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SOLUTION OF NP-COMPLETE PROBLEMS ON THE LANDAUER'S COMPUTER

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ABSTRACT

In this article a new kind of classical computer – Landauer's one is suggested. It is a computer which operates in agreement with Landauer's Principle (LP). It is characterized by clock rate which is exponentially large in comparison with clock rate of classical computers. It leads to the possibility to use Landauer's computer for solving of NP-complete problems in appropriate, i.e. polynomial time with the help of ordinary searching algorithms.

Keywords: Landauer's principle, Computational complexity, Clock rate, Searching algorithm.

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INTRODUCTION

A proposition of equivalence (or non-equivalence) of P and NP classes of problems is one of seven millennium's problems [1]. Let us remind that problems of NP – class are solving on classical computers (more precisely, in deterministic Turing machine, DTM) in non-polynomial time. An example of such problems is the problem of factoring of big integer.

A computational complexity of that problems forms a basis of reliability of contemporary cryptographic asymmetric systems, i.e. systems with open key. Non-possibility of that problems' solving on classical computers in appropriate, i.e. polynomial time provides stableness of such a cryptosystems.

Building of quantum computer which will be able to realize quantum Shor's factoring algorithm [2] for sufficiently large integers will provide a solution of factoring problem in polynomial time. This will mean an end of era of asymmetric cryptographic systems, and more widely of the existing system of information security.

In present article it is suggested to solve problems of NP-class with the help of new-type classical computer – Landauer's one in polynomial time. This in turn permits to prove an equivalence of P – and NP – classes in spite of common belief that this is not the case.

Landauer's Principle and Landauer's Computer

Landauer's computer is a computational device operating in accordance with Landauer's Principle (LP) which was formulated in 1961 by Landauer [3]. According LP all operations with bits of information, except operation of information deleting can, in principle, go with arbitrary small energy expenditures if their speeds are arbitrary small.

Operation of information deleting needs minimal energy expenditure *kTln2 J/bit(k* – Boltzmann constant, T- absolute temperature of thermostat which contains computing or measuring system). This statement is formulated in the following conditions:

1. Deleting of information in binary memory cells needs putting cells in a state in which every cell contains "ONE". This ground state which has no information was called by Landauer as "STATE "ONE""
2. All calculations are going at constant temperature of cells.

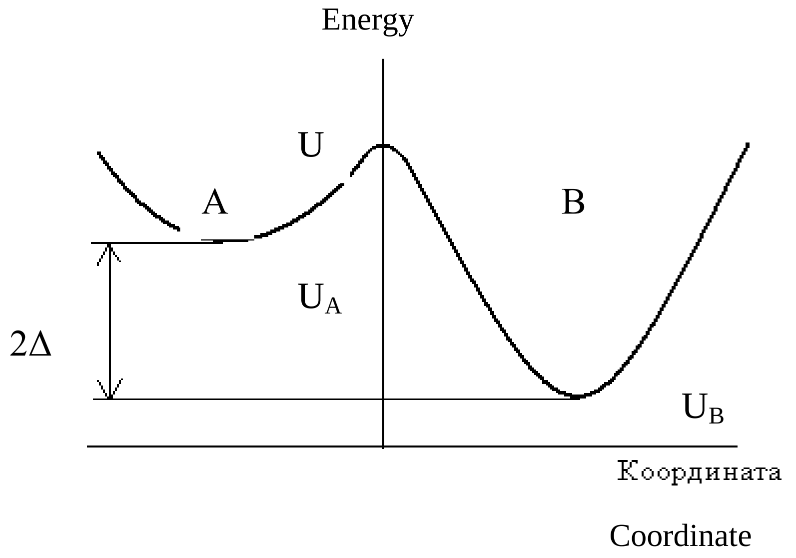
Let us say that condition 1 is not obligatory. In means only that memory cells (processor registers) have to be cleaned before new information will be written. It can be changed if one re-defines ground state so that entropy production *kln2 J/K·bit* will take place at another step, for example, at the step of information writing, not deleting [4, 5].

Author's formulation of the PL which asserts that "entropy must appear elsewhere as a heating effect" [3], obviously apart from computing system, is essential providing cells' operation in isothermal condition.

Landauer's Analysis of Computing Cell's Working

Under this condition Landauer was able to make analysis of bi-stable computing cell neglecting entropy dynamics. Let us remind a problem's formulation [3]. Landauer analyzed in Landauer [3] model of a bi-stable (double) potential well with two minima along one of the coordinates (information degree of freedom) (Fig. 1).

Figure-1. Model of switching cell analyzed by Landauer [3]



The behavior of statistical ensemble of cells is described by the following equations [3]:

$$\begin{aligned}\frac{dn_A}{dt} &= -\nu n_A \cdot \exp\left(-\frac{U-U_A}{kT}\right) + \nu n_B \cdot \exp\left(-\frac{U-U_B}{kT}\right) \\ \frac{dn_B}{dt} &= \nu n_A \cdot \exp\left(-\frac{U-U_A}{kT}\right) - \nu n_B \cdot \exp\left(-\frac{U-U_B}{kT}\right)\end{aligned} \tag{1}$$

where n_A and n_B are the numbers of ensemble members in wells *A* and *B*, respectively; *t* – time, *T* – temperature of thermostat, *ν*- is the frequency of transitions between the members of ensemble; U, U_A and U_B – are the energies at the maximum of the barrier and at the minima *A* and *B*, respectively. The difference $\Delta = ½(U_A - U_B)$ is a half of the energy dissipated in the course of switching, which is supplied by an external force controlling the switching. In symmetric equilibrium state which does not have information $U_A = U_B$ and $n_A = n_B$. Upon switching, the system goes out of that equilibrium and exhibits relaxation to a new equilibrium distribution

$$n_A = n_B \cdot \exp\left(\frac{U_B - U_A}{kT}\right) \tag{2}$$

This relaxation proceeds according to the exponential law ~ *exp(-λ t)* with a characteristic time *τ*, where $\tau^{-1} = \lambda$, and *λ* – is the characteristic value of Eqs (1):

$$\lambda = \nu \cdot \exp\left(-\frac{U-U_A}{kT}\right) + \nu \cdot \exp\left(-\frac{U-U_B}{kT}\right) \tag{3}$$

Using simple transformations, one can readily show that Landauer [3]:

$$\frac{1}{\tau}=\frac{1}{\tau_0}\cdot ch\left(\frac{\Delta}{kT}\right);\frac{1}{\tau_0}=2\nu\cdot\exp\left(-\frac{U-U_0}{kT}\right);U_0=\frac{1}{2}\cdot\left(U_A+U_B\right) \quad (4)$$

Where τ_0 – has a meaning of the information lifetime, τ - is the switching time defined above, and $ch(\Delta/kT)$ is the number of switching events (processor cycles), that is, the program length expressed in processor cycles. Under the condition (usually assumed) that $\Delta >> kT$, the number of cycles is large, and this fact shows the usefulness of computing devices of this type.

It was shown in the article [6] that these results are not valid for classical computers due to violation both conditions 1 and 2.

Implementation of the Landauer's Analysis to Classical Computers

In order to improve Landauer's model for its application to classical computers, let us supply Eqs. (1) with equation for entropy dynamics. This equation looks as follows:

$$\frac{dS}{dt}=\frac{2\Delta}{T\cdot\tau(T)} \quad (5)$$

where S is entropy, $\tau(T)$ – switching time depending on temperature T, and $2\Delta/T$ – is amount of entropy generated per one switch. Using the formula $dS/dt = (c/T)(dT/dt)$, where c is thermal heat capacity of cell, one can receive a solution of the Eqs. (1) and (5) and define temperature of cells as function of time - $T(t)$, which looks as follows [6]:

$$\begin{gathered}\exp\left(\frac{E}{kT}\right)=\exp\left(\frac{E}{kT_0}\right)-\alpha\cdot t\\ E=U-U_A,\alpha=\frac{2\nu E\Delta}{ckT_0^2},T_0=T(0)\end{gathered} \quad (6)$$

The frequency of switching $f=\alpha$ (processor clock rate) can be evaluated as:

$$f=\nu\cdot\frac{1}{N}\cdot\frac{E}{kT}\cdot\frac{2\Delta}{kT},N=\frac{c}{k},T\equiv T_0 \quad (7)$$

where N has a meaning of the number of particles (electrons) per cell (transistor). It leads from (7) that $Nf = const$, what is confirmed for CISC-processors (see fig.2) [6].

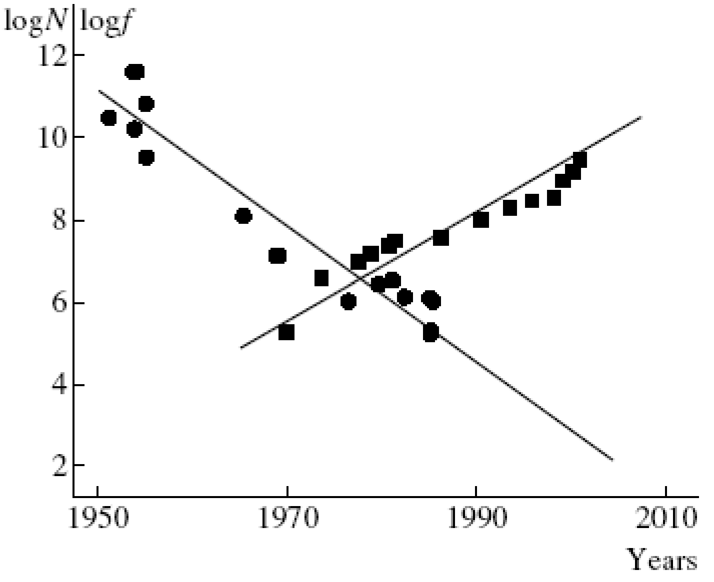


Figure 2. Plots of the number of impurities N in the base of bipolar transistors (circles, data from Keyes [7]) and the CISC processor clock rate f [Hz] (squares, data from Broido and Il'ina [8] versus year. Strait lines have been drawn to show the trends.

How Does Landauer's Computer Could Solve NP-Complete Problems?

In distinction with Landauer's result (4) processor's clock rate (7) is proportional to the value of Δ/kT, not to the $ch(\Delta/kT)$. This exponential decrease (for $\Delta/kT >> 1$) of the switching frequency (processor clock rate) of classical computers in comparison with (4) makes them useless for solving of NP – complete problems with the help of searching algorithms in appropriate polynomial time. Remind that time of solving these problems on classical computers (DTM) is evaluated as $\sim e^M$, where M – is a problem's dimension.

On the other hand Landauer's computer which has been defined earlier as a computer working in accordance with Landauer's Principle could solve NP – complete problems with the help of searching algorithms in polynomial time if $\Delta/kT > M$. An evidence of its possible existence has been demonstrated experimentally in recent work [9].

Another preference of the Landauer's computer in comparison with classical ones is that it utilizes a whole bit of information during one processor's cycle, while classical computer utilizes only a little part of a bit. It leads from evaluation of entropy production per cycle according to (6)

$$\Delta S = \frac{c\Delta T}{T_0} \approx c\frac{kT_0}{E}\exp\left(-\frac{E}{kT_0}\right) << k\ln 2 \qquad (8)$$

Nevertheless, problem of heat evacuation from crystals of small area is of independent interest and needs additional study.

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