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EXPLICIT NUMERICAL SOLUTION OF HIGH– DIMENSIONAL ADVECTION – DIFFUSION

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ABSTRACT

The Several numerical techniques have been developed and compared for solving the one-dimensional and three-dimentional advection-diffusion equation with constant coefficients. the subject has played very important roles to fluid dynamics as well as many other field of science and engineering. In this article, we will be presenting the of n-dimentional and we neglect the numerical examples.

Keywords: Finite difference methods, Advection–diffusion equation, Explicit and implicit techniques.

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INTRODUCTION

The significant applications of advection–diffusion equation lie in fluid dynamics [1], heat transfer [2] and mass transfer [3]. Various approaches are available for solving one-dimensional advection–diffusion partial differential equations. advection-diffusion equation illustrates many quantities such as mass, hate, energy, velocity, vorticity, etc. the solutions of equation model some of the phenomena such as the contaminat transport in groundwater, etc. the slow progress has been made towards the analytical solutions of the advection-diffusion equation when initial and boundry conditions are intricates since many of the analytical solutions have not much easy use, many attempts have been carried out on developing the accurate numerical techniques. In previous works [4] solved one-dimentional- diffuone-dimensional advection–diffusion phenomenon equation by using the temporal and spatial weighted parameters with following from:

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial x^2} \qquad (1^*)$$

begining requirements

$$c(x,0) = f(x) \qquad 0 \le x \le L \qquad (2^*)$$

and conditions

$$c(0,t) = g(t), \qquad 0 \le t \le T \qquad (3^*)$$

$$c(L,t) = h(t) \qquad 0 \le t \le T \qquad (4^*)$$

Where f, g and h are introduced functions. u and D are the speed of advection and diffusivity respectively. Soon after [4] extended this scheme.

. In this paper we shall consider the n-dimentional advection-diffusion equation

$$\frac{\partial c}{\partial t} + U_{x_1}\frac{\partial c}{\partial x_1} + U_{x_2}\frac{\partial c}{\partial x_2} + \cdots + U_{x_n}\frac{\partial c}{\partial x_n} = D_{x_1}\frac{\partial^2 c}{\partial x_1^2} + \cdots + Dx_n\frac{\partial^2 c}{\partial x_n} \quad (1)$$

With begining requirements

$$c(x_1, x_2, \dots, x_n, 0) = f(x_1, x_2, \dots, x_n) \qquad (2)$$

And conditions

$$c(0, x_2, x_3, \dots, x_{n-1}, t) = w_{01}(x_1, x_2, \dots, x_{n-1}, t)(3)$$

$$c(L_{x_1}, x_2, \dots, x_{n-1}, t) = w_{11}(x_1, x_2, \dots, x_{n-1}, t)(4)$$

$$c(x_1, 0, x_3, \dots, x_{n-1}, t) = w_{02}(x_1, x_3, \dots, x_{n-1}, t) \qquad (3)$$

$$c(x_1, L_{x_1}, \dots, x_{n-1}, t) = w_{12}(x_1, x_3, \dots, x_{n-1}, t) \qquad (4)$$

$$c(x_1, x_2, 0, x_4, \dots, x_{n-1}, t) = w_{03}(x_1, x_2, \dots, x_{n-1}, t) \qquad (5)$$

$$c(x_1, x_2, L_{x_3} \dots, x_{n-1}, t) = w_{13}(x_1, x_2, \dots, x_{n-1}, t) \qquad (6)$$

$$\vdots$$

$$c(x_1, x_2, \dots, 0, t) = w_{0n}(x_1, \dots, x_{n-2}, t) \qquad (7)$$

$$c(x_1, x_2, \dots, L_{x_n}, t) = w_{1n}(x_1, \dots, x_{n-2}, t) \qquad (8)$$

Where w_{01} ,$w_{11}, w_{02}, w_{12}, \dots, w_{on}, w_{in}$ are know functions $U_{x_i}$and $D_{x_i}$are the speed of advection and diffusinity respectively the domains are

$$o \le x_1 \le L_{x1}, 0 \le x_2 \le L_{x2}, \dots, 0 \le t \le T$$

By changing only the values of temporal ϕ , and spatial, weighted parameters, Eq(1) , can be solve by various explicit finite difference methodes

Numerical Solution

The lattice of grid- lines are nominated as :

$$x_1\theta_1 = \theta_1 \Delta x_1 \theta_1 = 0,1,2, \dots., M_{x_1} \qquad (9)$$

$$x_2\theta_2 = \theta_2 \Delta x_2 \theta_2 = 0,1,2, \dots., M_{x_2} \qquad (10)$$

$\vdots$

$$x_n\theta_n = \theta_n \Delta x_n \theta_n = 0,1,2, \dots., M_{x_n} \qquad (11)$$

$$t_n = n\Delta t \qquad\qquad t = 0, \dots, N \tag{12}$$

The fixed spatial and temporal grid – spacing are the fol

$\Delta x_1 = \frac{Lx_1}{\Delta x_1}$, $\Delta x_2 = \frac{Lx_2}{\Delta x_2}$, …, $\Delta x_n = \frac{Lx_n}{\Delta x_n}$, $\Delta t = \frac{T}{N}$, Karahan [5].

Where M denotes the total number of the in corporate time and space weightes ϕ ▯ $and\ \theta\ as\ follows:$

$$\frac{\partial c}{\partial t} = \frac{c(\theta_1, \theta_2, \dots, \theta_n, n+1) - c(\theta_1, \theta_2, \dots, \theta_n, n)}{\Delta t} \tag{13}$$

$$U_{x_1}\frac{\partial c}{\partial x_1} = \left(1-\phi\right)\left\{\frac{U_{x_1}}{\Delta x_1}[(1-\theta)c(\theta_1, \theta_2, \dots, \theta_n, n) + \theta c(\theta_1 + 1, \theta_2, \dots, \theta_n, n)\right.$$

$$-(1-\theta)c(\theta_1 - 1, \theta_2, \theta_n, n) - \theta c(\theta_1 - \theta_2, \dots, n)]\} \tag{14}$$

$$+\phi\left\{\frac{U_{x_1}}{\Delta x_1}[(1-\theta)c(\theta_1, \theta_2, \dots, n+1) + \theta c(\theta_1 + 1, \theta_2, \dots, n+1)\right.$$

$$-(1-\theta)c(\theta_1, \theta_2 - 1, \dots, \theta_n, n+1) - \theta c(\theta_1, \theta_2, \dots, \theta_n, n+1)]\}$$

$$U_{x_2}\frac{\partial c}{\partial x_2} = \left(1-\phi\right)\left\{\frac{U_{x_2}}{\Delta x_2}[(1-\theta)c(\theta_1, \theta_2, \dots, \theta_n, n) + \theta(\theta_1, \theta_2 + 1, \dots, \theta_n, n)\right.$$

$$-(1-\theta)c(\theta_1, \theta_2 - 1, \dots, \theta_n, n) - \theta c(\theta_1, \theta_2, \dots, \theta_n, n)]\} \tag{15}$$

$$+\phi\left\{\frac{U_{x_2}}{\Delta x_2}[(1-\theta)c(\theta_1, \theta_2, \dots, \theta_n, n+1) + \theta c(\theta_1 + \theta_2 + 1, \dots, \theta_n, n+1)\right.$$

$$-(1-\theta)c(\theta_1, \theta_2 - 1, \theta_n, n+1) - \theta c(\theta_1, \theta_2 + 1 \dots, \theta_n, n+1)]\}$$

$\vdots$

$$U_{x_n}\frac{\partial c}{\partial x_n} = \left(1-\phi\right)\left\{\frac{U_{x_n}}{\Delta x_n}[(1-\theta)c(\theta_1, \theta_2, \dots, \theta_n, n) + \theta c(\theta_1, \theta_2, \dots, \theta_n + 1, n)\right.$$

$$-(1-\theta)c(\theta_1, \theta_2, \dots, \theta_n - 1, n) - \theta c(\theta_1, \theta_2, \dots, \theta_n + 1, n+1)\} \tag{16}$$

$$+\phi\left\{\frac{U_{x_n}}{\Delta x_n}[(1-\theta)c(\theta_1, \theta_2, \dots, \theta_n, n+1) + \theta c(\theta_1 + \theta_2 + 1, \dots, \theta_n, n+1)\right.$$

$$-(1-\theta)c(\theta_1, \theta_2, \dots, \theta_n - 1, n+1) - \theta c(\theta_1, \theta_2, \dots, \theta_n, n+1)\}$$

$$D_{x_1}\frac{\partial^2 c}{\partial x_1^2} = \left(1-\phi\right)\left\{\frac{D_{x_1}}{\Delta x_1^2}[c(\theta_1 - 1, \theta_2, \dots, \theta_n, n) - 2c(\theta_1, \theta_2, \dots, \theta_n, n) + c(\theta_1 + 1, \theta_2, \theta_n, n)\}\right.$$

(17)

$$+\phi\left\{\frac{D_{x_1}}{\Delta x_1^2}[c(\theta_1 - 1, \theta_2, \dots, \theta_n, n+1) - 2c(\theta_1, \theta_2, \dots, \theta_n, n+1) + c(\theta_1 + 1, \theta_2, \dots, \theta_n, n+1)]\right\}$$

$$D_{x_2}\frac{\partial^2 c}{\partial x_2} = \left(1-\phi\right)\left\{\frac{D_{x_2}}{\Delta x_2^2}[c(\theta_1, \theta_2 - 1, \dots, \theta_n, n) - 2c(\theta_1, \theta_2, \dots, \theta_n, n) + c(\theta_1, \theta_2 + 1, \theta_n, n)]\right\}$$

(18)

$$+\phi\left\{\frac{D_{x_2}}{\Delta x_2^2}[c(\theta_1,\theta_2-1,\dots,\theta_n,n+1)-2c(\theta_1,\theta_2,\dots,\theta_n,n+1)-c(\theta_1,\theta_2+1,\dots,\theta_n,n+1)]\right\}$$

⋮

$$D_{x_n}\frac{\partial^2 c}{\partial x_n^2}=\left(1-\phi\right)\left\{\frac{D_{x_n}}{\Delta x_n^n}[c(\theta_1,\theta_2,,\dots,\theta_n-1,n)-2c(\theta_1,\theta_2,\dots,\theta_n,n)-2c(\theta_1,\theta_2,\dots,\theta_n,n)\right.$$
$$\left.+c(\theta_1,\theta_2,\dots,\theta_n+1,n)]\right\}$$

(19)

$$+\phi\left\{\frac{D_{x_n}}{\Delta x_n^2}[c(\theta_1,\theta_2,\dots,\theta_n-1,n+1)-2c(\theta_1,\theta_2,\dots,\theta_n,n+1)+c(\theta_1,\theta_2,\dots,\theta_n+1,n+1)]\right\}$$

Where ϕ is a time weighting factor and θ is the spatial weighting factore . substituting Equ (14-20) in to Eq(1) gives :

$$c(\theta_1,\theta_2,\dots,k,n+1)=-\frac{1}{A_1}[A_0C(\theta_1,\theta_2,\dots,\theta_n,n)+B_0C(\theta_1+1,\theta_2,\dots,\theta_n,n) \quad (20)$$

$$+B_1C(\theta_1+1,\theta_2,\dots,\theta_n,n+1)+D_0C(\theta_1-1,\theta_2,\dots,\theta_n,n)$$
$$+D_1C(\theta_1-1,\theta_2,\dots,\theta_n,n+1)\ ++E_0C(\theta_1,\theta_2+1,\dots,\theta_n,n)$$
$$+E_1C(\theta_1,\theta_2+1,\dots,\theta_n,n+1)+F_0C(\theta_1,\theta_2-1,\dots,\theta_n,n)+F_1C(\theta_1,\theta_2-1,\dots,\theta_n,n+1)$$

$$+\cdots+G_0C(\theta_1,\theta_2,\dots,\theta_n+1,n)+G_1C(\theta_1+1,\theta_2,\dots,\theta_n+1,n+1)$$
$$H_0C(\theta_1,\theta_2,\dots,\theta_n-1,n)+H_1C(\theta_1,\theta_2,\dots,\theta_n-1,n+1)]$$

Where

$$A_0=-1+(1-\phi)\{(a_{x_1}+a_{x_2}+\cdots+a_{x_n})(1-2\theta)+2(S_{x_1}+S_{x_2}+\cdots+S_{x_n})\} \quad (21)$$
$$A_1=1+\phi\{(a_{x_1}+a_{x_2}+\cdots+a_{x_n})(1-2\theta)+2(S_{x_1}+S_{x_2}+\cdots+S_{x_n})\} \quad (22)$$
$$B_0=(1-\phi)[a_{x_1}\theta-S_{x_1}] \quad (23)$$
$$B_1=\phi[a_{x_1}\theta-S_{x_1}] \quad (24)$$
$$D_0=-(1-\phi)[a_{x_1}(1-\theta)-S_{x_1}] \quad (25)$$
$$D_1=\phi[a_{x_1}(1-\theta)-S_{x_1}] \quad (26)$$
$$E_0=(1-\phi)[a_{x_2}\theta-S_{x_2}] \quad (27)$$
$$E_1=\phi[a_{x_2}\theta-S_{x2}] \quad (28)$$
$$F_0=-(1-\phi)[a_{x_2}(1-\theta)-S_{x_2}] \quad (29)$$
$$F_1-\phi[a_{x_2}(1-\theta)-S_{x_2}]= \quad (30)$$
$$G_0=(1-\phi)[a_{x_n}\theta-S_{x_n}] \quad (31)$$
$$G_1=\phi[a_{x_n}\theta-S_{x_n}] \quad (32)$$
$$H_0=-(1-\phi)[a_{x_n}(1-\theta)-S_{x_n}] \quad (33)$$
$$H_1=-\phi[a_{x_n}(1-\theta)-S_{x_n}] \quad (34)$$

And

$$a_{x_1} = U_{x_1}\frac{\Delta t}{\Delta x_1}$$

$$a_{x_2} = U_{x_2}\frac{\Delta t}{\Delta x_2}$$

$$\vdots$$

$$a_{x_n} = U_{x_n}\frac{\Delta t}{\Delta x_n}$$

,

$$s_{x_1} = D_{x_1}\frac{\Delta t}{\Delta x_1^2}$$

$$s_{x_2} = D_{x_2}\frac{\Delta t}{\Delta x_2^2}$$

$$\vdots$$

$$s_{x_n} = D_{x_n}\frac{\Delta t}{\Delta x_n^2}$$

The FTBSCS Technique

This technique – also called the explicit upwind – uses the forward – difference from for the time derivative , centered – difference forms for the difference forms for the diffusive derivatives and back wark differences forms for the spatial derivatives in the advection terms. Application of the technique on solving Eq (20) for ϕ=0 ,θ=0 is in following from.

$$C(\theta_1,\theta_2,\dots,\theta_n,n+1) = (s_{x_1}+a_{x_1})C(\theta_1-1,\theta_2,\dots,\theta_n,n)+(s_{x_2}+a_{x_2})\,C(\theta_1,\theta_2-1,\dots,\theta_n,n) +\dots+(s_{x_n}+a_{x_n})C(\theta_1,\theta_2,\dots,\theta_n-1,n)+s_{x_1}\,C(\theta_1+1,\theta_2,\dots,\theta_n,n)+s_{x_2}\,C(\theta_1,\theta_2+1,\dots,\theta_n,n)+\dots+s_{x_n}\,C(\theta_1,\theta_2,\dots,\theta_n+1,n)+(1-2S_{x_1}-2S_{x_2}-\cdots 2S_{x_n}-a_{x_1}-a_{x_2}-\cdots-a_{x_n})C(\theta_1,\theta_2,\dots,\theta_n,n)$$

CONCLUSIONS

The techniques based on high-order differences provide efficient and alternative methods for modeling the behavior of the problem. The present schemes are capable of solving the for a futher research. Restrictions of the stability of the high-order FD schemes for the advection-diffusion problems will be analyzed in a futher study in detail.

Symbols

C concentration

U_{x_i}advection coefficient

D_{x_i}diffusivity

L_{x_i}length

T total time

i space counter

n time counter

ϕ temporal weight

θ_i spatial weight

Δt time step

Δx_i space step

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