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EMERGENT BEHAVIOR OF "RATIONAL" AGENTS IN SOME FORGOTTEN N-PERSON GAMES

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ABSTRACT

This paper investigates N-person games with linear payoff functions, which are defined by four parameters each for the case when some of these parameters are equal to each other. Such cases represent transitions between different games. The participating agents are all greedy simpletons who imitate the behavior of the neighbor in their Moore neighborhood - nine neighbors, including themselves - that received the highest reward in the previous iteration. The results show that the solutions are non-trivial and represent quite irregular emergent behavior when the payoff functions are equal or cross each other.

Keywords: N-person games, Transitions, Emergence, Rationality, Agent-based simulation.

Contribution/ Originality

The paper contributes the first logical analysis of borderline N-person games.

1. INTRODUCTION

In simple games each participant has exactly two available choices of actions. It is generally accepted to call the first choice *cooperation* (C) and the other choice *defection* (D). A detailed discussion of such games can be found in reference (Szilagyi, 2012).

For two players, four outcomes are possible: CC, CD, DC, and DD (the first choice is that of the row player, the second is that of the column player). Each player receives a payoff (reward or punishment) for these situations. R is the reward for mutual cooperation, P is the punishment for mutual defection, T is the temptation to defect when the other player cooperates, and S is the sucker's payoff for cooperating when the other player defects. If the payoffs for the CD and DC outcomes (T and S) are the same for both players, the game is symmetric as shown in the following matrix (the first item in each pair of payoffs is for the row player, the second is for the column payer):

C D

C R, R S, T

D	T, S	P, P

If we assume that all four payoffs are different, then there are 4! = 24 symmetric preference orderings of the four parameters. Each ordering determines a different game. Seven of them are well known and have names:

P > R > S > T

P > R > T > S

P > S > R > T

P > S > T > R

P > T > R > S

P > T > S > R

R > P > S > T

R > P > T > S

R > S > P > T

R > S > T > P

R > T > P > S   Stag Hunt

R > T > S > P

S > P > R > T

S > P > T > R

S > R > P > T

S > R> T > P
S > T > P > R
S > T > R > P   Leader

T > P > R > S	Deadlock
T > P > S > R	
T > R > P > S	Prisoners' Dilemma
T > R > S > P	Chicken
T > S > P > R	Battle of the Sexes
T > S > R > P	Benevolent Chicken

The four parameters *P*, *R*, *S*, and *T* can be used to define two payoff functions of any N-person game: one for all cooperators and another for all defectors (*uniform* game). We define these payoffs as functions of the ratio of cooperators among all other players (Szilagyi, 2012). Points *P* and *S* correspond to no cooperators, points *R* and *T* to the case when all other agents cooperate. We now add two curves: one connects points *P* and *T*; the other connects points *S* and *R* (Figure 1). The former is called the defectors' payoff function D(x), the latter is the cooperators' payoff function C(x). Here x represents the ratio of cooperators among the neighbors of the agent currently being investigated. Obviously, these functions do not have to be linear as in Figure 1.

2. THE FORGOTTEN GAMES

We have been interested in the transitions between different games for a long time. The easiest way to investigate such transitions is to fix the values of three parameters and change the value of the fourth one (Zhao *et al.*, 2007). It will move between the inequalities and produce different games. Another approach is to express the values of the four parameters through a common variable (Szilagyi and Somogyi, 2008).

The classification of games according to the mutual relationships between the R, S, T, and P parameters ignores the cases when some of these parameters are equal to each other. Accordingly, investigators usually do not consider such cases. We call them the forgotten games. Two equal payoffs represent the transition between two different games. For example, *P* = *S* is the transition between Chicken and Prisoners' Dilemma (Nowak and May, 1992; Lloyd, 1995). It is interesting to find out what happens in cases when three or all four payoffs are the same. We will only consider linear payoff functions but even in this simple case the four parameters may have any values. Therefore, for a systematic investigation additional constraints are necessary. We decided to limit the payoff values to the region between -1 and 1. Only -1, 0, and 1 are the allowed values of the four parameters. This way, rewards and penalties are equally represented. The different payoff functions then are as follows:

$C_1 = 0 = \text{constant}$

$C_2 = - 1 = \text{constant}$

$C_3 = 1 = \text{constant}$

$C_4 = x$

$C_5 = - x$ (1)

$C_6 = 1 - x$

$C_7 = 1 - 2x$

$C_8 = - 1 + x$

$C_9 = - 1 + 2x$

and

$D_1 = 0 = \text{constant}$

$D_2 = - 1 = \text{constant}$

$D_3 = 1 = \text{constant}$

$D_4 = x$

$D_5 = - x$ (2)

$D_6 = 1 - x$

$D_7 = 1 - 2x$

$D_8 = - 1 + x$

$D_9 = - 1 + 2x$

These functions are shown in Figure 2. We have investigated all 81 combinations of these payoff functions. C(x) = D(x) happens in 9 cases, C(x) > D(x) in 27 cases, D(x) > C(x) in 27 cases, and the two functions cross each other in 18 cases. Note that all of them have at least two equal parameters because only -1, 0, and 1 are the allowed values of the four parameters.

3. SIMULATIONS

Agent-based computer simulation is the best way to investigate N-person games. We used our own software package *Dilemma* (Szilagyi and Szilagyi, 2000) for the simulation of the 81 games defined in Section 2. *Dilemma* is described in detail in Szilagyi (2012).

We assume that the games are uniform and iterated, the agents have no goals, know nothing about each other, and they cannot refuse participation in any iteration. They are distributed in and fully occupy a finite two-dimensional space and the updates are simultaneous. The initial ratio of cooperators is a variable. We started all simulations with 50% initial cooperation ratios.

The number of agents is 10,000 and initially half of them are cooperators and half of them are defectors randomly distributed in the two-dimensional 100*100 array. Therefore, all simulations were repeated at least three times and the average results were recorded. The repeated simulations always ended with approximately the same results.

The agents are all greedy simpletons who imitate the behavior of the neighbor in their Moore neighborhood – nine neighbors, including themselves - that received the highest reward in the previous iteration. In case of a tie when two or more neighbors with different behaviors received the highest reward, there is no change in their behavior. (The number of neighbors depends on the location of the agent: an agent at an edge of the array and especially in a corner has fewer neighbors than one in the inside.) Some people would consider such agents rational but we will see that they do not always act rationally.

If the neighborhood of such agents extends to the entire society, they will all cooperate or defect immediately at the first iteration because they will all imitate the behavior of those agents who received the higher rewards for their initial action. The situation is completely different, however, for a one-layer deep neighborhood. We must remember that in this case the payoff functions depend on the ratio of cooperators in the immediate neighborhood only.

3.1. C(x) < D (x)

If the cooperators' payoff function is below that for the defectors, then obviously the ratio of cooperators will decrease through subsequent iterations, ending with total defection. This is what happens in all 27 cases.

3.2. C(x) > D (x)

If the cooperators' payoff function is above that for the defectors, then obviously the ratio of cooperators will increase through subsequent iterations, ending with total cooperation. This is what happens in all 27 cases.

3.3. Games with Crossing Payoff Functions

There are 18 different combinations of the payoff functions when they cross each other. In these cases the results are very interesting.

In nine cases the results are stable. This always happens when the C function is below the D function for $x < 0.5$ and above it for $x > 0.5$. Using the notations of Equations (1) and (2), the stable solutions are near total cooperation in three cases (C4-D7, C9-D1, and C9-D5), near total defection in three cases (C1-D7, C8-D7, and C9-D6), and intertwined clusters with approximately equal cooperation and defection for three cases (C4-D6, C8-D5, and C9-D7).

If the C function is above the D function for $x < 0.5$ and below it for $x > 0.5$, the outcomes of the games are wildly fluctuating between two different fixed values of X (C5-D8, Figure 3) or show oscillations with varying amplitudes (C1-D9, Figure 4 and C7-D1, Figure 5). Figure 4 shows more fluctuation than Figure 5 in spite of the fact that these two cases are mirror images of each other because in the first case C is constant while in the second case D is constant. For rational players the intersection point x_{cross} = 0.5 of the two payoff functions is a Nash equilibrium. This is not always true for our greedy simpletons because x refers to the immediate neighbors only while the total ratio of cooperators X represents the ratio of the total number of cooperators in the entire array.

Graphics outputs are shown in two subsequent iterations for C(x) =-x and D(x) = -1+ x in Figures 6 and 7.

3.4. C(x) = D (x)

If the cooperators' payoff function is the same as that is for the defectors, one would expect that the behavior does not change and the ratio of cooperators remains constant through subsequent iterations. This, however, only happens when C(x) = D(x) = 0 or 1. When C(x) = D(x) = -1, the result is total defection. This may be caused by the punishment for any action.

If we start the simulation with a 50% initial cooperation ratio, we receive amazing results. For C(x) = D(x) = - 1, - x, (1 – x), or (1 – 2 x) the final ratio of cooperators in the entire two-dimensional array is equal or very close to zero. For C(x) = D(x) = x, (- 1 + x), or (- 1 + 2 x) the final ratio of cooperators is equal or very close to one.

Are these results surprising? Not at all if we remember that we are dealing here with cellular automata (Wolfram, 1994) where anything can happen. The local interactions cause emergent behavior in the entire array of agents. This behavior cannot be the consequence of the initial random distribution of the agents because repeated simulations always ended with the same results within the range of accuracy. We are dealing here with strong emergence in which the emergent property is irreducible to its individual constituents.

4. CONCLUSION

The results show that the solutions are non-trivial and represent quite irregular emergent behavior when the payoff functions are equal or cross each other.

REFERENCES

Lloyd, A.L., 1995. Computing bouts of the prisoner's Dilemma. Scientific American, 272(6): 110-115.

Nowak, M.A. and R.M. May, 1992. Evolutionary games and spatial chaos. Nature, 359: 826-829.

Szilagyi, M.N., 2012. Investigation of N-person games by agent-based modeling. Complex Systems, 21: 201-243.

Szilagyi, M.N. and I. Somogyi, 2008. Agent-based simulation of N-person games with crossing payoff functions. Complex Systems, 17: 427-439.

Szilagyi, M.N. and Z.C. Szilagyi, 2000. A tool for simulated social experiments. Simulation, 74(1): 4-10.

Wolfram, S., 1994. Cellular automata and complexity: Collected papers. Reading, MA: Addison-Wesley.

Zhao, J., F. Szidarovszky and M.N. Szilagyi, 2007. Finite neighborhood binary games: A structural study. Journal of Artificial Societies and Social Simulation. [Accessed 10/3/3].

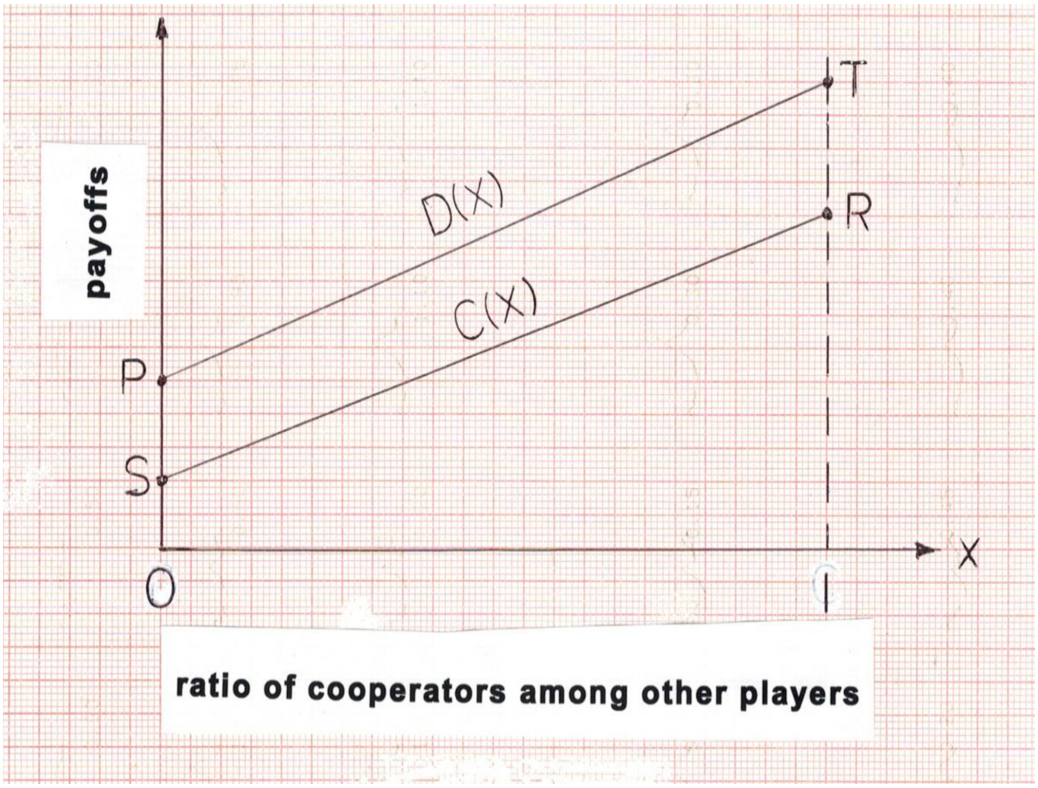

Figure-1. The payoff functions. Here x is the ratio of cooperators among the eight neighbors.

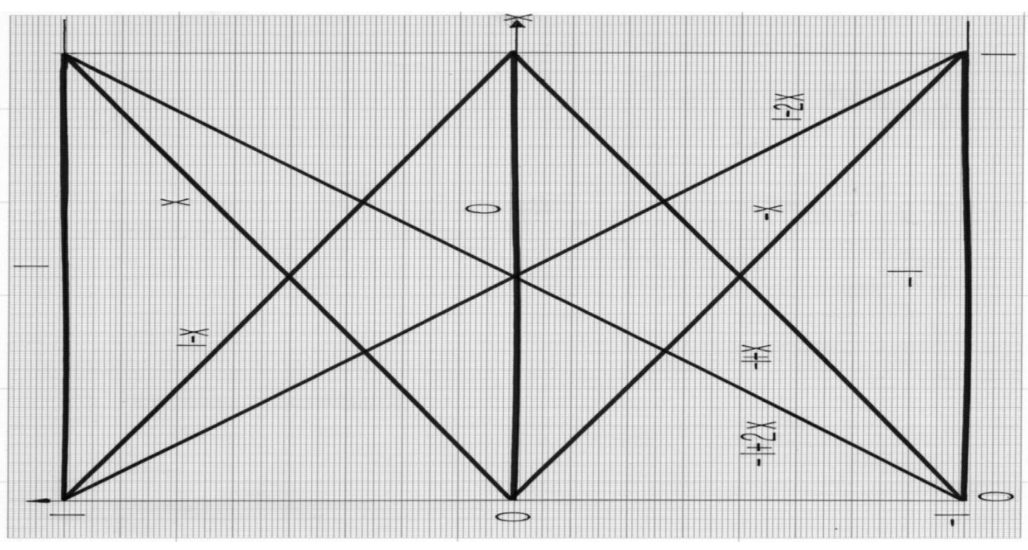

Figure-2. The nine different payoff functions investigated in this paper.

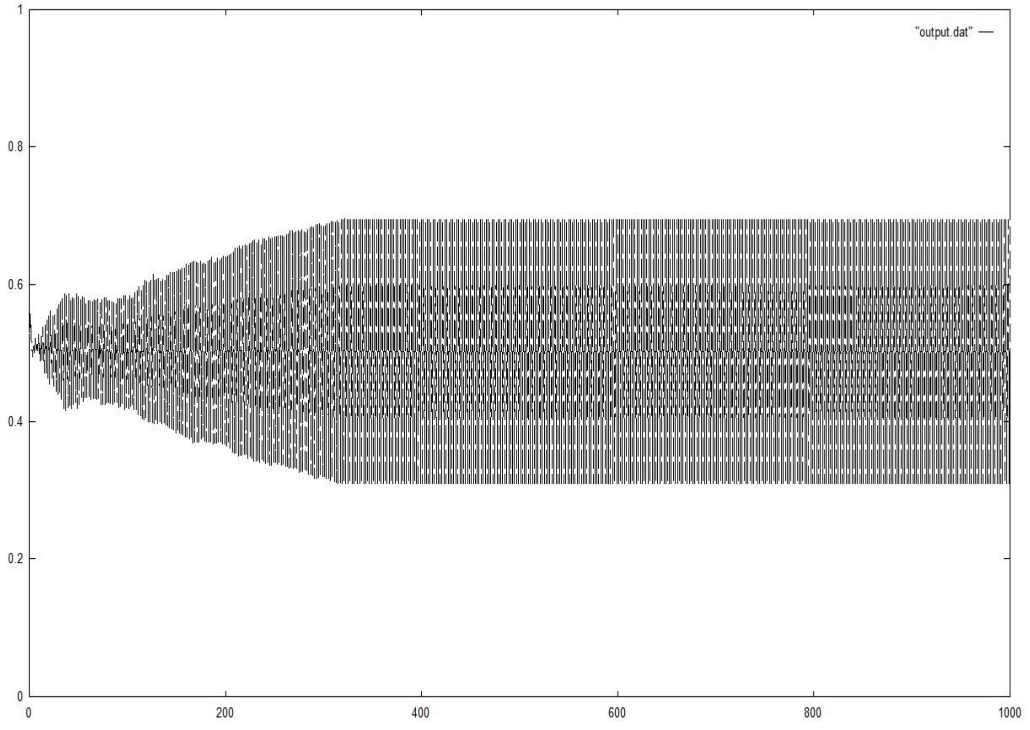

Figure-3. The total ratio of cooperators as a function of the iterations when C(x) = - x and D(x) = - 1 + x. The solution oscillates around X = 0.5 where X is the total ratio of cooperators.

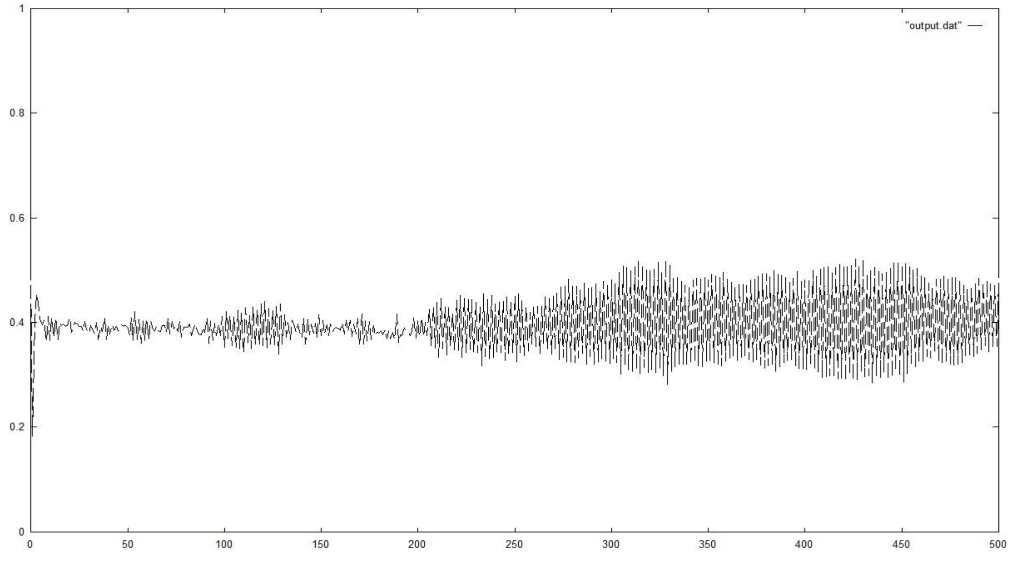

Figure-4. The total ratio of cooperators as a function of the iterations when C(x) = 0 and D(x) = - 1 + 2 x. The solution oscillates around X = 0.4 where X is the total ratio of cooperators.

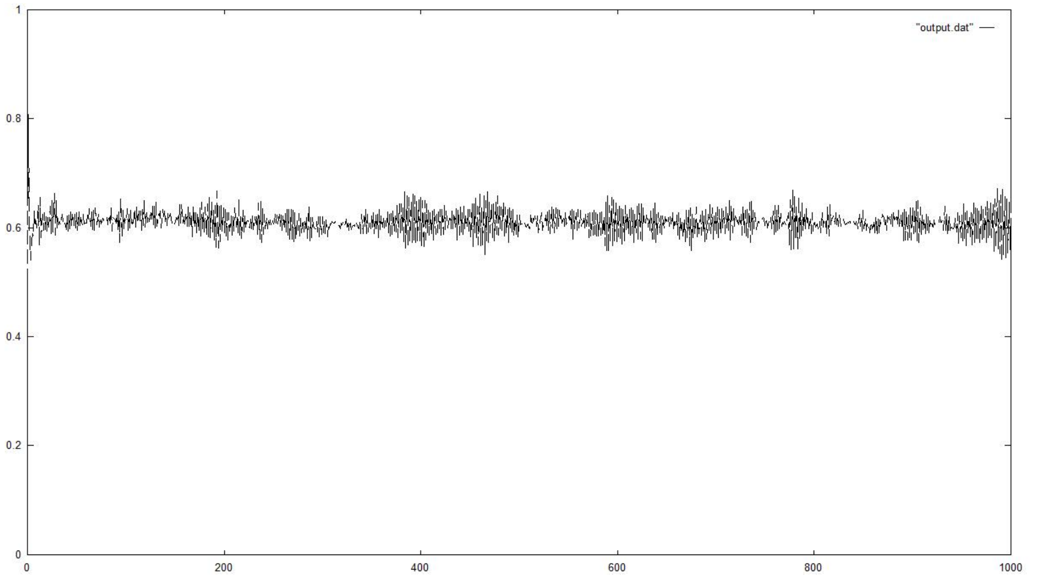

Figure-5. The chaotic behavior of the total ratio of cooperators as a function of the iterations when C(x) = 1 − 2 x and D(x) = 0. The solution oscillates around X = 0.6 where X is the total ratio of cooperators.

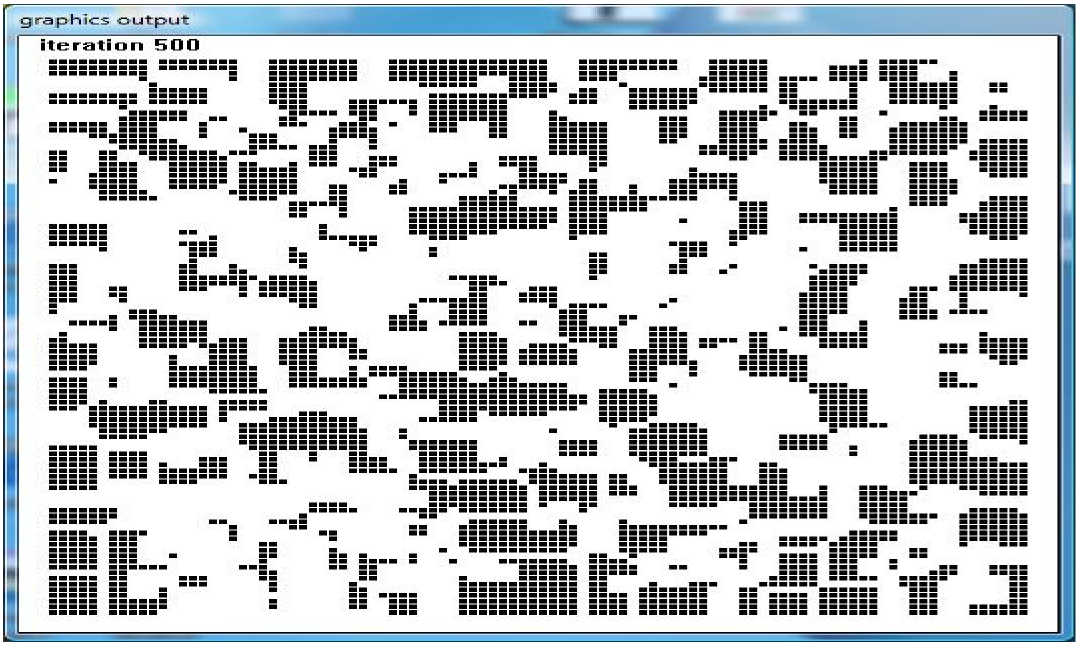

Figure-6. The graphics output of the game shown in Figure 3 after the 500th iteration. The black spots represent cooperators, the white ones represent defectors.

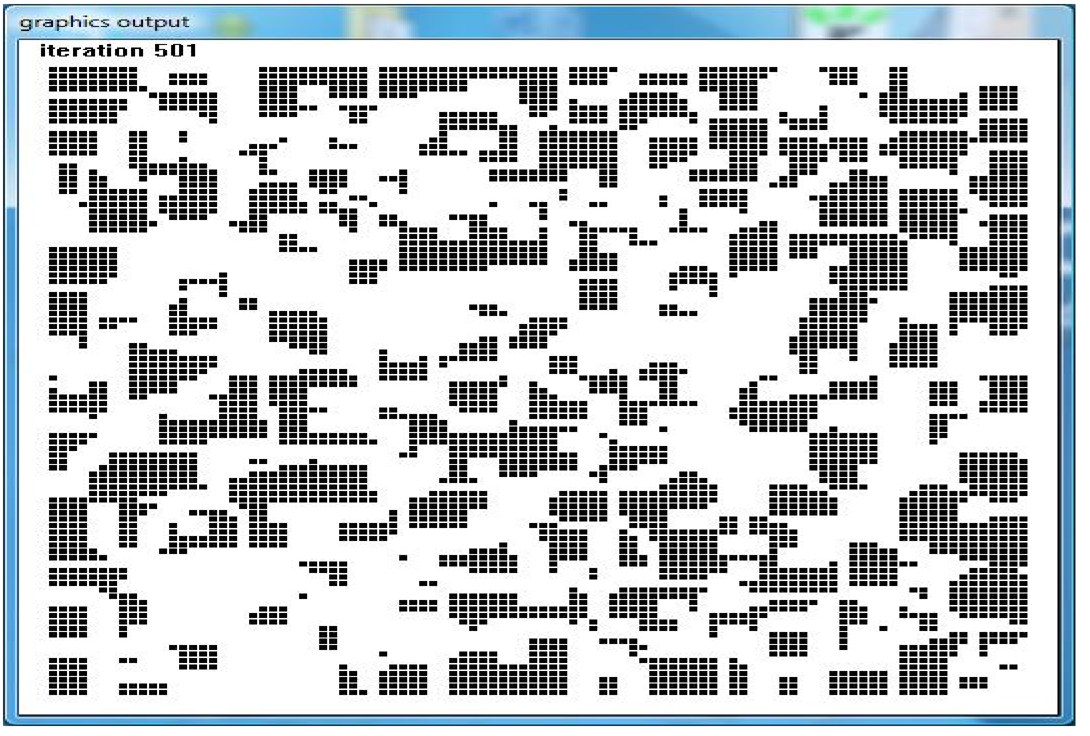

Figure-7. The graphics output of the game shown in Figure 3 after the 501st iteration. The black spots represent cooperators, the white ones represent defectors.