

## Games Review

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## AGENT-BASED SIMULATION OF CONTRIBUTION GAMES

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### ABSTRACT

*Agent-based simulation was performed for various contribution games. The amount of contribution can be constant or variable and the first few contributions are less, more, or equally important than the last few. We found that the results strongly depend on the participating agents' personalities. Two types of personalities were investigated: Pavlovian and greedy. Our simple formula (Equation 2) provides valuable information about the outcomes of N-person games for Pavlovian agents.*

**Keywords:** Agent-based simulation, N-person games, Contribution game, Public goods game, Prisoners' Dilemma.

### Contribution/ Originality

This study uses a new approach to the study of contribution games and provides new results.

## 1. INTRODUCTION

The contribution game (or public goods game) is an N-person game where the participating agents have two choices: to cooperate by contributing to some public good or use the benefits of other agents' contributions, i.e. defect (Hamburger, 1979). The greater proportion of the agents chooses to contribute, the better is the outcome for everyone. From the point of view of Game Theory, this is an N-person Prisoners' Dilemma like the Tragedy of the Commons (Hardin, 1968). Indeed, if P corresponds to the punishment when no one contributes, R is the reward when all agents contribute, T is the temptation not to contribute when everybody else does, and S is the sucker's payoff for contributing alone, then for this game

$$T > R > P > S \quad (1)$$

As a result of its choice, each agent receives a reward or punishment (payoff) that is dependent on its choice as well as the choices of all the others. The payoff functions are given as two curves: C(x) for contributors and D(x) for defectors where x is the ratio of contributors with respect to the total number of agents. S and R are the end points of the C(x) function; P and T are the end points of the D(x) function. Regardless of what the other agents do, each agent receives a lower payoff for contributing than for refusing to contribute but all agents receive a lower payoff

if all defect than if all contribute. Therefore, the defectors' payoff function  $D(x)$  is above that of the contributors  $C(x)$  (Figure 1).

Theoretical investigations of contribution games are available from the literature (Bagnoli and Lipman, 1989; Gale, 2001; Lockwood and Thomas, 2002; Duffy *et al.*, 2007; Matthews, 2013). Agent-based simulation will be used in this paper to investigate these games. The simulations were performed by the use of our agent-based simulation tool (Szilagyi and Szilagyi, 2000) that is suitable for any iterated N-person game with a wide range of user-defined parameters. The agents are stochastic learning cellular automata situated on a two-dimensional grid and their number is equal to 10,000. We consider the contribution game as an iterated game. The aggregate proportion of contributors changes over subsequent iterations. At each iteration, every agent chooses an action according to the payoff received for its previous action. The updating occurs simultaneously for all agents. With each iteration, the software tool draws the array of agents in a window on the computer's screen, with each agent in the array colored according to its most recent action. The experimenter can view and record the evolution of the society of agents as it changes in time. After a certain number of iterations the proportion of cooperators usually stabilizes to either a constant value or oscillates around such a value. The outcome of any N-person game strongly depends on the agents' personalities and the depth of the agents' neighborhood (Szilagyi, 2003a). In this paper, two personality types will be used: Pavlovian and greedy.

For Pavlovian agents the probability of choosing the previously chosen action again changes by an amount proportional to the reward/penalty for the previous action. If an action is followed by a reward, then the tendency of the agent to produce that particular action is reinforced. For such agents we considered the case when the neighborhood is the entire collective of agents and also the case of the one-layer deep neighborhood.

Greedy agents imitate the choice of their neighbor who received the highest reward for its previous action. For such agents if the neighborhood extends to the entire collective of agents, they will all defect immediately at the first iteration because the defectors always receive a higher reward than the contributors. Therefore, we considered only the case when each agent looks at its immediate neighbors only (one-layer deep neighborhood).

In summary, we performed the following simulations for each case:

- a) Pavlovian agents when the neighborhood is the entire collective of agents. In this case the validity of Equation 2 was also checked.
- b) Pavlovian agents when the neighborhood is one layer deep.
- c) Greedy agents when the neighborhood is one layer deep.

Each simulation was performed for the entire range of the initial ratio of cooperators ( $0 < x_0 < 1$ ).

## 2. SIMULATIONS

### 2.1. The Amount of Contribution is Constant and the Contributions are Equally Important

Let us first assume that 1) the amount of contribution (the difference between the two payoff functions) is constant, 2) the first contributions are as important as the last ones (both payoff functions are linear). We investigated this case for Pavlovian agents in [Szilagyi \(2003b\)](#). These assumptions correspond to the case when the amount of contribution is constant and the contributions are equally important.

This is a clear case of the N-person Prisoners' Dilemma. Various aspects of this game have been investigated in the literature ([Weil, 1966](#); [Hamburger, 1973](#); [Schelling, 1973](#); [Anderson, 1974](#); [Goehring and Kahan, 1976](#); [Nowak and May, 1992](#); [Szilagyi and Szilagyi, 2002](#)). This game has great practical importance because its study may lead to a better understanding of the factors stimulating or inhibiting cooperative behavior within social systems.

We have shown ([Szilagyi, 2012](#)) that for Pavlovian agents if the neighborhood is the entire collective of agents and the cooperators receive the same total payoff as the defectors, i.e.,

$$x C(x) = (1 - x) D(x), \quad (2)$$

an equilibrium occurs. If this equation has two real solutions  $x_1$  and  $x_2$  ( $x_2 > x_1$ ) in the interval  $0 < x < 1$ , then  $x_1$  is a stable attractor and  $x_2$  is an unstable repulsor. When the initial cooperation ratio is below  $x_2$ , the solution of the game converges toward  $x_1$  as an oscillation while it stabilizes exactly when the initial cooperation ratio is above  $x_2$ .

The payoff functions shown in Figure 1 are

$$C(x) = 2x - 1 \quad \text{and} \quad D(x) = 2x - 0.5 \quad (3)$$

Substituting these into Equation (2) we obtain the quadratic equation

$$4x^2 - 3.5x + 0.5 = 0 \quad (4)$$

Its solutions are  $x_1 = 0.1798$  and  $x_2 = 0.6952$ . Our simulations confirmed this result. We also investigated the case when the neighborhood is one layer deep ([Szilagyi, 2003b](#)).

For greedy agents, however, Equation (2) is not applicable. For the case of the one-layer-deep neighborhood the result is shown in Figure 2. The horizontal axis represents the number of iterations, the vertical axis shows the ratio of contributors  $x$ . The different curves correspond to different values of the initial ratio of contributors  $x_0$ . As we see, the ratio of contributors jumps down at the first iteration but then gradually increases to a high value (above 0.8). It is very strange that the lower is the initial value the higher is the final result  $x_{\text{final}}$  (the curves all cross each other). The results are shown in the following table:

$x_0$	$x_{\text{final}}$
0.1	0.96
0.2	0.96
0.6	0.87
0.9	0.83

These results completely coincide with those obtained in [Szilagyi \(2003b\)](#).

**2.2. The Amount of Contribution is Variable but the Contributions are Equally Important**

We investigated two extreme cases. In the first, the amount of contribution grows when the number of contributors increases (Figure 3). In the second it is the other way around (Figure 4).

The payoff functions shown in Figure 3 are

$$C(x) = 2x - 1 \quad \text{and} \quad D(x) = 2.5x - 1 \quad (5)$$

Substituting these into Equation (2) we obtain the quadratic equation

$$4.5x^2 - 4.5x + 1 = 0 \quad (6)$$

Its solutions are  $x_1 = 0.3333$  and  $x_2 = 0.6667$ .

For Pavlovian agents when the neighborhood is the entire collective of agents the results of the simulation are shown in Figure 5. One can see that Equation 2 is exactly satisfied. For Pavlovian agents when the neighborhood is one layer deep the simulations lead to a different picture (Figure 6). The final results are as follows:

$x_0$	$x_{final}$
0.00	0.59
0.30	0.60
0.45	0.60
0.55	0.65
0.65	0.82
0.70	0.89

For greedy agents when the neighborhood is one layer deep the results are similar to the previous case (Figure 7). The numerical final values are:

$x_0$	$x_{final}$
0.1	0.98
0.5	0.94
0.9	0.92

The payoff functions shown in Figure 4 are

$$C(x) = 2x - 1 \quad \text{and} \quad D(x) = 1.5x - 0.5 \quad (7)$$

Substituting these into Equation (2) we obtain the quadratic equation

$$3.5x^2 - 3x + 0.5 = 0 \quad (8)$$

Its solutions are  $x_1 = 0.2265$  and  $x_2 = 0.6306$ .

For Pavlovian agents when the neighborhood is the entire collective of agents the results of the simulation are shown in Figure 8. One can see that Equation 2 is exactly satisfied again. For Pavlovian agents when the neighborhood is one layer deep the simulations are shown in Figure 9. The final results are as follows:

$x_0$	$x_{final}$
0.0	0.31
0.5	0.32
0.6	0.80

0.7 0.97

For greedy agents when the neighborhood is one layer deep the results are different now (Figure 10). The final numerical values are approaching total cooperation for any initial ratio of contributors:

$x_0$	$x_{final}$
0.1	1.00
0.5	0.99
0.9	0.98

### 2.3. The Amount of Contribution is Constant but the Values of the Contributions are Variable

Let us first consider the case when the amount of contribution is constant but the first few contributions are more important than the last few (Hamburger, 1979). The payoff functions shown in Figure 11 are

$$C(x) = -2x^2 + 4x - 1 \quad \text{and} \quad D(x) = -2x^2 + 4x - 0.5 \quad (9)$$

Substituting these into Equation (2) we obtain the cubic equation

$$-4x^3 + 10x^2 - 5.5x + 0.5 = 0 \quad (10)$$

It has three real solutions. The two solutions in the interval  $0 < x < 1$  are  $x_1 = 0.113$  and  $x_2 = 0.628$ .

For Pavlovian agents when the neighborhood is the entire collective of agents the results of the simulation are shown in Figure 12. We can see that Equation 2 is satisfied for the region  $0 < x < 0.5$  but for  $x_0 = 0.6$  the final value is  $x_{final} = 0.429$  instead of 0.113.

For Pavlovian agents when the neighborhood is one layer deep the simulations are shown in Figure 13. The final results are as follows:

$x_0$	$x_{final}$
0.0	0.19
0.5	0.26
0.6	0.55
0.65	0.69
0.8	0.94
0.9	0.99

For greedy agents when the neighborhood is one layer deep the results are different now (Figure 14). The solutions wildly fluctuate. For the case of  $x_0 = 0.2$  we continued the simulation for 5000 iterations (Figure 15). After the first iteration, the value of  $x$  almost reaches zero, then shoots up to 0.5, after that starts a wild fluctuation between  $x = 0.35$  and  $x = 0.47$ . Figure 16 shows the graphics output after the 500th iteration. The black dots represent contributors, the white ones free riders. Looking at the 501st iteration (Figure 17), we see the large difference. The final numerical values are:

$x_0$	$x_{final}$
0.1	0.96
0.15	0.96

0.2 0.43 (wild fluctuations around this value)

0.5 0.42 (wild fluctuations around this value)

0.9 0.39 (wild fluctuations around this value)

We will now investigate the case when the amount of contribution is constant but the contributions are of little use if there are too few of them [1]. The payoff functions shown in Figure 18 are

$$C(x) = 2x^2 - 1 \quad \text{and} \quad D(x) = 2x^2 - 0.5 \quad (11)$$

Substituting these into Equation (2) we obtain the cubic equation

$$4x^3 - 2x^2 - 1.5x + 0.5 = 0 \quad (12)$$

It has three real solutions. The two solutions in the interval  $0 < x < 1$  are  $x_1 = 0.287$  and  $x_2 = 0.776$ .

For Pavlovian agents when the neighborhood is the entire collective of agents the results of the simulation are shown in Figure 19. Equation 2 is strictly satisfied for the entire region  $0 < x < 1$ .

For Pavlovian agents when the neighborhood is one layer deep the simulations are shown in Figure 20. The results are quite interesting: for the region  $0 < x < x_2$  the solutions oscillate around  $x_{\text{final}} = 0.32$ . When the initial ratio of contributors is 0.8, the same final value is reached only after 5000 iterations (Figure 21). For greedy agents when the neighborhood is one layer deep the results are shown in Figure 22. The values of  $x$  first go deeply down again and by the 50<sup>th</sup> iteration they all reach their final values above 0.9 and again the smaller the initial value  $x_0$  the higher the final value  $x_{\text{final}}$ . Wild fluctuations were not observed in this case.

### 3. CONCLUSION

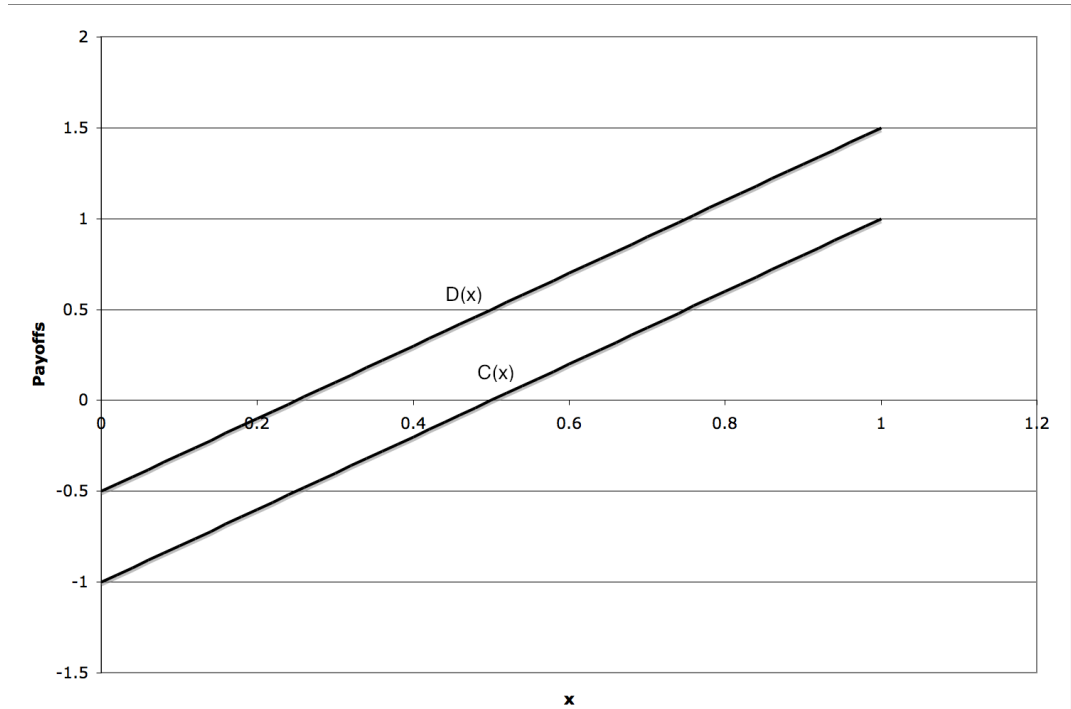
Agent-based simulation was performed for various contribution games. The amount of contribution can be constant or variable and the first few contributions are less, more, or equally important than the last few. We found that the results strongly depend on the participating agents' personalities. For Pavlovian agents when the neighborhood is the entire collective of agents we found that the value of the attractor  $x_1$  is highest when the contributions are equally important and the amount of contribution grows when the number of contributors increases. The value of  $x_1$  is lowest when the amount of contribution is constant but the first few contributions are more important than the last few. For Pavlovian agents when the neighborhood is one layer deep the final solutions  $x_{\text{final}}$  are usually higher than  $x_1$  and strongly depend on the initial value  $x_0$ . In the case when the amount of contribution is constant but the first few contributions are less important than the last few, the solutions start to oscillate around  $x_{\text{final}}$  that is a little higher than  $x_1$  and this oscillation starts much slower than when the neighborhood is the entire collective of agents. For greedy agents when the neighborhood is one layer deep the results are less regular. The final values of  $x$  are usually very high: close to contribution by all agents. Interestingly, the smaller the initial value  $x_0$  the higher the final value  $x_{\text{final}}$ . The results are different for the case when the amount of contribution is constant but the first few contributions are more important than the last few. We found wild fluctuations in this case. This study clearly showed that the behavior of the agents in N-person games is strongly

dependent on their personalities. Our simple formula (Equation 2) provides valuable information about the outcomes of N-person games for Pavlovian agents.

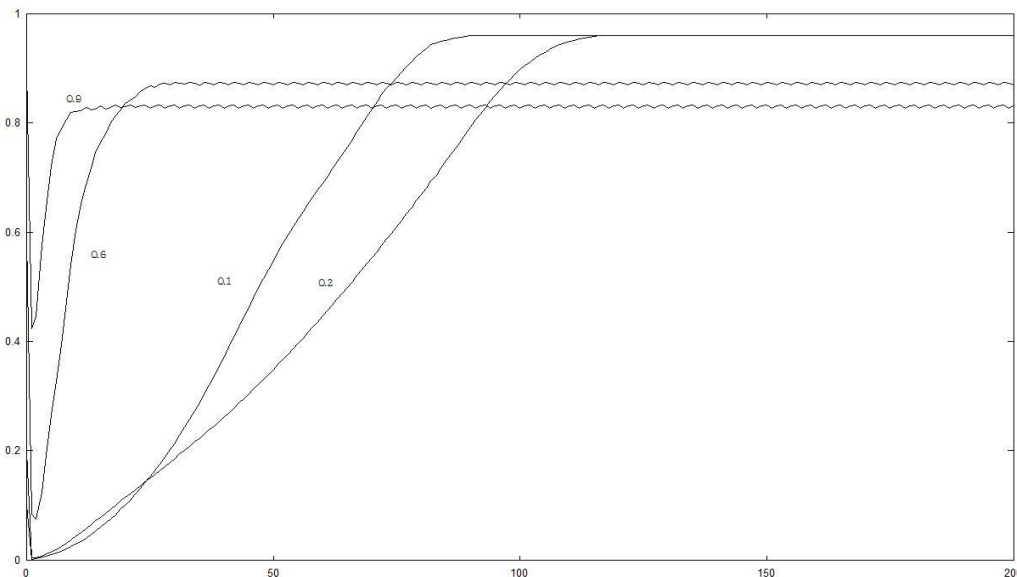
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FIGURE CAPTIONS

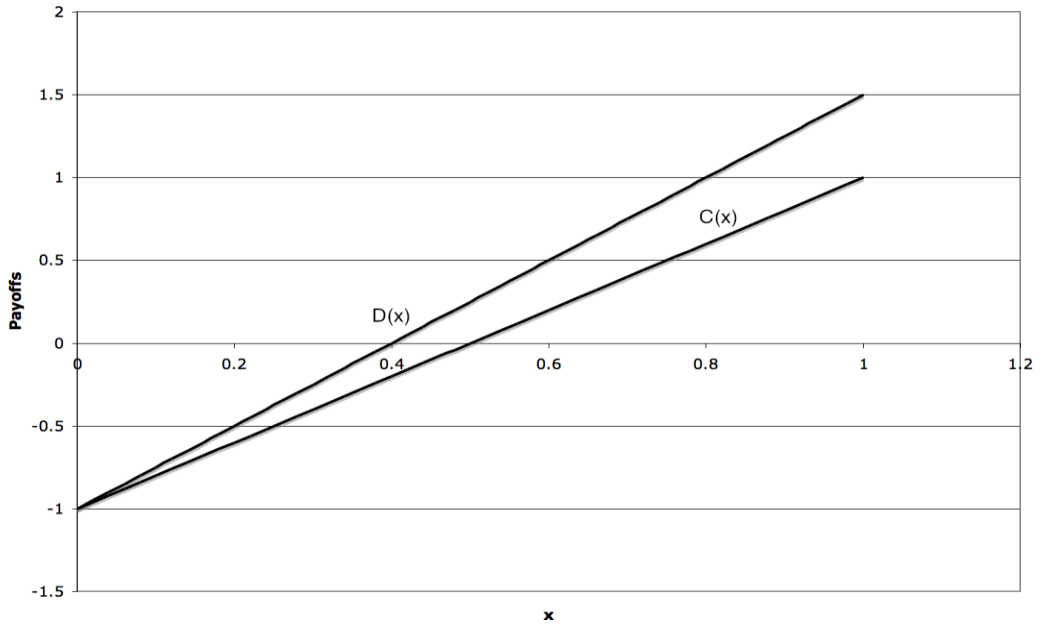


**Figure-1.** Payoff (reward/penalty) functions for contributors (C) and defectors (D) when the amount of contribution is constant and the contributions are equally important.  $C(x) = 2x - 1$  and  $D(x) = 2x - 0.5$ . The horizontal axis ( $x$ ) represents the ratio of the number of contributors to the total number of agents; the vertical axis is the reward/penalty provided by the environment.

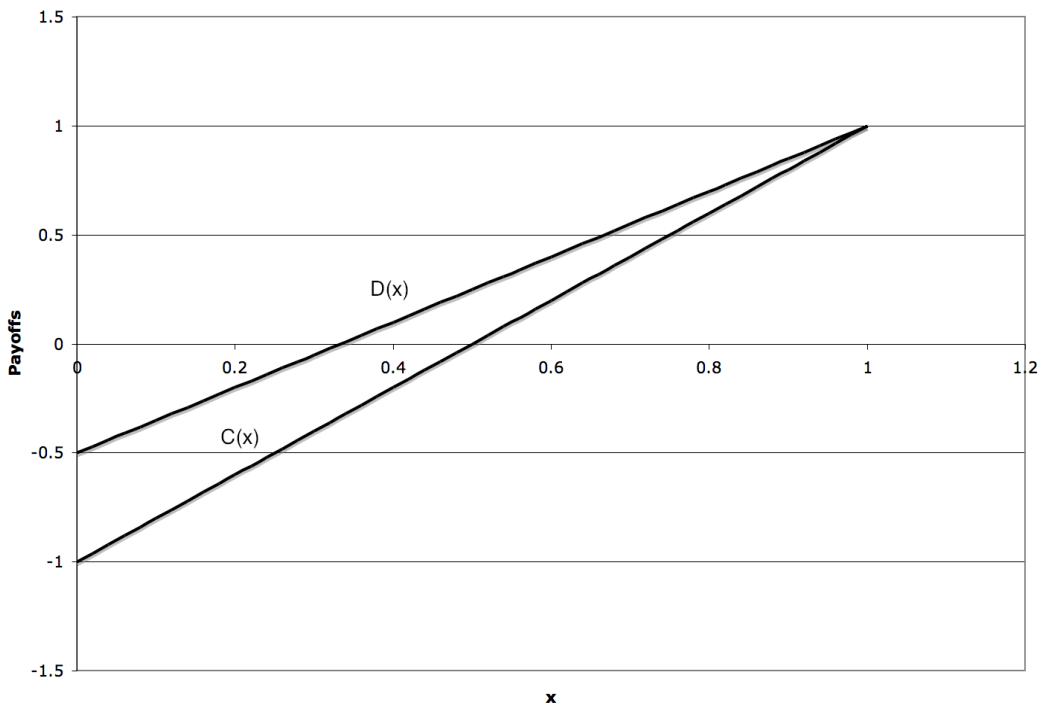


**Figure-2.** Evolution of the game for the case when all agents are greedy, Figure 1 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.

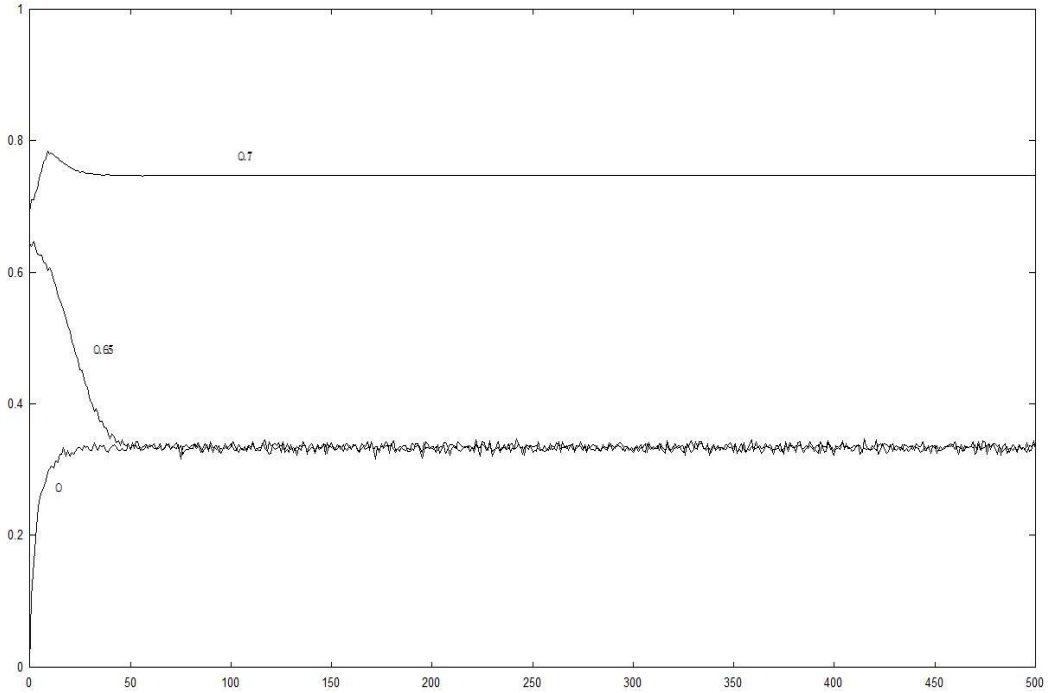




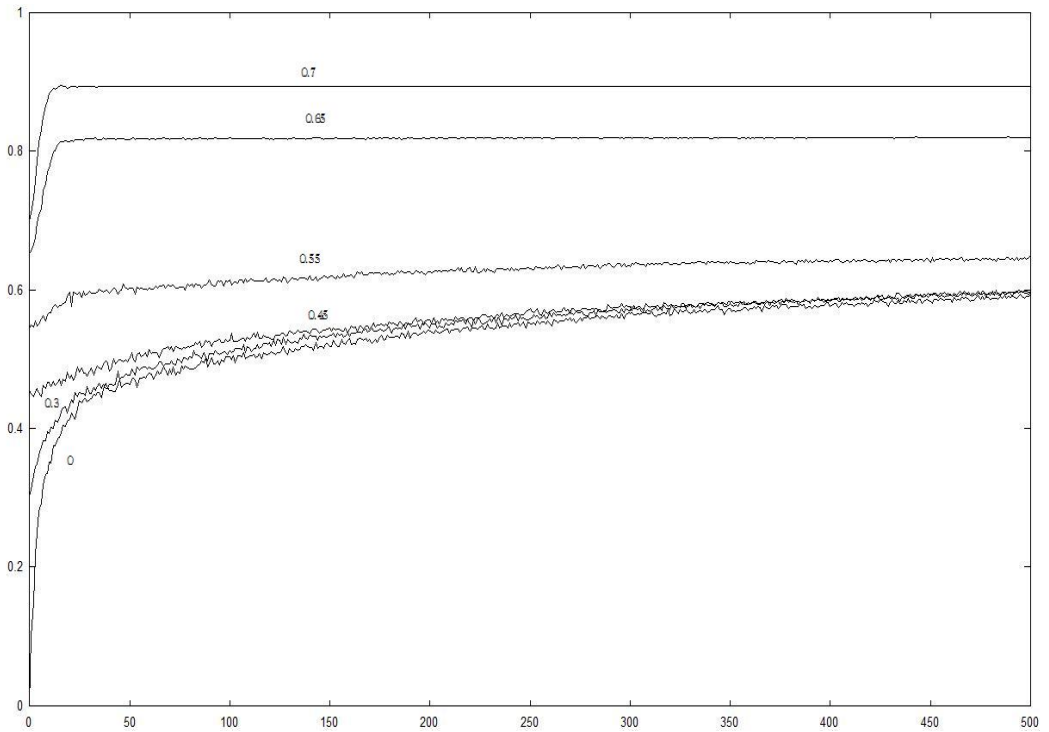
**Figure-3.** Payoff (reward/penalty) functions for contributors (C) and defectors (D) when the amount of contribution grows when the number of contributors increases.  $C(x) = 2x - 1$  and  $D(x) = 2.5x - 1$ . The horizontal axis (x) represents the ratio of the number of contributors to the total number of agents; the vertical axis is the reward/penalty provided by the environment.



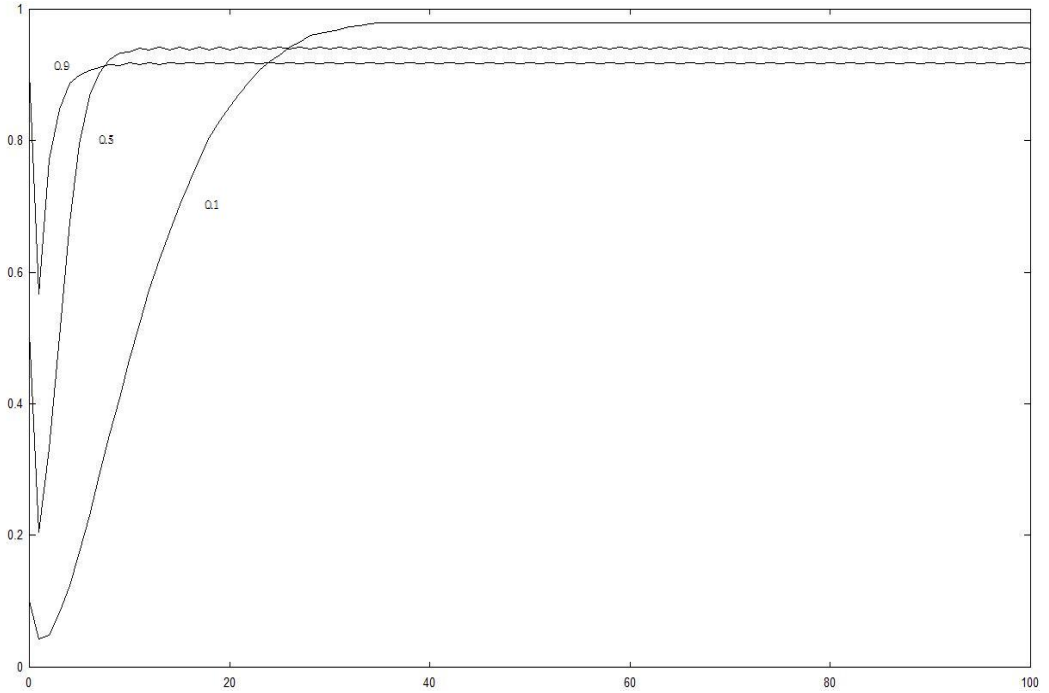
**Figure-4.** Payoff (reward/penalty) functions for contributors (C) and defectors (D) when the amount of contribution decreases when the number of contributors increases.  $C(x) = 2x - 1$  and  $D(x) = 1.5x - 0.5$ . The horizontal axis (x) represents the ratio of the number of contributors to the total number of agents; the vertical axis is the reward/penalty provided by the environment.



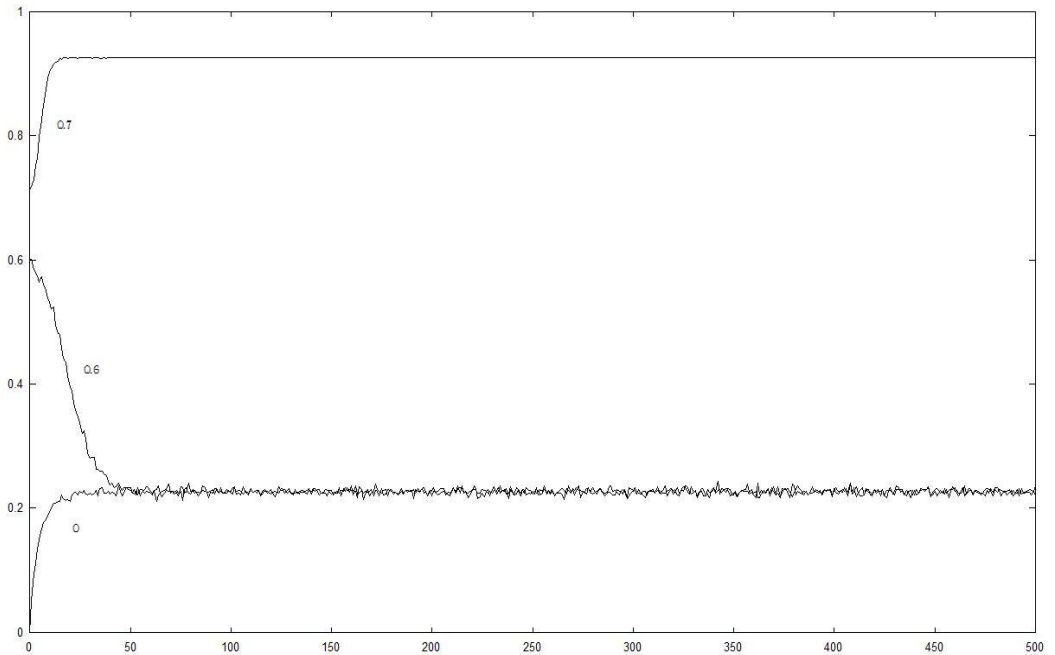
**Figure-5.** Evolution of the game for the case when all agents are Pavlovian, Figure 3 gives the payoff curves, and the neighborhood is the entire collective of agents. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



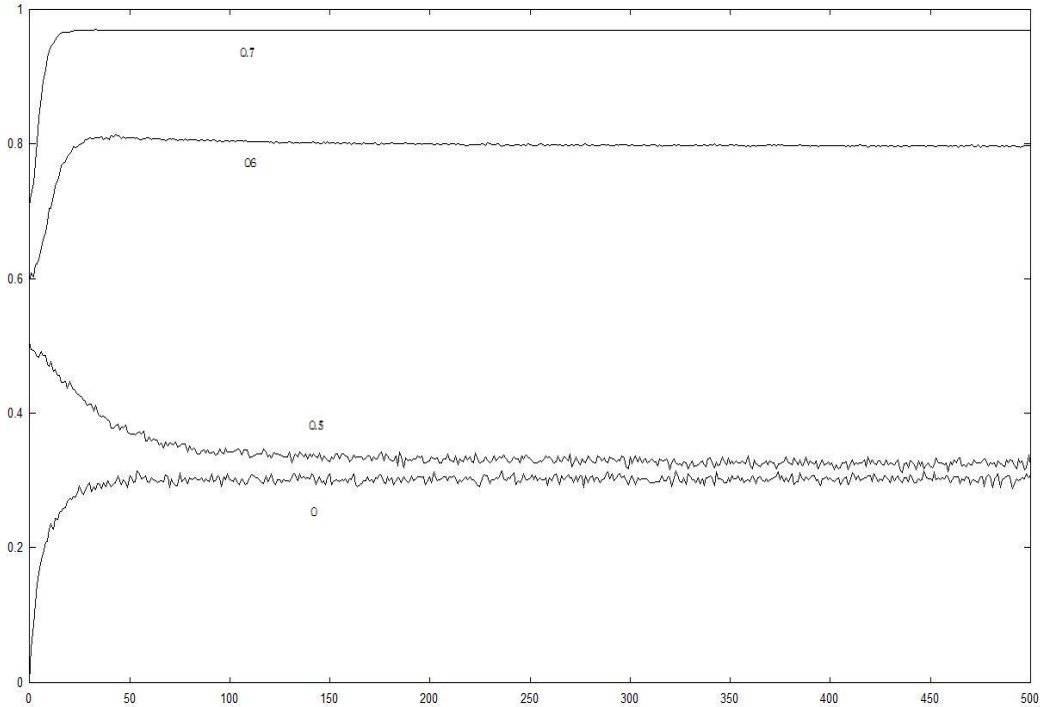
**Figure-6.** Evolution of the game for the case when all agents are Pavlovian, Figure 3 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



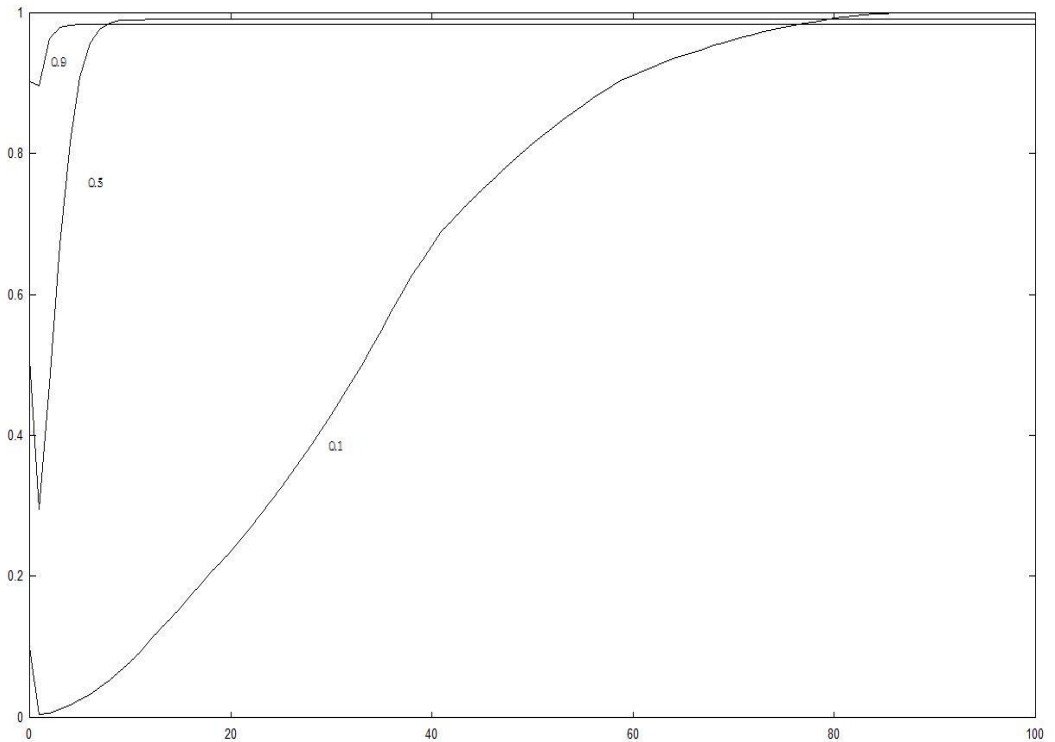
**Figure-7.** Evolution of the game for the case when all agents are greedy, Figure 3 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



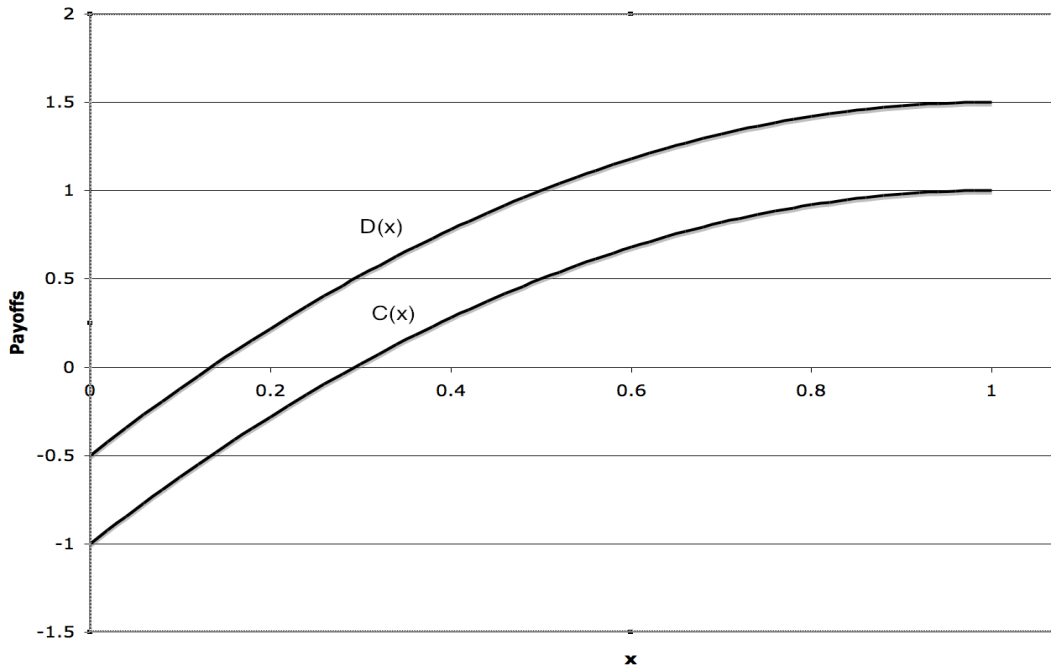
**Figure-8.** Evolution of the game for the case when all agents are Pavlovian, Figure 4 gives the payoff curves, and the neighborhood is the entire collective of agents. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



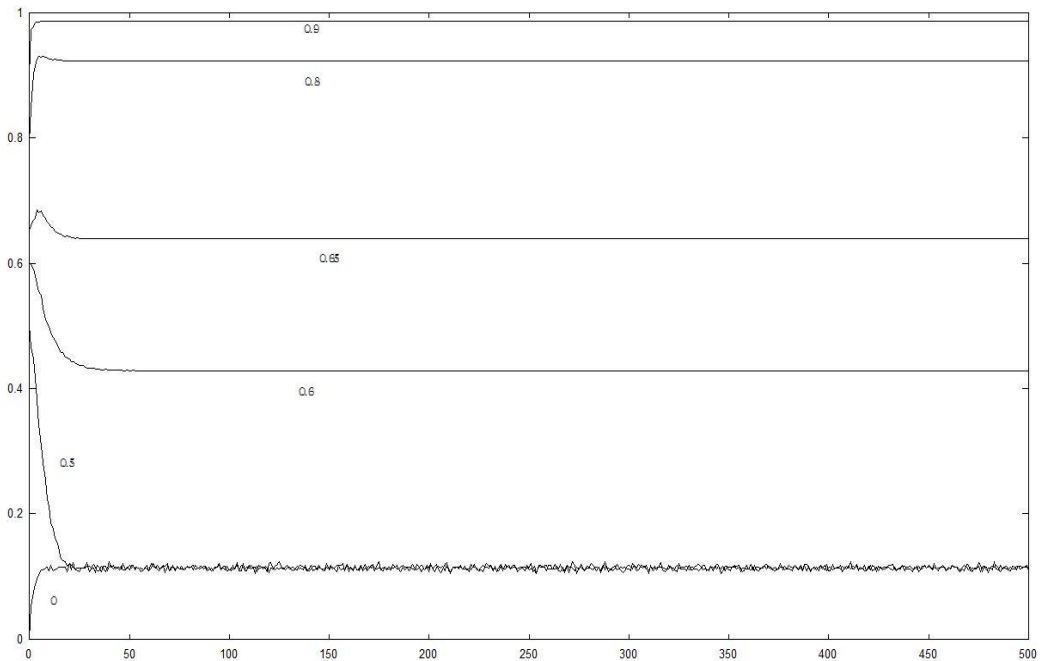
**Figure-9.** Evolution of the game for the case when all agents are Pavlovian, Figure 4 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



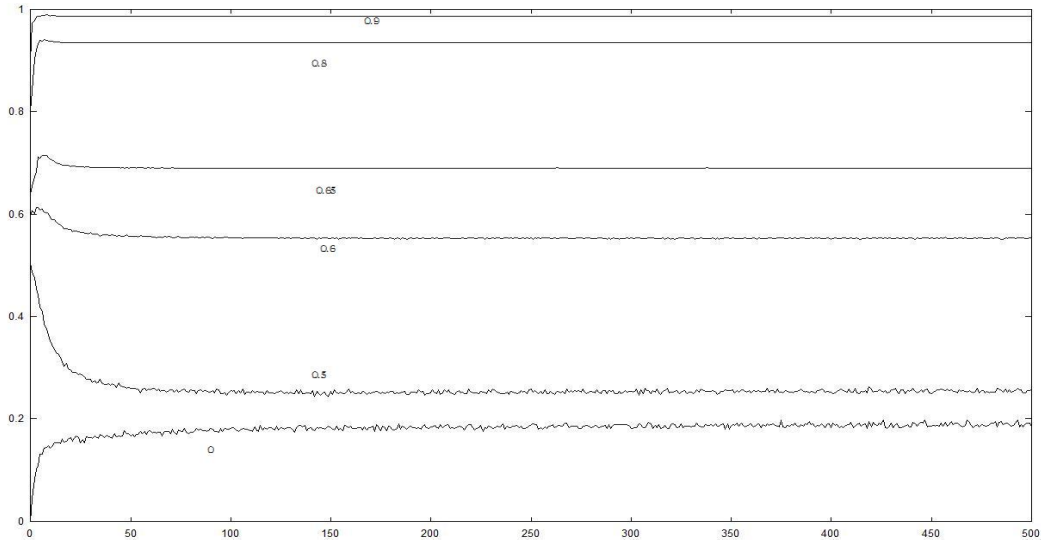
**Figure-10.** Evolution of the game for the case when all agents are greedy, Figure 4 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



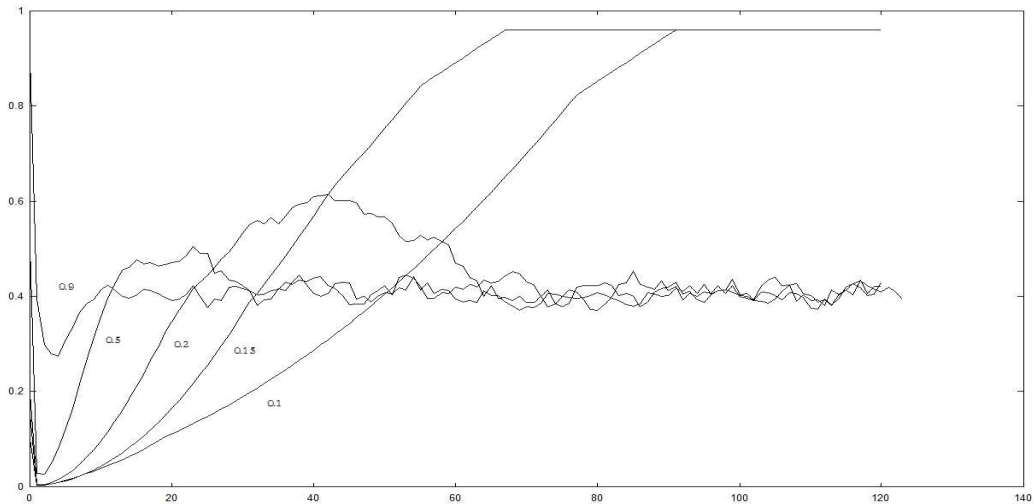
**Figure-11.** Payoff (reward/penalty) functions for contributors (C) and defectors (D) when the amount of contribution is constant but the first few contributions are more important than the last few.  $C(x) = -2x^2 + 4x - 1$  and  $D(x) = -2x^2 + 4x - 0.5$ . The horizontal axis ( $x$ ) represents the ratio of the number of contributors to the total number of agents; the vertical axis is the reward/penalty provided by the environment.



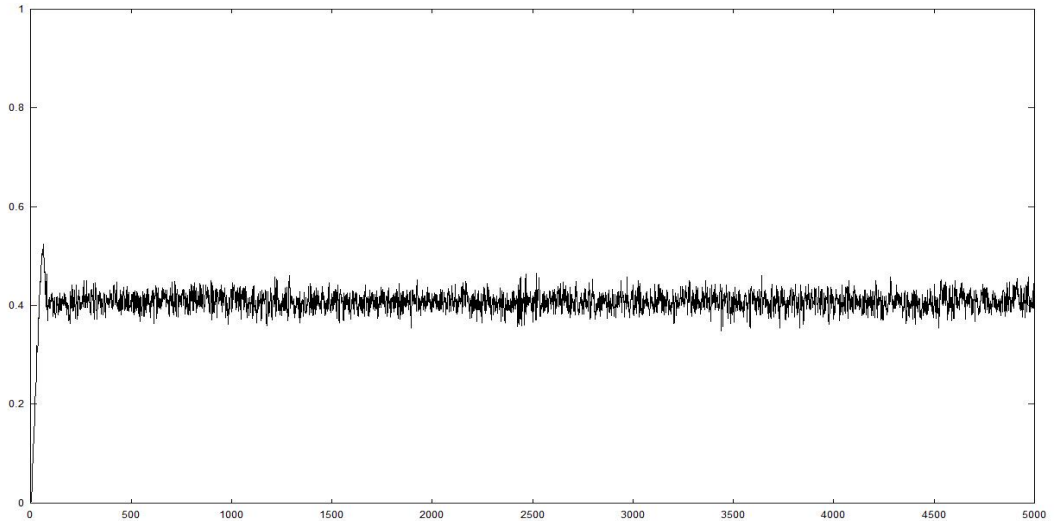
**Figure-12.** Evolution of the game for the case when all agents are Pavlovian, Figure 11 gives the payoff curves, and the neighborhood is the entire collective of agents. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



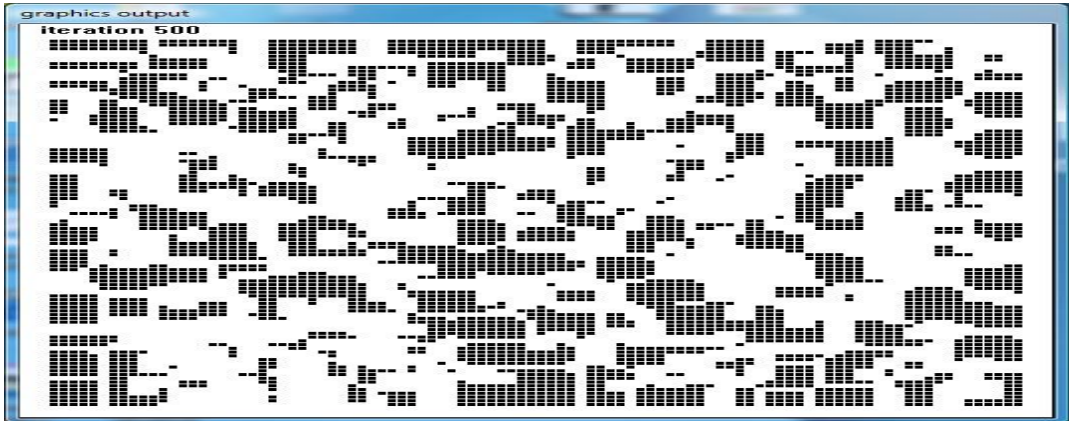
**Figure-13.** Evolution of the game for the case when all agents are Pavlovian, Figure 11 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



**Figure-14.** Evolution of the game for the case when all agents are greedy, Figure 11 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



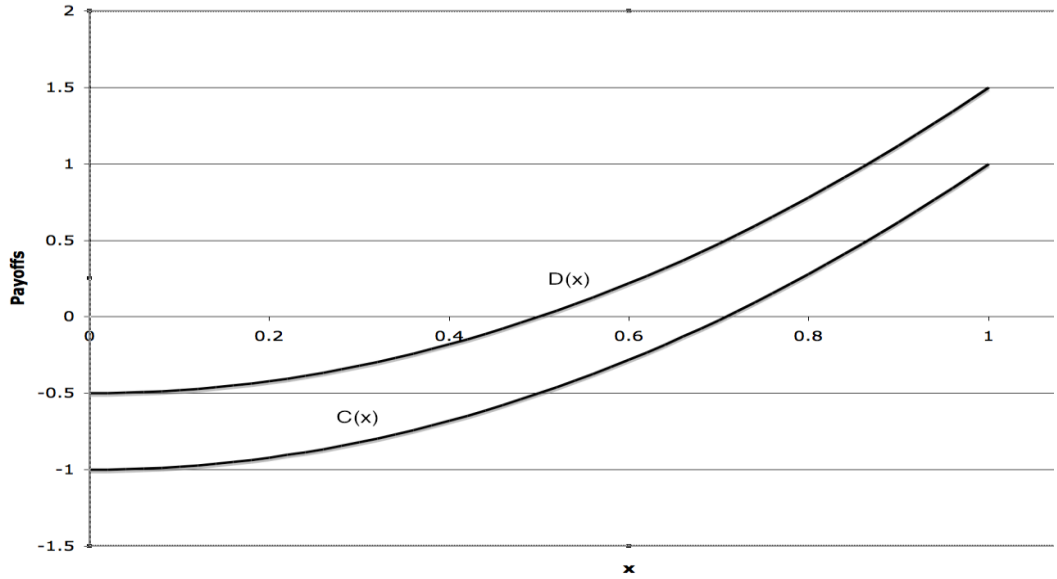
**Figure-15.** Evolution of the game for the case when all agents are greedy, Figure 11 gives the payoff curves, and the neighborhood is one layer deep. The graph shows the proportion of contributing agents as a function of the number of iterations for 5000 iterations. The initial cooperation ratio is  $x_0 = 0.2$ .



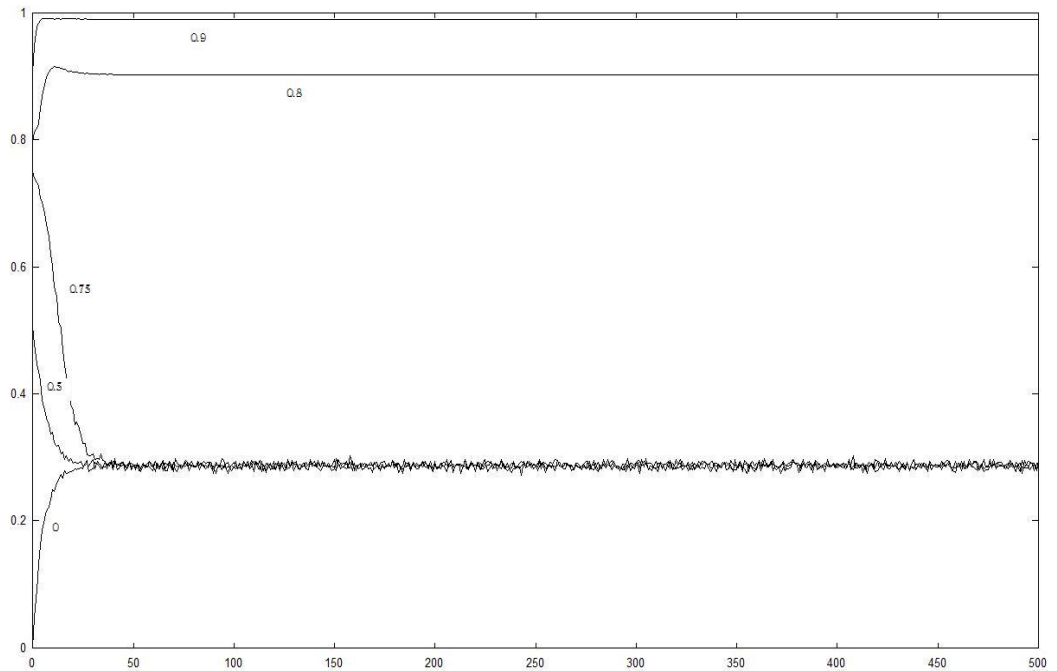
**Figure-16.** A snapshot of the graphics output of the simulation shown in Figure 15 after the 500th iteration. The black dots represent contributors, the white ones are free riders.



**Figure-17.** A snapshot of the graphics output of the simulation shown in Figure 15 after the 501st iteration. The black dots represent contributors, the white ones are free riders.

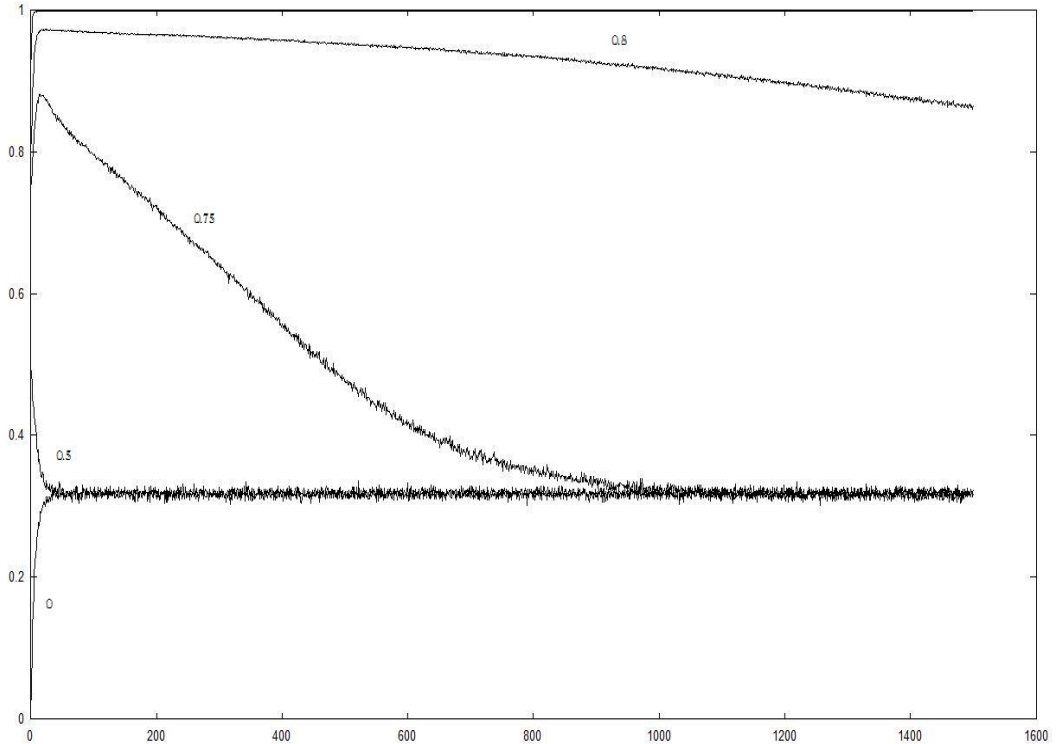


**Figure-18.** Payoff (reward/penalty) functions for contributors (C) and defectors (D) when the amount of contribution is constant but the first few contributions are less important than the last few.  $C(x) = 2x^2 - 1$  and  $D(x) = 2x^2 - 0.5$ . The horizontal axis ( $x$ ) represents the ratio of the number of contributors to the total number of agents; the vertical axis is the reward/penalty provided by the environment.

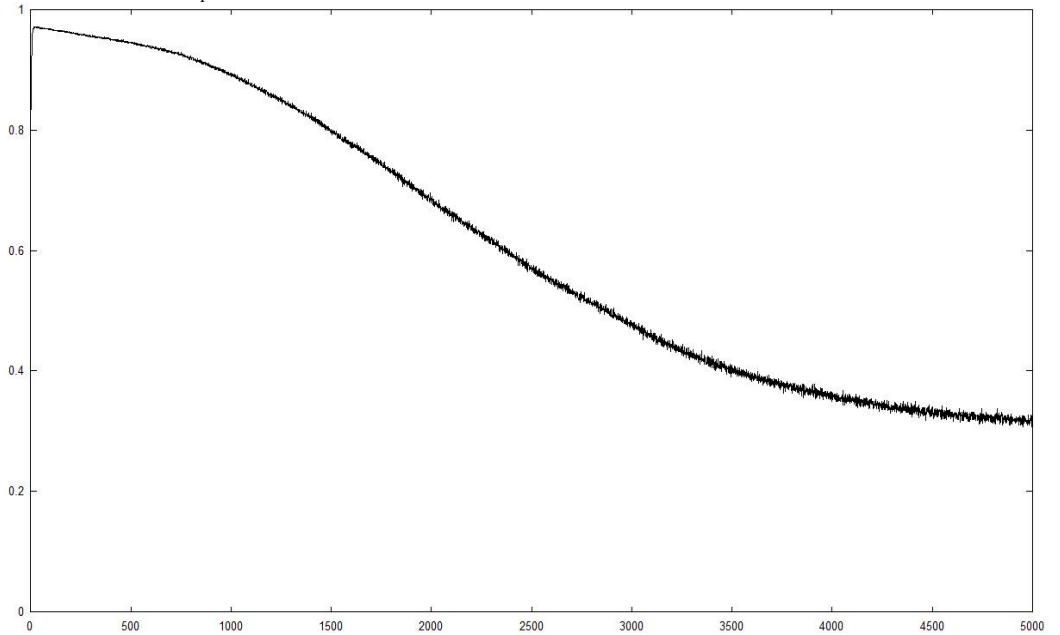


**Figure-19.** Evolution of the game for the case when all agents are Pavlovian, Figure 18 gives the payoff curves, and the neighborhood is the entire collective of agents. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.

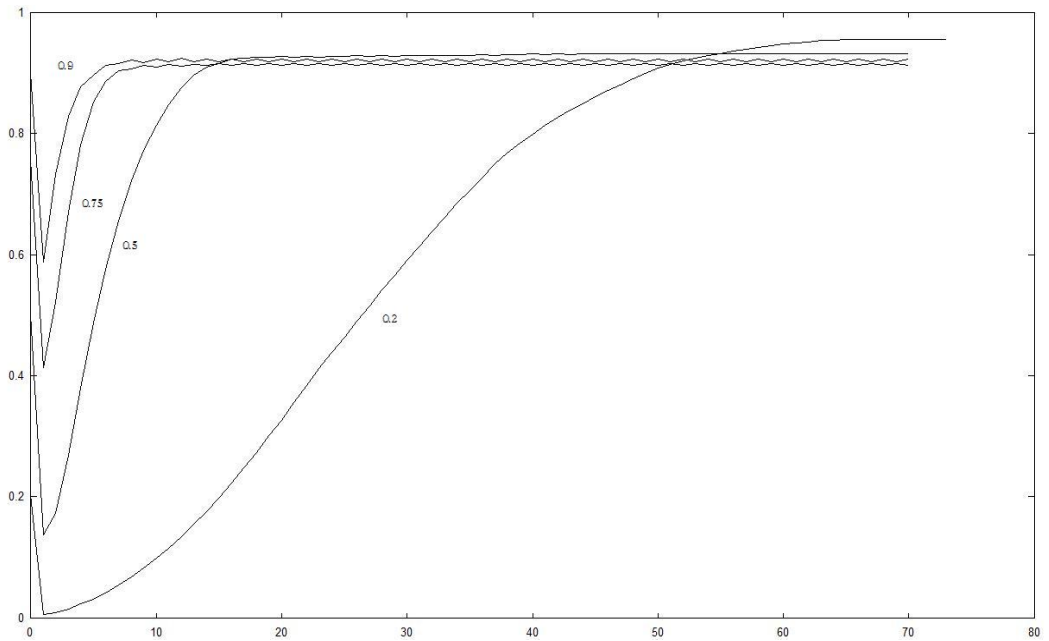




**Figure-20.** Evolution of the game for the case when all agents are Pavlovian, Figure 18 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.



**Figure-21.** Evolution of the game for the case when all agents are Pavlovian, Figure 18 gives the payoff curves, and the neighborhood is one layer deep. The graph shows the proportion of contributing agents as a function of the number of iterations for 5000 iterations. The initial cooperation ratio is  $x_0 = 0.8$ .



**Figure-22.** Evolution of the game for the case when all agents are greedy, Figure 18 gives the payoff curves, and the neighborhood is one layer deep. The graphs show the proportions of contributing agents as functions of the number of iterations. The initial cooperation ratio  $x_0$  is shown at each curve.

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