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A FRACTIONAL COINTEGRATION ANALYSIS OF PURCHASING POWER PARITY: EVIDENCE OF ELW

Nadhem Selmi[1]

[1]*Department of Quantitative Methods, Faculty of Economics and Management of Sfax - Tunisia*

ABSTRACT

This study examines the long-run relationship between exchange rates and relative prices. We use a long memory techniques that allow for persistence of chock relationships across real exchange rate to examine the existence of weak-form and strong-form Purchasing Power Parity (PPP) between the Tunisian and five partner countries of Tunisia, namely, (Germany, the United States, France, Italy, the UK, Morocco and Libya. The empirical results obtained through the R/S, Modified R/S, GPH and ELW tests; make us consider the PPP as an event in the long run if significant short-term deviations from the PPP cannot exist. Therefore, the analysis of the fractional cointegration makes the deviations, regarding equilibrium, follow a slightly integrated process and therefore capture a much wider group of research parity or mean-reverting behavior.

Keywords: Exchange rate, ELW, Fractional cointegration, long memory, GPH, Robinson.

JEL Classification: C12, C14.

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Contribution/ Originality

This study uses new estimation methodology. The application of fractional cointegration technique is very important in resolving puzzle of purchasing power parity (PPP). By allowing deviations from equilibrium to follow a fractionally integrated process, the fractional cointegration analysis can explain the persistence of a wider range of mean-reversion behaviour better than the standard cointegration analyses.

1. INTRODUCTION

The Purchasing power parity (PPP) is a basis of the exchange rate theory and a standard for evaluating the exchange rates in strategy negotiations. It asserts at least in the long run, the nominal exchange in relation to the units of a chosen foreign currency per unit of the domestic one. The PPP varies proportionally through its relative price ratio; it is defined as the fraction of the foreign price level out of the domestic one. Over the years, the frequent econometric studies have attempted to verify the PPP intention with mean-reversion. The previous studies which were carried out since the 1980s tend to assess that even if the PPP exists, it does so in the short run, through significant long-run deviations. To assess the PPP short run validity, the absolute version was used, but its long-run validity seemed to be out of question to debate. So, the most significant evidence of the PPP was considered by numerous authors to be very 'fragile'.

The equilibrium exchange rate is defined as the rate that allows for both internal and external balance of trade balance. Several theories seek to determine this rate as the theory of interest rate parity and the quantity theory of money. However, the theory of Purchasing Power Parity (PPP) remains the basic theory as it was introduced by Frenkel (1978), Dridi and Kugler (1995). PPP is a very important concept in international economic modeling; it is used for important goals and specific. Indeed, it serves as a guide to monetary authorities when they intervene in the foreign exchange market to move the exchange rate to a level consistent in equilibrium. The deviation of the exchange rate market given by the theory of PPP can result in either overvalued or undervalued currency. These two phenomena, and on the undervaluation may have adverse effects on exports, imports and the unemployment rate.

Abuaf and Jorian (1990), Ariff and Baharumshah (1997), Taylor (1999), Caner and Kilian (2001) and Mark (2003) studied the parity of purchasing power based on both absolute and relative versions. The absolute version was tested by regression of the nominal exchange rate on the ratio of domestic and foreign prices.

The potential presence of stochastic long memory in financial and economic time series has been an important subject of both theoretical and empirical researches. The long-memory phenomenon describes the high-order correlation function of a series. If these series present long memory, there is a persistent temporal dependence among observations widely separated over time horizon. Such series exhibit hyperbolically decaying autocorrelations and low-frequency spectral density. Fractionally integrated processes have a long memory (Mandelbrot, 1977; Granger and Joyeux, 1980; Hosking, 1981). Alternatively, the short-memory, property describes the low-order correlation function of these series. Short-memory series are characterized by quickly declining autocorrelations and high-frequency spectral density. An autoregressive moving average model cannot exhibit long-run low frequency dependence as they can only describe the short-run high-frequency behavior of a time series. During the recent years, there have been some views towards a significant extension of nonlinear modeling.

But check this absolute version requires the completion of several assumptions, where the use relative version of the PPP as it is introduced by Fisher and Park (1991) and Glen (1992). This version is checked on the regression of the real exchange rate on the order of auto-regression for each country studied.

The objective of this paper is to test the theory of purchasing power parity in distinguishing these two main versions absolute and relative, and indicate both the non-validity of the PPP: the overvaluation and undervaluation of currency. In the empirical part we will estimate the nominal exchange rate on the ratio of domestic and foreign prices and the real exchange rate on the order autoregressive using Box-Jenkins techniques for each country while indicating the nature of each process. Finally, we will test the fractional integration parameter.

This paper differs from Robertson *et al.* (2009) in three important ways. First, Robertson *et al.* (2009) evaluate PPP by estimating half-lives of adjustment back to long-run PPP rather than the cointegration approach used in this paper. Second, Robertson *et al.* (2009) focus on the importance of aggregation bias rather than cross-section differences across goods. The importance of using cross-section differences across goods to investigate PPP has been well-established in the trade literature, as conclusions can be affected by trade costs, local taxes, varying shares of traded inputs (Crucini *et al.*, 2005). Hernandez Vega (2012) uses a highly disaggregated sample of coordinated prices of individual goods between the US and Mexico in examining competing theories of real exchange rate determination, based upon the variable's volatility.

The objective of this paper is to test the presence of long memory in the exchange rate of five partner countries of Tunisia, namely Germany, the United States, France, Italy, the UK, Morocco and Libya countries. A number of studies have used various procedures to test for long-term dependence in composite and common stock returns. The deductions of this study are mixed relying on the testing procedure, sample period, frequencies of the series, exchange rate and others used in these studies.

This paper is divided into three sections: the first one analyzes the econometric methodology, the second describes the empirical results finally, and the third section presents the conclusion remarks.

2. ECONOMETRIC METHODOLOGY

2.1. A Model of Fractional Cointegration

We consider the p-vector fractional process X_t generated by the process

$$\Delta\left(L, d_1, \ldots, d_p\right) X_t = u_t I\left\{t \geq 1\right\}, t = 1, 2, \ldots \quad (1)$$

where $I\{.\}$ is the indicator function, $\Delta\left(L, d_1, \ldots, d_p\right) = diag\left(\left(1-L\right)^{d_1}, \ldots, \left(1-L\right)^{d_p}\right)$, and $u_t = C\left(L\right)\varepsilon_t$ is a p-vector stationary zero mean model with spectral density matrix $f_u\left(\lambda\right)$. The covariance matrix of ε_t has full rank, so without loss of generality we normalize it to I_p

(the $p \times p$ identity matrix). The rank of $C(1)$ is $p - r \leq p$.

The rank condition on $C(1)$ determines the cointegrating rank of X_t. As in the standard scenario, this implies that the number of cointegrating vectors is r or equivalently that the system is determined by $p - r$ common stochastic trends. Thus, the system might be generated by a triangular form like the process

$$(1-L)^{d-b}\left(X_{1t} - \alpha' X_{2t}\right) = v_{1t} I\{t \geq 1\}, t = 0,1,2,..., \quad (2)$$

$$(1-L)^{d} X_{2t} = v_{2t} I\{t \geq 1\}, t = 0,1,2,..., \quad (3)$$

Where X_{1t} is an r-vector X_{2t} is a $(p-r) \times r$ -vector, and α is a $(p-r) \times r$ matrix. For simplicity, the process in (2)-(3) has equal integration orders for all the observed variables (d) and for the cointegrating errors $(d-b)$. The triangular form has a straightforward interpretation as equilibrium relations given by (2) and stochastic trends specified as equation (3). Note that in this representation, the cointegrating vectors are the rows of the $(r \times p)$ matrix $(I_r; -\alpha')$. Also note that (1) is more general than the triangular representation and also incorporates, for example the possibility of fractional multi-cointegration and/or polynomial cointegration which is not present in (2)-(3). Though, the triangular system is simple and easy to interpret as a possible generating mechanism for X_t. There are two main characterizations of fractional integration that have been used in the literature, see, for example. Marinucci and Robinson (1999) and Robinson (2005). The process (1) is a convenient and unified classification that applies for both (asymptotically) stationary and nonstationary processes. The generating models for X_{at} is

$$(1-L)^{d_a} X_{at} = u_{at} I\{t \geq 1\}, t = 1,2,..., \quad (4)$$

Expanding the binomial in (4) gives the form

$$\sum_{k=0}^{t} \frac{(-d_a)}{k!} X_{at-k} = u_{at} I\{t \geq 1\} \quad (5)$$

Where $(d)_k = \Gamma(d+k)/\Gamma(d) = (d)(d+1)....(d+k-1)$, is the forward factorial function and $\Gamma(.)$ is the gamma function. When d_a is a positive integer, the series in (4) terminates, giving the usual equation in terms of the differences and higher order differences of X_{at}. Inverting (4) gives a valid linear representation of X_{at} for all coefficients of d_a,

$$X_{at} = (1-L)^{-d_a} u_{at} I\{t \geq 1\} = \sum_{k=0}^{t} \frac{(-d_a)}{k!} X_{at-k} \quad (6)$$

2.2. Local Whittle Method

The classes of semiparametric frequency domain estimators follow the local Whittle approach suggested by Künsch (1987) which was analyzed by Robinson (1995) (who called it a Gaussian semi-parametric estimator). The Local Whittle estimator is defined as the maximization of the local Whittle likelihood purpose:

$$Q(g,d) = -\frac{1}{m}\sum_{j=1}^{m}\left[\log\left(g\lambda_j^{-2d}\right) + \frac{I(\lambda_j)}{g\lambda_j^{-2d}}\right] \tag{7}$$

Where $m = m(T)$ is a bandwidth number which tends to infinity $T \to \infty$ except at a slower speed than T ;

$I(\lambda) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{it\lambda}\right|^2$, is the periodogram of X_t ,

$g_x(\lambda)$ is the spectral density of X_t , $\lambda_j = \frac{2\pi j}{n}$, and $j = 1,\ldots,n$.

One disadvantage compared to log-periodogram estimation is that a statistical optimization is needed. On the other hand, the assumptions underlying this estimator are weaker than the log-periodogram regression (LPR) estimator. Robinson (1995) showed that while $d \in \left(-\frac{1}{2}, \frac{1}{2}\right)$;

$$\sqrt{m}\left(\hat{d}_{LW} - d\right) \xrightarrow{d} N(0, 1/4) \tag{8}$$

Therefore, the asymptotic distribution is extremely simple, facilitating easy asymptotic inference. In particular the estimator is more efficient than the LPR estimator. The ranges of reliability and asymptotic normality for the Local Whittle estimator have been shown by Velasco (1999) and by Shimotsu and Phillips (2005) to be the same as those of the LPR estimator.

The asymptotic distribution under the null hypothesis is not known if the real errors X_t are observable. The search for fractional cointegration has been the subject of various studies focusing on financial time series and using mostly the method Geweke and Porter-Hudak (1983).

The fractional cointegration test of Robinson (1994) followed a similar problem in the cointegration test of Cheung and Lai (1993) for the regression fallacy. To negotiate this problem we obtain the empirical dimension of the cointegration test samples done using a simulation approach.

3. DATA AND SUMMARY STATISTICS

Financial series considered are formed by the indices nominal exchange rate, the real exchange rate and price indices of general consumption French Franc / Dollar U.S, dinar Tunisian / Dollar U.S, British Pound / Dollar U.S, Italian Lira / Dollar U.S. German Deutschmark / Dollar U.S, Moroccan Dirham / Dollar U.S and Libyan Dinar / Dollar U.S.

The data are monthly. The period of analysis for indices exchange rate from January 1990 to December 2006 for all series that is to say 204 observations. Price indices for general consumption, the period begin in January 1990 and ending in June 2006 for all series that is to say 198 observations.

Table-1. Descriptive statistics of the real exchange rate

	Nb obs	Mean	Variance	t-stat	Skewnes	Kurtossis	Jarque-Bera
Germany	96	2.254	0.011	205.52	0.256	-0.026	1.057
France	108	7.541	0.068	299.82	0.064	-0.491	1.163
U.K	225	1.230	0.014	152.71	1.021	0.381	40.479
Italy	108	2.258	45382.06	110.154	-0.198	-0.796	3.564
Libya	33	1.005	0.127	16.168	0.649	-1.564	5.612
Marocoo	204	1.3620	0.454	288.52	-0.019	-0.445	1.699
Tunisia	224	1.711	0.033	140.701	0.451	-1.095	18.807

The descriptive statistics show that an average series, the exchange rate is close to 2. Developed markets are characterized by a lower variance compared to emerging markets. Skewness is nonzero and positive for most of the series reflecting the asymmetric behavior of the series studied (except for Italy and Libya). The kurtosis is much higher than the normal value exposing the occurrence of extreme values and the existence of heteroscedasticity in the series. We also note the presence of serial autocorrelation of first and second order. These linear dependencies confirm the imperfect markets. The auto-correlation of the second order also explains the heteroscedasticity series yields (Table 1).

Table-2. Results of long memory analysis

	R/S Statistic	Modified R/S Statistic	GPH			LW	ELW
			$\mu = 0.45$	$\mu = 0.5$	$\mu = 0.55$		
Germany	0.553	0.687	0.265	0.387	0.421	0.476	0.879
France	0.489	0.658	0.325	0.452	0.341	0.478	0.958
U.K	0.532	0.689	0.358	0.389	0.378	0.477	0.776
Italy	0.544	0.676	0.268	0.356	0.259	0.468	0.879
Libya	0.529	0.679	0.389	0.298	0.341	0.479	0.911
Marocoo	0.517	0.683	0.387	0.347	0.375	0.428	0.974
Tunisia	0.543	0.715	0.347	0.256	0.358	0.410	0.958

Notes: The modified R/S test for long memory suggested by Lo (1991) is performed on the exchange rate series for each of the following countries: Germany, France, U.K, Italy, Libya, Marocoo and Tunisia. The lag parameter q used for the modified R/S test is determined by Andrews (1993) data-dependent rule (see equation (3)). At the 5% significance level, the null hypothesis of a short-memory process is rejected if the modified R/S statistic does not fall within the confidence interval [0.809, 1.862]. Statistical significance is indicated by an asterisk (*) for the 5% level.

Furthermore Table 2 displays the estimates of d calculated using the Gaussian semi-parametric estimator of GPH and ELW for exchange rate series. There is a weak suggestion of long memory in the German and Libya series, and strong evidence for France. The GPH approach provides no evidence for long memory in the Libya France and Italy exchange rate, which contrasts with the LW evidence. The remaining series display big evidence of long memory. Interestingly a number of the estimates of d are low predominantly through Germany which is suggestive of anti-persistence. Nevertheless none of the negative estimates are statistically significant.

Table 2 also displays the estimates of d calculated using the Whittle (1956) estimator for both exchange rate series. Kang and Yoon (2008) using the test for short memory versus long memory in the high frequency data of KOSPI 200 using the FIAPARCH model. The asymmetric long memory property is observed in the four different frequencies of KOSPI 200 returns, in which the intraday data of KOSPI 200 were taken, every 10-min interval, for the entire two calendar years, commencing on January 2, 2003 and ending on December 30, 2004. The 10-min KOSPI 200 prices consist of 36 intervals per day from 9 a.m. to 3 p.m., covering 17,832 data points. There is a weak suggestion of long memory in Japan, the United Kingdom, France and Canada. However, none of the negative estimates are statistically significant. Significant evidence of long memory can be found in all the returns of the United States, Germany and Italy with a monthly and quarterly frequency.

Table 2 indicates that the long memory property in the series appears to be slightly lower than that 0.5. This may be due to the more asymmetric exchange rate in these series. It can be seen that the exchange rate is highly persistent in all series during the full period. We note that the highest values for exchange rate study (0.41 up to 0.47 respectively for all series study), indicating the persistence of shocks of goods that of the banking courses and stock markets.

4. CONCLUSION

To make the relative version, using the real exchange rate, the nature of the process it has been determined for each country based on Correlogram total and partial autocorrelations for each process. We looked the validity of the relative version of the app using the autoregressive part of the model to determine the stationary and the moving average part to determine the invertibility of the model; it was shown that the relative version is checked only between the Tunisian dinar and Libyan dinar. It was estimated later differentiation fractional values "d" based on the methods of Geweke and Porter-Hudak (1983), Whittle (1956), Hurst (1951) and Lo (1991). It has proved the existence of long memory for series of exchange rates of the countries studied by the methods of Whittle (1956) and Lo (1991) and short memories by the methods of Geweke and Porter-Hudak (1983) and Hurst (1951).

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