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# A NEGATIVE RATIONAL PIGS GAME AND ITS APPLICATIONS TO WEBSITE MANAGEMENT

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## ABSTRACT

*In order to study the balance relation between individual interests and its negative effects, based on a simple rational pigs game to allow robbing food whose negative game system is given by the axiomatic method. The research results are that a system regulation-free will lead to malignant states due to individuals' immediate interests and that in a system with regulation, a bad equilibrium situation can change into a equilibrium situation with the best public welfare by adjusting some indices. An application to website management shows that the conclusion of this paper conforms to actual situations.*

**Keywords:** Negative game, Rational pigs game, Inhaling poisonous gas quantity, Supervision mechanism, Punishment for gas emission, Recompense for absorbing gas, Public welfare, Website management.

## Contribution/ Originality

This paper establishes a new axiomatic theory on rational pigs game which is a negative game of a rational pigs game to allow robbing food. The theory can be used to control and to adjust players' the selfish behavior to injure the public interest to benefit their own private interest.

## 1. INTRODUCTION

Based on famous experiment of two psychologists, Baldwin and Meese (1979); Rasmusen (1989) introduced an example in his book on game theory called *Boxed Pigs*. John (1992) rewrote *Boxed Pigs* as *Rational Pigs*. But he wrote a new story. Susan (1996) said that “The big pig can run faster and eat faster than the little pig, so the little pig knows it will do better if it gets to the dispenser before the big pig” (p.701). Fabac *et al.* (2014) created a rational game extended (RPGE), in which the introduction of a third glutton entails significant structural changes. Based on situation analysis of double action games with entropy, e.g., Jiang (2008); Jiang (2008); Jiang (2010); Jiang (2010); Jiang (2010); Jiang (2011); Jiang (2012) and Jiang (2015); Jiang (2012); Jiang (2012); Jiang (2013); Jiang *et al.* (2013) and Jiang *et al.* (2014) used axiomatic method to research

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simple boxed pigs (or rational pigs) game theory with peace levels 0 and 1 respectively. Jiang (2015) established L-system of axiomatic theory of boxed pigs (or rational pigs) game and some deductive sub-systems were obtained from it, such as simple K-systems, instant K-systems and timing K-systems,  $K=0,1$ . Finally, the method to change a game among many pigs (can be infinitely many) into a game between a big pig and a small pig and applicable degree of the method are given. Jiang (2015) defines that a formal game is called the negative game of another one if sum of their payoff functions is equal to zero. In Jiang (2015) the negative boxed pigs game (or rational pigs game) is relative to 0-peace system in Jiang (2015). Li *et al.* (2015) gave an axiomatic system of boxed pigs (or rational pigs) game, i.e., a boxed pigs game (or rational pigs game) to allow robbing food.

In this paper, we try to research negative game of the game given by Li *et al.* (2015) and we will give an application to website management. The organization of this paper is follows. In section 2, our new game model will be given, either pig's inhaling poisonous gas quantities theorem will be proved, and some basic inequalities will be given. Set of pure Nash equilibria in this game will be discussed in section 3, which is divided into the two cases with and without supervision mechanism. In section 4, we want to introduce public welfare function on the game. How should a "government" let public welfare function take the greatest value and activate social members' labor enthusiasm? That will be discussed in section 5. Finally, in section 6, an application example to website management will be given.

## 2. MODEL AND PIG P'S INHALING POISONOUS GAS QUANTITIES THEOREM

By imitating Section 1.5 in Jiang (2015) we give another model as follows. In a sealed room within air, one pig P and one pig Q are put in two mesh cages. There is a trough in each cage and  $c$  unit of pig food are in each trough. The two cages close to the left wall and right wall of the room respectively. Two spray nozzles are installed on the two walls, which are close to the cages. When the pig P eats its food,  $q$  units of poisonous gas A erupt from the spray nozzle close the pig P. When the pig Q eats its food,  $q$  units of poisonous gas B erupt from the spray nozzle close the pig Q. We assume units of pig food and units of the poisonous gas are converted into an appropriate equivalent and  $q > c$ , and similarly for the following statement. When the two pigs eat their own food at the same time,  $q$  units of poisonous gas A erupt from the spray nozzle close the pig P and  $q$  units of B does from Q. The kind of gas emitted by all two nozzles is the same, but the spread velocity of the gas differs. Suppose the spread velocities of the poisonous gases are  $u_p$  and  $u_q$  respectively. The pig P inhales  $v_p$  units of poisonous gas in one time unit, and the pig Q inhales  $v_q$  units of poisonous gas in one time unit. The distance between the two pigs is  $d$ . The

quantity of the poisonous gas inhaled by the pig P is  $P$  units,  $T$  units, and  $Q$  units, respectively, when the pig P eats his food alone, the two pigs eat their food at the same time, and the pig Q eats his food alone.

Now let us give a basic assumption: The pig P to eat alone is favorable to himself and the quantity of the poisonous gas inhaled by him is less than the quantity of the poisonous gas erupted out, i.e.,  $P < \min\{c, q\}$ .

**Theorem 2.1** (Pig P's Inhaling Poisonous Gas Quantities Theorem) *The pig P's inhaling poisonous gas quantities are*

$$P = \frac{(qu_P + dv_Q)v_P}{u_P(v_P + v_Q)}, T = \frac{2v_Pq}{v_P + v_Q} + \frac{(u_Q - u_P)v_Pv_Qd}{u_Pu_Q(v_P + v_Q)}, \text{ and } Q = \frac{(qu_Q - dv_Q)v_P}{u_P(v_P + v_Q)}. \quad (2-1)$$

*When the pig P eats alone, the two pigs eat at the same time, and the pig Q eats alone respectively.*

**Proof:** (1) When the pig P eats alone, it inhales poisonous gas A at once. The poisonous gas A need time  $d/u_P$  to spread to the pig B. By this time, the pig P has inhaled  $dv_P/u_P$  units. There are still  $q - dv_P/u_P$  units in the room. The remaining poisonous gases are inhaled by both pigs together. Therefore, the quantity of poisonous gas inhaled by the pig P is

$$P = \frac{dv_P}{u_P} + \left( q - \frac{dv_P}{u_P} \right) \frac{v_P}{v_P + v_Q} = \frac{(qu_P + dv_Q)v_P}{u_P(v_P + v_Q)}.$$

(2) When the two pigs eat their pig food at the time, poisonous gases A and B with total quantity  $2q$  spray from the two spray nozzle. Poisonous gas A needs time  $t_1 = d/u_P$  to spread to the pig B. By this time, the pig P has inhaled  $dv_P/u_P$  units. The remaining  $q - dv_P/u_P$  units of poisonous gas A is inhaled by both pigs together. Poisonous gas B needs time  $t_1 = d/u_Q$  to spread to the pig A. By this time, the pig Q has inhaled  $dv_Q/u_Q$  units. The remaining  $q - dv_Q/u_Q$  units of poisonous gas A is inhaled by both pigs together.

Therefore, the quantity of poisonous gas inhaled by the pig P is

$$T = \frac{v_Pd}{u_P} + \frac{v_P}{v_P + v_Q} \left( 2q - \frac{v_Pd}{u_P} - \frac{v_Qd}{u_Q} \right) = \frac{2v_Pq}{v_P + v_Q} + \frac{(u_Q - u_P)v_Pv_Qd}{u_Pu_Q(v_P + v_Q)}.$$

(3) When the pig Q eats alone,  $q$  units of poisonous gas B is sprayed out from the spray nozzle close to the pig Q and the pig Q inhales it at once. Poisonous gas B needs time  $d/u_Q$  to spread to the pig P. By this time, the pig Q has inhaled  $dv_Q/u_Q$  units. There are still  $q-dv_Q/u_Q$  units in the room. The remaining poisonous gases are inhaled by both pigs together. Therefore, the quantity of poisonous gas inhaled by the pig P is

$$Q = \frac{q-dv_Q/u_Q}{v_P+v_Q} v_P = \frac{(qu_Q-dv_Q)v_P}{u_P(v_P+v_Q)}. \text{ Q.E.D.}$$

**Definition 2.1** The pig P is called a big pig and Q small one if  $u_P \geq u_Q$  and  $v_P > v_Q$ .

For the following research, it is assumed that  $u_P = u_Q = u$ , i.e., spread velocities of poisonous gases A and B are  $u$ . Then  $v_P$  and  $v_Q$  are written as  $v_b$  and  $v_s$ , respectively. And P, T, and Q are written as b, t and s to stand for that the big pig's inhaling poisonous gas quantities when the big pig eats alone, the two pigs eat at the same time, and the small pig eats at alone, respectively. Where  $b$ ,  $t$ , and  $s$  are prefixes of the words "big", "two", and "small" respectively. By Theorem 2.2.1, we obtain immediately that

$$b = \frac{v_b(qu + dv_s)}{u(v_b + v_s)}, t = \frac{2v_bq}{v_b + v_s}, \text{ and } s = \frac{v_b(qu - dv_s)}{u(v_b + v_s)}. \tag{2-2}$$

) **Remark:** Theorem 1 (The big pig's incomes) in Li *et al.* (2015) gave us the following result. The big pig's incomes are the follows when he alone presses his own panel, when the two pigs press their own panels together, and when the small pig alone presses his own panel, respectively.

$$b = \frac{v_b(qu_s + dv_s)}{u_s(v_b + v_s)}, t = \frac{2v_bq}{v_b + v_s} + \frac{(u_b - u_s)v_bv_s d}{u_b u_s (v_b + v_s)}, s = \frac{v_b(u_bq - v_s d)}{u_b(v_b + v_s)}.$$

When that  $u_b = u_s = u$ , the above formulas are simplified as (2-2). This shows that when

$u_b = u_s = u$  our new game is a negative game of that in Li *et al.* (2015).

**Theorem 2.2 (Basic Inequalities)** we have the basic inequalities

$$0 < d < \frac{qu}{v_b}, \text{ and } 0 < s < b < q < t = b + s < b + q < 2q.$$

**Proof:** Those basic inequalities can be obtained by the following inequalities.

$$0 < q - b = \frac{v_s(qu - dv_b)}{(v_b + v_s)u}, \quad b - s = \frac{2dv_bv_s}{(v_b + v_s)u} > 0,$$

$$t - q = \frac{q(v_b - v_s)}{v_b + v_s} > 0, \quad \text{and} \quad t - b = \frac{v_b(qu - dv_s)}{(v_b + v_s)u} = s. \quad \text{Q.E.D.}$$

### 3. SET OF PURE NASH EQUILIBRIA

#### 3.1. Games without Supervision Mechanism

The game is said to be without supervision mechanism if the two pigs' behaviors are not interposed.

In the following matrix, the big pig's actions are denoted by row and the small pig's by column. The first row denotes the big pig eating and the second row denotes the big pig not eating. Similarly, the first column denotes the small pig eating and the second column denotes the small pig not eating.

		Small		Pig
		eating		not eating
Big Pig	eating	$\left[ \begin{array}{cc} (c-t, c-2q+t) & (c-b, b-q) \end{array} \right]$		
	not eating	$\left[ \begin{array}{cc} (-s, c-q+s) & (0, 0) \end{array} \right]$		

**Theorem 3.1** *Set of pure Nash equilibria of a game without supervision mechanism is  $\text{PNE}(\Gamma) = \{(eating, eating)\}$ .*

**Proof:** By Theorem 2.2 and basic assumption, it can be obtained that  $q - s < b < c$ . Thus we have  $c - t + s = c - b > 0$ , i.e.,  $c - t > -s$  and  $q < b + s < c + s$ . The two inequalities imply that  $c - 2q + t - b + q = c + s - q > 0$ . Hence  $c - 2q + t > b - q$ . Based on marking method of finding pure Nash equilibria, it can be obtained that

$$\left[ \begin{array}{cc} (c-t, c-2q+t) & (c-b, b-q) \\ (-s, c-q+s) & (0, 0) \end{array} \right].$$

This proves that  $\text{PNE}(\Gamma) = \{(eating, eating)\}$ . Q.E.D.

Theorem 3.1 tells us that each of two pigs will eat, which is careless of the consequences, if there is no supervision mechanism. Therefore the two pigs' behaviors must be supervised in order to maintain a benign state of the system.

### 3.2. Games with Supervision Mechanism

Suppose a pig to discharge poisonous gas should be punished  $\alpha q$  units. Where  $\alpha > 0$  is said to be a punishment coefficient and  $\alpha q$  is said to be poisoning punishment quantity (or briefly, PPQ). We suppose also a pig to inhales poisonous gas should be compensated  $\delta q$  units. Where

$\delta > 0$  is called compensation coefficient and  $\delta q$  is called inhaled compensation quantity (or briefly, ICQ). Then the game can be written as

$$\left[ \begin{array}{cc} (c - \alpha q - (1 - \delta)t, c - \alpha q - (1 - \delta)(2q - t)) & (c - \alpha q - (1 - \delta)b, -(1 - \delta)(q - b)) \\ (-(1 - \delta)s, c - \alpha q - (1 - \delta)(q - s)) & (0, 0) \end{array} \right].$$

**Lemma 3.1** Let  $\alpha q > c$ . Then  $0 < \frac{\alpha q - c + b}{b} < \frac{q - s + \alpha q - c}{q - s}$ .

**Proof:** Theorem 2.2 and the assumption  $c < \alpha q$  tell us that

$$\frac{q - s + \alpha q - c}{q - s} - \frac{\alpha q - c + b}{b} = \frac{(b + s - q)(\alpha q - c)}{q - s} > 0. \text{ Q.E.D.}$$

Since  $b + s = t$ , we have that  $\delta > (=, <) \frac{\alpha q - c + b}{b}$  if and only if

$$c - \alpha q - (1 - \delta)t + (1 - \delta)s = c - \alpha q - (1 - \delta)b > (=, <) 0.$$

And  $\delta > (=, <) \frac{q - s - c + \alpha q}{q - s}$  if and only if

$$c - \alpha q - (1 - \delta)(2q - t) + (1 - \delta)(q - b) = c - \alpha q - (1 - \delta)(q - s) > (=, <) 0.$$

Thus we can obtain

**Theorem 3.2** If a pig's PPQ is more than the quantity of food eaten by him, then the set of pure Nash equilibria of the game with supervision mechanism is

$$\text{PNE}(\Gamma(\alpha q)) = \begin{cases} \{(\text{eating}, \text{eating})\}, & \text{if } \delta > \frac{\alpha q - c + q - s}{q - s}, \\ \{(\text{eating}, \text{eating}), (\text{eating}, \text{not eating})\}, & \text{if } \delta = \frac{\alpha q - c + q - s}{q - s}, \\ \{(\text{eating}, \text{not eating})\}, & \text{if } \frac{\alpha q - c + b}{b} < \delta < \frac{\alpha q - c + q - s}{q - s}, \\ \{(\text{eating}, \text{not eating}), (\text{not eating}, \text{not eating})\}, & \text{if } \delta = \frac{\alpha q - c + b}{b}, \\ \{(\text{not eating}, \text{not eating})\}, & \text{if } \delta < \frac{\alpha q - c + b}{b}. \end{cases} \quad \text{Proof: (1)}$$

Let  $\delta > \frac{\alpha q - c + q - s}{q - s}$ . Then Lemma 3.1 shows that

$$\delta > \frac{\alpha q - c + q - s}{q - s} > \frac{\alpha q - c + b}{b}.$$

That is

$$\begin{cases} c - \alpha q - (1 - \delta)b > 0, \\ c - \alpha q - (1 - \delta)t + (1 - \delta)s > 0, \\ c - \alpha q - (1 - \delta)(2q - t) + (1 - \delta)(q - b) > 0, \\ c - \alpha q - (1 - \delta)(q - s) > 0. \end{cases}$$

By marking method of finding pure Nash equilibria, it is obtained that

$$\left[ \begin{array}{cc} (\underline{c - \alpha q - (1 - \delta)t}, \underline{c - \alpha q - (1 - \delta)(2q - t)}) & (\underline{c - \alpha q - (1 - \delta)b}, \underline{-(1 - \delta)(q - b)}) \\ (\underline{-(1 - \delta)s}, \underline{c - \alpha q - (1 - \delta)(q - s)}) & (0, 0) \end{array} \right].$$

Thus  $\text{PNE}(\Gamma(\alpha q)) = \{(\text{eating}, \text{eating})\}$ .

(2) Let  $\delta = \frac{\alpha q - c + q - s}{q - s}$ . Then Lemma 3.1 shows that

$$\delta = \frac{\alpha q - c + q - s}{q - s} > \frac{\alpha q - c + b}{b}.$$

That is

$$\begin{cases} c - \alpha q - (1 - \delta)b > 0, \\ c - \alpha q - (1 - \delta)t + (1 - \delta)s > 0, \\ c - \alpha q - (1 - \delta)(2q - t) + (1 - \delta)(q - b) = 0, \\ c - \alpha q - (1 - \delta)(q - s) = 0. \end{cases}$$

It can be obtained that  $\text{PNE}(\Gamma(\alpha q)) = \{(\text{eating}, \text{eating}), (\text{eating}, \text{not eating})\}$  by imitating (1).

(3) Let  $\frac{\alpha q - c + b}{b} < \delta < \frac{\alpha q - c + q - s}{q - s}$ . Then

$$\left\{ \begin{array}{l} c - \alpha q - (1 - \delta)b > 0, \\ c - \alpha q - (1 - \delta)t + (1 - \delta)s > 0, \\ c - \alpha q - (1 - \delta)(2q - t) + (1 - \delta)(q - b) < 0, \\ c - \alpha q - (1 - \delta)(q - s) < 0. \end{array} \right.$$

It can be obtained that  $\text{PNE}(\Gamma(\alpha q)) = \{(\text{eating}, \text{not eating})\}$  by imitating (1).

(4) Let  $\delta = \frac{\alpha q - c + b}{b}$ . By Lemma 3.1, we can obtain that

$$\frac{\alpha q - c + q - s}{q - s} > \frac{\alpha q - c + b}{b} = \delta,$$

which is equivalent to

$$\left\{ \begin{array}{l} c - \alpha q - (1 - \delta)b = 0, \\ c - \alpha q - (1 - \delta)t + (1 - \delta)s = 0, \\ c - \alpha q - (1 - \delta)(2q - t) + (1 - \delta)(q - b) < 0, \\ c - \alpha q - (1 - \delta)(q - s) < 0. \end{array} \right.$$

It is obtained that  $\text{PNE}(\Gamma(\alpha q)) = \{(\text{eating}, \text{not eating}), (\text{not eating}, \text{not eating})\}$  by imitating

(1).

(5) Let  $\delta < \frac{\alpha q - c + b}{b}$ . By Lemma 3.1, we obtain that

$$\frac{\alpha q - c + q - s}{q - s} > \frac{\alpha q - c + b}{b} > \delta,$$

which is equivalent to

$$\left\{ \begin{array}{l} c - \alpha q - (1 - \delta)b < 0, \\ c - \alpha q - (1 - \delta)t + (1 - \delta)s < 0, \\ c - \alpha q - (1 - \delta)(2q - t) + (1 - \delta)(q - b) < 0, \\ c - \alpha q - (1 - \delta)(q - s) < 0. \end{array} \right.$$

It can be obtained that  $\text{PNE}(\Gamma(\alpha q)) = \{(\text{not eating}, \text{not eating})\}$  by imitating (1). Q.E.D.

Theorem 3.2 tells us that the stable situation of a game with supervision mechanism can be adjusted. A regulator can let pigs' strategy selection change into an expected one by some methods. However, a regulator should find maximization of the two pigs' benefits.

#### 4. PUBLIC WELFARE FUNCTION

The total benefit created by the two pigs brings positive effect to the public welfare of two pigs but discharging poisonous gas brings negative effect to that. It is obvious that each pig hates



poisonous gas and degree of hate is different. Let  $\lambda$  denote an aversion index about poisonous gas. Then  $\lambda > 0$  and  $\lambda q$  shows negative effect brought by poisonous gas.

Now let us give definition of public welfare function as follows.

**Definition 4.1** *Difference obtained by that total benefit created by the two pigs subtract negative effect brought by poisonous gas is called public welfare function of the game with supervision mechanism.*

Since regulators' function is to let public welfare function of the two pigs taking its greatest value, the public welfare function about the situation (not eating, not eating) has no meaning. Based on this, we consider only the two cases as follows.

(1) Public welfare function with type (eating, eating)

$$\begin{aligned} F_1(\lambda) &= c - \alpha q - (1 - \delta)t + c - \alpha q - (1 - \delta)(2q - t) - 2\lambda q \\ &= 2c - 2\alpha q - 2q(1 - \delta) - 2\lambda q. \end{aligned}$$

(2) Public welfare function with type (eating, not eating)

$$\begin{aligned} F_2(\lambda) &= c - \alpha q - (1 - \delta)b - (1 - \delta)(q - b) - \lambda q \\ &= c - \alpha q - q(1 - \delta) - \lambda q. \end{aligned}$$

**Theorem 4.1** *Let  $c < \alpha q$ . Then the two pigs' public welfare function takes the greatest value if and only if one of the two cases holds.*

$$\begin{aligned} (1) \quad & \delta > \frac{\alpha q - c + q - s}{q - s}, \lambda < \frac{c - \alpha q - q(1 - \delta)}{q}, \text{ and} \\ (2) \quad & \frac{\alpha q - c + b}{b} < \delta < \frac{\alpha q - c + q - s}{q - s}, \lambda > \frac{c - \alpha q - q(1 - \delta)}{q}. \end{aligned}$$

Where the unique pure Nash equilibrium of the case (1) is (eating, eating), and the unique pure Nash equilibrium of the case (2) is (eating, not eating).

Proof: (1) Since  $c < \alpha q$ , when

$$\delta > \frac{\alpha q - c + q - s}{q - s}, \text{ and } \lambda < \frac{c - \alpha q - q(1 - \delta)}{q},$$

on the one hand, by Theorem 3.2, the game has a unique pure Nash equilibrium (eating, eating);

and on the other hand, we can obtain that  $F_1(\lambda) > F_2(\lambda)$ , because

$$F_1(\lambda) - F_2(\lambda) = 2c - 2\alpha q - 2q(1 - \delta) - 2\lambda q - c + \alpha q + q(1 - \delta) + \lambda q > 0.$$

At this time, the two pigs' public welfare function has the greatest value.

The inverse proposition can be proved by the similar method.

(2) It can be proved by imitating the case (1). Q.E.D.

Theorem 4.1 tells us the fact. If poisoning punishment quantity is more than the quantity of food eaten, then the two pigs' public welfare function is the greatest one when either (1) the inhaled compensation quantity is higher and the aversion index about poisonous gas is lower, or (2) the inhaled compensation quantity is moderate and the aversion index about poisonous gas is higher.

## 5. ADJUSTMENT AND SETTING OF PARAMETERS

**Definition 5.1** It is said to be the critical value of aversion index about poisonous gas that

$$\lambda_0 = \frac{c - \alpha q - q(1 - \delta)}{q}. \text{ We said that the two pigs are not very sensitive to the poisonous gas if}$$

$\lambda < \lambda_0$  and the two pigs dislike the poisonous gas if  $\lambda > \lambda_0$ .

### 5.1. When the Aversion Index about Poisonous Gas is Lower

(1) Adjusting compensation coefficient  $\delta$  : By Theorem 4.1, each pig to eat his food lets public welfare function be the greatest only if compensation coefficient  $\delta$  is greater than

$$\frac{\alpha q - c + q - s}{q - s}.$$

(2) Adjusting punishment coefficient  $\alpha$  :

Let games with poisoning punishment quantities  $\alpha q$  and  $k\alpha q$  be  $\Gamma(\alpha q)$  and  $\Gamma(k\alpha q)$ , respectively. Now let us discuss what value  $k$  should be taken such that

$$\text{PNE}(\Gamma(\alpha q)) = \{(\text{eating}, \text{not eating}), (\text{not eating}, \text{not eating})\} \text{ and}$$

$$\text{PNE}(\Gamma(k\alpha q)) = \{(\text{eating}, \text{eating})\}.$$

**Lemma 5.1** If  $c < \alpha q$ , then  $\frac{c}{\alpha q} < \frac{\alpha q(q - s) + c(s + b - q)}{b\alpha q} < 1$ .

**Proof:** It is obvious that

$$\frac{\alpha q(q - s) + c(s + b - q)}{b\alpha q} - \frac{c}{\alpha q} = \frac{(\alpha q - c)(q - s)}{b\alpha q} > 0,$$

$$\frac{\alpha q(q - s) + c(s + b - q)}{b\alpha q} - 1 = \frac{(c - \alpha q)(s + b - q)}{b\alpha q} < 0. \text{ Q.E.D.}$$

**Theorem 5.1** We have the equivalent propositions as follows.

$\text{PNE}(\Gamma(\alpha q)) = \{(eating, not eating), (not eating, not eating)\}$  and  $\text{PNE}(\Gamma(k\alpha q)) = \{(eating, eating)\}$  if and only if

$$\frac{c}{\alpha q} < k < \frac{\alpha q(q-s) + c(s+b-q)}{b\alpha q} (< 1).$$

**Proof:** It is obvious that  $\delta > \frac{k\alpha q - c + q - s}{q - s}$  iff  $k < \frac{\alpha q(q-s) + c(s+b-q)}{b\alpha q}$ . Q.E.D.

Theorem 5.1 shows the fact. Ones can use the method of deducing poisoning punishment quantity to adjust the big pig eating alone to both eating and to let public welfare be optimal.

**Lemma 5.2** If  $c < \alpha q$ , then  $\frac{b\alpha q - c(s+b-q)}{\alpha q(q-s)} > 1$ .

**Proof:** By the condition  $c < \alpha q$ , it can be obtained that

$$\frac{b\alpha q - c(s+b-q)}{\alpha q(q-s)} - 1 = \frac{(\alpha q - c)(s+b-q)}{\alpha q(q-s)} > 0. \text{ Q.E.D.}$$

**Theorem 5.2** When the two pigs are not very sensitive to the poisonous gas, there exist no  $k$ , such that

$$\text{PNE}(\Gamma(\alpha q)) = \{(eating, eating)\} \text{ and}$$

$$\text{PNE}(\Gamma(k\alpha q)) = \{(eating, not eating), (not eating, not eating)\}. \quad (5-1)$$

**Proof:** Assume there exists  $k$  such that (5-1) holds. Let  $\alpha' = k\alpha$ , and  $k' = \frac{\alpha}{\alpha'}$ . Then

$$\alpha q = k' \alpha' q \text{ and } k\alpha q = \alpha' q. \text{ So } k' = \frac{\alpha}{\alpha'} = \frac{\alpha}{k\alpha} = \frac{1}{k}. \text{ Then}$$

$\text{PNE}(\Gamma(\alpha q)) = \{(eating, eating)\}$  and  $\text{PNE}(\Gamma(k\alpha q)) = \{(eating, not eating), (not eating, not eating)\}$ .

$$\text{PNE}(\Gamma(k' \alpha' q)) = \{(eating, eating)\} \text{ and}$$

$$\text{PNE}(\Gamma(\alpha' q)) = \{(eating, not eating), (not eating, not eating)\}.$$

By Theorem 5.1, it can be obtained that

$$\frac{c}{\alpha' q} < k' < \frac{\alpha' q(q-s) + c(s+b-q)}{b\alpha' q} (< 1).$$

It is clear that  $k > \frac{b\alpha q - c(s+b-q)}{\alpha q(q-s)}$  is equivalent to

$$k' < \frac{\alpha'q(q-s) + c(s+b-q)}{b\alpha'q},$$

and  $k' > \frac{c}{\alpha'q}$  is equivalent to  $\alpha > \frac{c}{q}$ . Thus

$$\begin{aligned} & \frac{c - \lambda q - q(1-\delta)}{\alpha q} - \frac{b\alpha q - c(s+b-q)}{\alpha q(q-s)} \\ &= \frac{b(c - \alpha q) - \lambda q(q-s) - q(1-\delta)(q-s)}{\alpha q(q-s)} < 0. \end{aligned}$$

Therefore it can be obtained that

$$k > \frac{b\alpha q - c(s+b-q)}{\alpha q(q-s)} > \frac{c - \lambda q - q(1-\delta)}{\alpha q}.$$

As a result, we have

$$\lambda > \frac{c - k\alpha q - q(1-\delta)}{q} = \frac{c - \alpha'q - q(1-\delta)}{q}.$$

However, it contradicts the condition that the two pigs are not very sensitive to the poisonous gas. Q.E.D.

### 5.2. When the Aversion Index about Poisonous Gas is Higher

We have the two methods as follows.

(1) Increasing the compensation coefficient  $\delta$  :

By Theorem 4.1(2), each of the two pigs eats and public welfare function is the greatest if and

only if the compensation coefficient  $\delta$  is between  $\frac{\alpha q - c + b}{b}$  and  $\frac{\alpha q - c + q - s}{q - s}$ .

(2) Decreasing punishment coefficient  $\alpha$  :

**Lemma 5.3** Let  $c < \alpha q$ . Then  $\frac{c}{\alpha q} < \frac{(\alpha q - c)(q - s) + cb}{b\alpha q} < 1$ .

**Proof:** By the condition, it can be obtained that

$$\begin{aligned} & 1 - \frac{(\alpha q - c)(q - s) + cb}{b\alpha q} = \frac{(b + s - q)(\alpha q - c)}{b\alpha q} > 0, \text{ and} \\ & \frac{(\alpha q - c)(q - s) + cb}{b\alpha q} - \frac{c}{\alpha q} = \frac{(\alpha q - c)(q - s)}{b\alpha q} > 0. \text{ Q.E.D.} \end{aligned}$$

**Theorem 5.3** Let  $c < \alpha q$ . Then

$$\frac{(\alpha q - c)(q - s) + cb}{b\alpha q} < k < 1$$

if and only if

$$\text{PNE}(\Gamma(\alpha q)) = \{(\text{eating}, \text{ not eating}), (\text{not eating}, \text{ not eating})\}, \tag{5-2}$$

$$\text{PNE}(\Gamma(k\alpha q)) = \{(\text{eating}, \text{not eating})\}. \tag{5-3}$$

3)

**Proof:** By Theorem 3.2, (5-2) is equivalent to  $\delta = \frac{\alpha q - c + b}{b}$ , and (5-3) is equivalent to

$$\frac{k\alpha q - c + b}{b} < \delta < \frac{k\alpha q - c + q - s}{q - s}. \text{ Therefore each of both (5-2) and (5-3) is satisfied if and}$$

only if

$$\frac{k\alpha q - c + b}{b} < \frac{\alpha q - c + b}{b} < \frac{k\alpha q - c + q - s}{q - s}.$$

That is  $\frac{(\alpha q - c)(q - s) + cb}{b\alpha q} < k < 1$ . By Theorem 5.3, it is feasible. Q.E.D.

Theorem 5.3 shows that appropriate reduction of poisoning punishment quantities can let public welfare function be the greatest and can delete the situation that each pig does not eat.

**Theorem 5.4** Let  $c < \alpha q$ ,  $0 < \delta \leq 1$ . Then  $1 < k < \frac{(\alpha q - c)b + c(q - s)}{(q - s)\alpha q}$  if and only if

$$\text{PNE}(\Gamma(\alpha q)) = \{(\text{eating}, \text{not eating})\}, \tag{5-4}$$

$$\text{PNE}(\Gamma(k\alpha q)) = \{(\text{eating}, \text{not eating}), (\text{not eating}, \text{not eating})\}, \tag{5-5}$$

and the two pigs dislike the poisonous gas.

**Proof:** Let  $k\alpha = \alpha'$ , and  $k' = \frac{1}{k}$ . Then we have that  $\alpha = \frac{\alpha'}{k} = k'\alpha'$ . So we have that

$$\alpha q = k'\alpha'q \text{ and } k\alpha q = \alpha'q.$$

**Necessity:** Let

$$1 < k' < \frac{(\alpha'q - c)b + c(q - s)}{(q - s)\alpha'q}.$$

Then

$$\frac{(\alpha q - c)(q - s) + cb}{b\alpha q} < k < 1.$$

By  $\alpha' > \frac{c}{q}$  and the condition  $\delta \leq 1$ , we have that  $\delta \leq 1 < \frac{(1+\alpha')q-c}{q}$ . Thus it can be

obtained that  $\lambda > 0 > \frac{c-q(1-\delta+\alpha')}{q}$ . This proves that

$$1 - \frac{c-q(1-\delta)-\lambda q}{\alpha'q} = \frac{\alpha'q-c+q(1-\delta)+\lambda q}{\alpha'q} > 0.$$

This implies that  $k' > 1 > \frac{c-q(1-\delta)-\lambda q}{\alpha'q}$ . Therefore

$$\lambda > \frac{c-k'\alpha'q-q(1-\delta)}{q} = \frac{c-\alpha q-q(1-\delta)}{q}.$$

This proves that the two pigs dislike the poisonous gas.

On the other hand,  $k' > 1$  implies that  $\alpha = k'\alpha' > \alpha' > \frac{c}{q}$ . By Theorem 5.3 and the

inequality  $\frac{(\alpha q-c)(q-s)+cb}{b\alpha q} < k < 1$ , we obtain both (5-2) and (5-3).

Let  $k'$  and  $\alpha'$  be replaced by  $k$  and  $\alpha$  respectively, both (5-4) and (5-5) can be obtained.

**Sufficiency:** Let the two pigs dislike the poisonous gas and let both (5-4) and (5-5) be satisfied.

Let  $k$  and  $\alpha$  be replaced by  $k'$  and  $\alpha'$  in (5-4), (5-5) and the condition  $\alpha > \frac{c}{q}$ , respectively. It

can be obtained that

$$\alpha' > \frac{c}{q}, \text{ PNE}(\Gamma(\alpha'q)) = \{(\text{eating}, \text{not eating})\}, \text{ and}$$

$$\text{PNE}(\Gamma(k'\alpha'q)) = \{(\text{eating}, \text{not eating}), (\text{not eating}, \text{not eating})\}.$$

Based on Theorem 5.3, it can be obtained that

$$\frac{(\alpha'q-c)(q-s)+cb}{b\alpha'q} < k' < 1.$$

By the relations  $k\alpha = \alpha'$  and  $k' = \frac{1}{k}$ , we can obtain that

$$1 < k < \frac{(\alpha q - c)b + c(q - s)}{(q - s)\alpha q}. \text{ Q.E.D.}$$

## 6. AN APPLICATION TO WEBSITE MANAGEMENT

A website has two plates, one big and one small. In order to motivate the two plates to compete against each other, the website implements the management system of independent economic accounting. Either one of the two plates can increase 8 units of profit if the plate releases some of the false news to increase click rate. In this case that one plate so does, the website would suffer indirect economic losses of 10 units and the two plates so do, the website would suffer indirect economic losses of 20 units, due to impaired reputation. If only the big plate does so, it would suffer losses  $23/3$  units. If only small one does so, the big plate would suffer losses  $17/3$  units. If the two plates do so, the big one would suffer losses  $40/3$  units. Our problems are the follows.

(1) Find the relation among spread speed of the bad reputation, economic damage speed of the big plate, and economic damage speed of the small plate.

(2) If a plate would be fined 90% of reputation, loss equivalent of the website for releasing some of the false news and compensation coefficient due to the website's reputation loss is not more than 100%, how should the website delete the small plate's selfish behavior such that public welfare of the website is the best?

**Solution:** (1) Since  $c = 8$ ,  $b = 23/3$ ,  $t = 40/3$ ,  $s = 17/3$ , and  $q = 10$ , by (2-2), we can obtain a system of equations.

$$\frac{v_b(10u + v_s d)}{u(v_b + v_s)} = \frac{23}{3}, \frac{20v_b}{v_b + v_s} = \frac{40}{3}, \text{ and } \frac{v_b(10u - v_s d)}{u(v_b + v_s)} = \frac{17}{3}.$$

It can be simplified as

$$d = \frac{23uv_s - 7uv_b}{3v_b v_s}, \quad d = \frac{13uv_b - 17uv_s}{3v_b v_s}, \quad v_b = 2v_s.$$

Its solution is  $u = v_b = 2v_s$ . It can be explained as that, in an average sense, spread speed of the bad reputation and speed of the big plate's economic damage are 2 times of speed of the small plate's economic damage.

(2) Since

$$\alpha = \frac{9}{10} > \frac{4}{5} = \frac{c}{q}, \quad 0 < \delta \leq 1, \quad \text{and} \quad \frac{(\alpha q - c)b + c(q - s)}{(q - s)\alpha q} = \frac{127}{117},$$

By Theorem 5.4, fined quantity of a plate who releases some of the false news is increased by  $k$  times ( $1 < k < 127/117$ ) again if and only if the website can delete the small plate's selfish behavior such that public welfare of the website is the best.

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