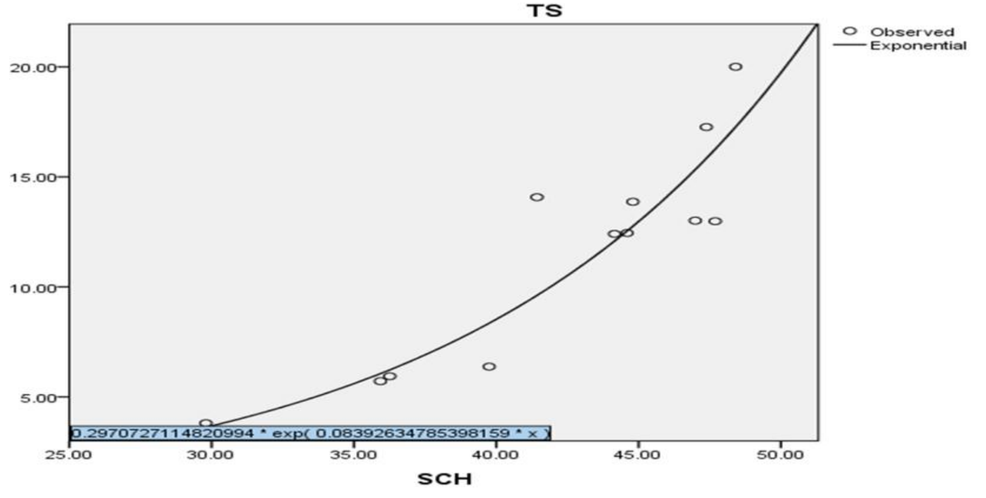

Figure-4. Exponential function used to show the relation between TS and SCH for a typical rock type

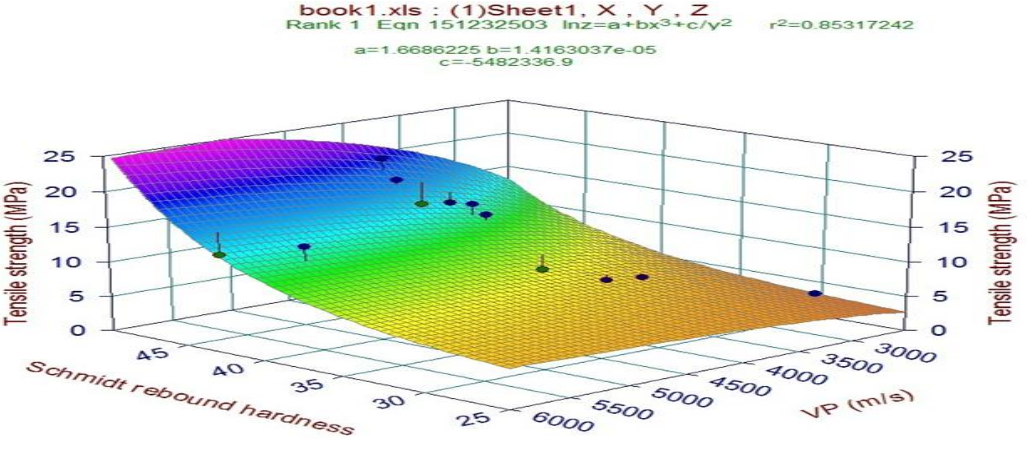

Figure-5. Three-dimensional diagram of the nonlinear multivariate regression model (Obtained by Table curve 3D)

Table-1. Characteristics of cylindrical samples for doing sound velocity and Schmidt hammer tests

Sample number	Diameter (mm)	Length (mm)
11	51	143.76
12	51	171.5
21	51	124.18
22	51	110.92
31	51	105.91
32	51	112.28
41	51	177
51	51	150.92
61	51	83.12
62	51	90.33
63	51	84.96
71	51	77.82

Table-2. Characteristics of the disc samples for Brazilian testing

Sample number	Diameter (mm)	Length (mm)
111	51	26.29
112	51	24.78
121	51	25.5
122	51	26.3
211	51	26.18
221	51	25.64
222	51	26.32
311	51	25.55
312	51	25.74
321	51	25.78
411	51	26.29
412	51	26.15
511	51	26.44
512	51	25.74
611	51	25.18
612	51	26.43
621	51	25.86
622	51	25.8
631	51	25.93
711	51	25.69
712	51	25.81

Table-3. Correction of Schmidt hardness for non-horizontal rebounds (Fahimifar and Soroush, 2001)

	Correction of Schmidt hardness			
	directions of downward		directions of upward	
Schmidt hardness	**90-**	**45-**	**45+**	**90+**
10	3.2+	2.4+	-	-
20	3.4+	2.5+	3.5-	5.4-
30	3.1+	2.3+	3.1-	4.7-
40	2.7+	2+	2.6-	3.9-
50	2.2+	1.6+	2.1-	3.1-
60	1.7+	1.3+	1.6-	2.3-

Table-4. Results of the sound velocity, Brazilian and Schmidt hammer tests

Sample number	σ_t (MPa)	SCH	V_P (m.s)
11	5.93878691	36.26	3558.416
12	6.374940905	39.75	3641.189
21	17.26257134	47.38	3880.625
22	20.00530234	48.41	3864.808
31	12.44707517	44.6	5755.978
32	14.07657555	41.43	5450.485
41	3.804808834	29.8	2595.308
51	13.86786312	44.8	3559.434
61	12.40856578	44.15	3522.034
62	13.01220504	46.99	3460.92
63	12.98518636	47.69	3615.319
71	5.714031279	35.93	3283.544

Table-5. Summary of the functions and their parameters to examine the relationship between TS and VP Dependent Variable: TS

Equation	Model Summary					Parameter Estimates			
	R Square	F	df1	df2	Sig.	Constant	b1	b2	b3
Linear	.168	2.024	1	10	.185	2.589	.002		
Logarithmic	.223	2.876	1	10	.121	-79.91	11.101		
Inverse	.280	3.891	1	10	.077	24.67	-48679.765		
Quadratic	.454	3.748	2	9	.065	-55.96	.031	-3.254E-006	
Cubic	.467	3.946	2	9	.059	-41.594	.019	.000	-2.762E-010
Compound	.234	3.058	1	10	.111	3.472	1.000		
Power	.306	4.412	1	10	.062	.000	1.348		
S	.382	6.176	1	10	.032	3.929	-5892.832		
Growth	.234	3.058	1	10	.111	1.245	.000		
Exponential	.234	3.058	1	10	.111	3.472	.000		
Logistic	.234	3.058	1	10	.111	.288	1.000		

The independent variable is Vp.

Table-6. A summary of the functions and their parameters to examine the relationship between TS and SCH

Equation	Model Summary					Parameter Estimates			
	R Square	F	df1	df2	Sig.	Constant	b1	b2	b3
Linear	.794	38.582	1	10	.000	-20.928	.767		
Logarithmic	.770	33.544	1	10	.000	-99.115	29.618		
Inverse	.738	28.125	1	10	.000	38.349	-1112.628		
Quadratic	.818	20.188	2	9	.000	16.890	-1.174	.024	
Cubic	.817	20.132	2	9	.000	4.723	-.224	.000	.000
Compound	.884	76.475	1	10	.000	.0297	1.088		
Power	.879	72.480	1	10	.000	4.942E-005	3.280		
S	.863	62.811	1	10	.000	5.345	-124.752		
Growth	.884	76.475	1	10	.000	-1.214	.084		
Exponential	.884	76.475	1	10	.000	.0297	.084		
Logistic	.884	76.475	1	10	.000	3.366	.919		

The independent variable is SCH.

Table-7. A summary of the linear model of multivariate Regression

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Sum of Squares	df	Mean Square	F	Sig.
Regression	.896[a]	.803	.759	2.44784	219.967	2	109.983	18.355	.001[b]

a. Predictors: (Constant), Vp, SCH

Table-8. Coefficients of the linear model of multivariate Regression

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
(Constant)	-21.796	5.594		-3.896	.004
SCH	.735	.137	.854	5.387	.000
Vp	.001	.001	.101	.640	.538

Dependent Variable: TS

Table-9. Values obtained from Brazilian test and the equations for the estimation of tensile strength

Sample number	$TS_{VP,SCH}$	TS_{SCH}	TS_{VP}	σ_t
11	6.758963	6.245223	13.57093	5.938787
12	8.539168	8.372704	14.25481	6.374941
12	16.62613	15.89329	15.99694	17.26257
22	18.32717	17.32961	15.89298	20.0053
31	15.79423	12.58338	15.09743	12.44708
32	12.07632	9.641694	17.2424	14.07658
41	3.419505	3.629797	2.888594	3.804809
51	12.29731	12.79657	13.57959	13.86786
61	11.53645	12.11661	13.25751	12.40857
62	14.59138	15.38106	12.71366	13.01221
63	16.20544	16.31258	14.04547	12.98519
71	6.153856	6.074483	11.01529	5.714031

Table-10. The comparison of relations related to the estimation of tensile strength

Predicted TS	R^2	RMSE	VAF
TS_{VP}	0.47	2.20	46.05
TS_{SCH}	0.88	0.09	81.49
$TS_{VP,SCH}$	0.85	0.24	80.91