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INTEGRALS INVOLVING H-FUNCTION AND SOME COMMONLY USED FUNCTIONS

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ABSTRACT

The aim of this paper is to obtain some integrals involving Fox's H-function.
Key words: Arguments, Poles, Integrals equation, Gamma function

INTRODUCTION

H-function of one variable, which was introduced by Fox [1], is represented as follows:

$$H^{m,n}_{p,q}\left[x\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right] = (1/2\pi i)\int_L \theta(s)\, x^s\, ds \qquad (1)$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\ \prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\ \prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)}$$

x is not equal to zero and an empty product is interpreted as unity; p, q, m, n are integers satisfying $1 \le m \le q$, $0 \le n \le p$, α_j $(j = 1, \ldots, p)$, β_j $(j = 1, \ldots, q)$ are positive numbers and a_j $(j = 1, \ldots, q)$ are complex numbers. L is a suitable contour of Barnes type such that poles of $\Gamma(b_j - \beta_j s)$ $(j = 1, \ldots, m)$ lie to the right and poles of $\Gamma(1 - a_j + \alpha_j s)$ $(j = 1, \ldots, n)$ to the left of L. These assumptions for the H-function will be adhered to throughout this paper.

According to Braakasma

$$H^{m,n}_{p,q}\left[x\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right] = O(|x|^{\alpha}) \text{ for small } x,$$

where $\sum_{j=1}^{p}\alpha_j - \sum_{j=1}^{q}\beta_j \le 0$ and $\alpha = \min R(b_h/\beta_h)$ $(h = 1, .., k)$

and

$$H^{m,n}_{p,q}\left[x\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right] = O(|x|^{\beta}) \text{ for large } x,$$

where

$$\square \quad \sum_{j=1}^{n} \square\square\square\square\square\square\square\square\square\square\square\square\square\square_j - \sum_{j=n+1}^{p} \square\square\square_j + \sum_{j=1}^{m} \square\square\square_j - \sum_{j=m+1}^{q} \square\square\square_j \; \square\square A > \square 0, \qquad (2)$$

$$\sum_{j=1}^{p} \square\square\square\square_j - \sum_{j=1}^{q} \square\square\square_j < \square 0$$

|arg x| < ½ A□ and □□= max R[(a_j − 1)/□$_j$] (j = 1, .., n)

FORMULA USED

In the present investigation we require the following formula:
From Sharma and Rathi [2], we have

$$\int_0^{\pi/2} e^{i(2\rho+1)\theta}(\sin\theta)^{\rho-1}(\cos\theta)^{\rho}\ {}_2F_1[\alpha,\beta;\frac{\alpha+\beta+2}{2};e^{i(\theta-\pi/2)}\sin\theta]d\theta$$

$$= \frac{e^{\frac{i\pi\rho}{2}}\Gamma(\rho)\Gamma(\rho-\alpha/2-\beta/2)\Gamma(\alpha/2+\beta/2+1)}{2^{2\rho-\alpha-\beta+2}(\alpha-\beta)\Gamma(\alpha)\Gamma(\beta)} \times$$

$$\times \left[\frac{(2\rho-\alpha+\beta)\Gamma\left(\frac{\alpha}{2}+\frac{1}{2}\right)\Gamma\left(\frac{\beta}{2}\right)}{\Gamma\left(\rho-\frac{\alpha}{2}+1\right)\Gamma\left(\rho-\frac{\beta}{2}+\frac{1}{2}\right)} + \frac{(2\rho+\alpha-\beta-2)\Gamma(\alpha/2)\Gamma(\beta/2+1/2)}{\Gamma(\rho-\beta/2+1/2)\Gamma(\rho-\beta/2+1)}\right], \qquad (3)$$

where Re(□) > 0, Re(□□□□□□□□□□□) > 0;

$$\int_0^1 x^{\rho-1}(1-x)^{\rho}\ {}_2F_1[\alpha,\beta;\frac{\alpha+\beta+2}{2};x]dx$$

$$= \frac{2^{\alpha+\beta+\rho-2}\Gamma(\rho)\Gamma(\rho-\alpha/2-\beta/2)\Gamma(\alpha/2+\beta/2+1)}{(\alpha-\beta)\Gamma(\alpha)\Gamma(\beta)} \times$$

$$\times \left[\frac{(2\rho-\alpha+\beta)\Gamma(\alpha/2+1/2)\Gamma(\beta/2)}{\Gamma(\rho-\alpha/2+1)\Gamma(\rho-\beta/2+1/2)} + \frac{(2\rho+\alpha-\beta)\Gamma(\alpha/2)\Gamma(\beta/2+1/2)}{\Gamma(\rho-\beta/2+1)\Gamma(\rho-\alpha/2+1/2)}\right], \qquad (4)$$

where Re(□) > 0, Re(□□□□□□□□□□□) > 0;

MAIN INTEGRALS

$$\int_0^{\pi/2} e^{i(2\rho+1)\theta}(\sin\theta)^{\rho-1}(\cos\theta)^{\rho}\ {}_2F_1[\alpha,\beta;\frac{\alpha+\beta+2}{2};e^{i\left(\theta-\frac{\pi}{2}\right)}\sin\theta]$$

$$\times H_{p,q}^{m,n}[ze^{-2i\lambda\theta}(\sin\theta)^{-\lambda}(\cos\theta)^{-\lambda}|_{(b_j,\beta_j)_{1,q}}^{(a_j,\alpha_j)_{1,p}}]d\theta$$

$$= \frac{e^{\frac{i\pi\rho}{2}}\Gamma\left(\frac{\alpha}{2}+\frac{\beta}{2}+1\right)}{2^{2\rho-\alpha-\beta+2}\Gamma(\alpha)\Gamma(\beta)}(\Gamma(\alpha/2+1/2)\Gamma(\beta/2) \times$$

$$\times H_{p+3,q+3}^{m+3,n}$$

$$[\frac{ze^{-\frac{i\pi\lambda}{2}}}{4^{-\lambda}}|^{,(a_j,\alpha_j)_{1,p},(1+\rho-\alpha/2,\lambda),(1/2-\beta/2+\rho,\lambda),(1+2\rho-\alpha+\beta,2\lambda)}_{(\rho,\lambda),(\rho-\alpha/2-\beta/2,\lambda),(2\rho-\alpha+\beta+1,2\lambda),(b_j,\beta_j)_{1,q},}]$$

$$-\Gamma(\alpha/2)\Gamma(\beta/2+1/2)\times$$

$$H^{m+3,n}_{p+3,q+3}[\frac{ze^{-\frac{i\pi\lambda}{2}}}{4^{-\lambda}}|^{(a_j,\alpha_j)_{1,p},(1+\rho-\beta/2,\lambda),(1/2-\alpha/2+\rho,\lambda),(2\rho+\alpha-\beta,2\lambda)}_{(\rho,\lambda),(\rho-\alpha/2-\beta/2,\lambda),(1+2\rho+\alpha-\beta,2\lambda),(b_j,\beta_j)_{1,q}}]),$$

(5)

provided that Re(□) > 1, Re (□□□□□□□□□□□) > 2, $\lambda \geq 0$, $|\arg z| < \frac{1}{2}\pi A$, where A is given by equation (2).

$$\int_0^1 x^{\rho-1}(1-x)^{\rho}\ {}_2F_1[\alpha,\beta;\frac{\alpha+\beta+2}{2};x]\times$$

$$H^{m,n}_{p,q}[zx^{-\lambda}(1-x)^{-\lambda}|^{(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}]dx$$

$$=\frac{2^{\alpha+\beta-2\rho-2}\Gamma(\alpha/2+\beta/2+1)}{(\alpha-\beta)\Gamma(\alpha)\Gamma(\beta)}(\Gamma(\alpha/2+1/2)\Gamma(\beta/2)\times$$

$$\times H^{m+3,n}_{p+3,q+3}[\ z4^{\lambda}|^{,(a_j,\alpha_j)_{1,p},(1+\rho-\alpha/2,\lambda),(1/2-\beta/2+\rho,\lambda),(2\rho-\alpha+\beta,2\lambda)}_{(\rho,\lambda),(\rho-\alpha/2-\beta/2,\lambda),(2\rho-\alpha+\beta+1,2\lambda),(b_j,\beta_j)_{1,q},}]$$

$$-\Gamma(\alpha/2)\Gamma(\beta/2+1/2)\times$$

$$\times H^{m+3,n}_{p+3,q+3}[z4^{\lambda}|^{(a_j,\alpha_j)_{1,p},(1+\rho-\beta/2,\lambda),(1/2-\alpha/2+\rho,\lambda),(2\rho+\alpha-\beta,2\lambda)}_{(\rho,\lambda),(\rho-\alpha/2-\beta/2,\lambda),(2\rho+\alpha-\beta+1,2\lambda),(b_j,\beta_j)_{1,q}}]),$$

(6)

provided that Re(□) > 0, Re (□□□□□□□□□□□) > 0, $\lambda \geq 0$, $|\arg z| < \frac{1}{2}\pi A$, where A is given by equation (2).

Proof

To establish (5), on the left hand side replace the H-function of one variable by its equivalent counter integral as given in (1), change the order of integration which is valid under the given conditions, we get

$$\frac{1}{2\pi\omega}\int_L\ z^{\xi}\,\theta(\xi)[\int_0^{\pi/2}e^{i[2(\rho-\lambda\xi)+1]\theta}\,(\sin\theta)^{(\rho-\lambda\xi)-1}(\cos\theta)^{(\rho-\lambda\xi)}$$

$$\cdot\ {}_2F_1[\alpha,\beta;\frac{\alpha+\beta+2}{2};e^{i\left(\theta-\frac{\pi}{2}\right)}\sin\theta]d\theta]d\xi.$$

Now, we evaluate the inner integral with the help of (3) and finally interpreting it with (1), we obtain (5). The result (6) can be established in the view of (4) exactly as the same given above.

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