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NOTES ON Q-INTUITIONISTIC FUZZY SUBSEMIRING OF A SEMIRING

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ABSTRACT

In this paper, we make an attempt to study the algebraic nature of Q-intuitionistic fuzzy subsemiring of a semiring and some properties of Q-intuitionistic fuzzy subsemiring of a semiring are investigated. AMS Mathematics Subject Classification (2010): 06D72, 08A72, 03F55.

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1. INTRODUCTION

An algebraic structure (R,+,.) is said to be a semiring if (R,+) and (R,.) are semigroups satisfying a.(b+c) = a.b + a.c and (a+b).c = a.c + b.c for all a, b, c ϵ R. A semiring (R,+,.) is said to be additively commutative, if a+b = b+a for all a,b ϵ R. A semiring has an identity 1, defined by a.1 = a = 1.a and zero 0 defined by a+0 = a = 0+a and 0.a = 0 = a.0 for all a ϵ R. After the introduction of fuzzy sets by Zadeh [1], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov [2, 3], as a generalization of the notion of fuzzy set. A study on anti Q-fuzzy subsemiring of a semiring has been introduced by Vanathi, et al. [4]. In this paper, we introduce some theorems in Q-intuitionistic fuzzy subsemiring of a semiring.

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2. PRELIMINARIES

2.1 Definition

Let X be a non empty set and Q be a non empty set. A Q-fuzzy subset A of X is a function $A : X\times Q \rightarrow [0, 1]$.

2.2 Definition

Let (R,+,.) be a semiring. A Q-fuzzy subset A of R is said to be a Q-fuzzy subsemiring of R if it satisfies the following conditions:

(i) $\mu_A(x+y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$,

(ii) $\mu_A(xy, q) \geq \min\{ \mu_A(x, q), \mu_A(y, q) \}$, for all x and y in R and q in Q.

2.3 Definition

Let (R,+,.) be a semiring. A Q-fuzzy subset A of R is said to be an anti Q-fuzzy subsemiring of R if it satisfies the following conditions:

(i) $\mu_A(x+y, q) \leq \max\{ \mu_A(x, q), \mu_A(y, q) \}$,

(ii) $\mu_A(xy, q) \leq \max\{ \mu_A(x, q), \mu_A(y, q) \}$, for all x and y in R and q in Q.

2.4 Definition

A Q-intuitionistic fuzzy subset A in X is defined as an object of the form $A = \{\langle (x, q), \mu_A(x, q), \nu_A(x, q) \rangle / x\in X$ and q in Q$\}$, where $\mu_A : X\times Q \rightarrow [0,1]$ and $\nu_A : X\times Q \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element x in X and q in Q respectively and for every x in X and q in Q satisfying $0 \leq \mu_A(x, q) + \nu_A(x, q) \leq 1$.

2.5 Definition

If A is a Q-intuitionistic fuzzy subset of X, then the complement of A, denoted A^c is the Q-intuitionistic fuzzy set of X, given by $A^c(x, q) = \{< (x, q), \nu_A(x, q), \mu_A(x, q) > / x\in X \text{ and } q\in Q \}$.

2.6 Definition

Let (R,+,.) be a semiring. A Q-intuitionistic fuzzy subset A of R is said to be a Q-intuitionistic fuzzy subsemiring of R if it satisfies the following conditions:

(i) $\mu_A(x+y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$,

(ii) $\mu_A(xy, q) \geq \min\{ \mu_A(x, q), \mu_A(y, q) \}$,

(iii) $\nu_A(x+y, q) \leq \max\{\nu_A(x, q), \nu_A(y, q) \}$,

(iv) $\nu_A(xy, q) \leq \max\{ \nu_A(x, q), \nu_A(y, q) \}$, for all x and y in R and q in Q.

2.7 Definition

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Let A and B be Q-intuitionistic fuzzy subsets of sets G and H respectively. The product of A and B, denoted by A×B, is defined as A×B = {⟨ ((x, y), q), $\mu_{A\times B}$((x, y), q), $\nu_{A\times B}$((x, y), q) ⟩ / x in G and y in H and q in Q }, where $\mu_{A\times B}$((x, y), q) = min{ μ_A(x, q), μ_B(y, q)} and $\nu_{A\times B}$((x, y), q) = max{ ν_A(x, q), ν_B(y, q) }.

2.8 Definition

Let A be a Q-intuitionistic fuzzy subset in a set S, the strongest Q-intuitionistic fuzzy relation on S, that is a Q-intuitionistic fuzzy relation on A is V given by μ_V((x, y), q) = min{ μ_A(x, q), μ_A(y, q) } and ν_V((x, y), q) = max{ ν_A(x, q), ν_A(y, q) }, for all x and y in S and q in Q.

2.9 Definition

Let (R, +, ·) and (R^{I}, +, ·) be any two semirings. Let f : R → R^{I} be any function and A be a Q-intuitionistic fuzzy subsemiring in R, V be a Q-intuitionistic fuzzy subsemiring in f(R) = R^{I}, defined by $\mu_V(y, q) = \sup_{x\in f^{-1}(y)} \mu_A(x, q)$ and $\nu_V(y, q) = \inf_{x\in f^{-1}(y)} \nu_A(x, q)$, for all x in R and y in R^{I}. Then A is said to be the preimage of V under f and is denoted by $f^{-1}(V)$.

2.10 Definition

Let A be a Q-intuitionistic fuzzy subsemiring of a semiring(R, +, ·) and a in R. Then the pseudo Q-intuitionistic fuzzy coset $(aA)^p$ is defined by ($(a\mu_A)^p$)(x, q) = p(a)μ_A(x, q) and ($(a\nu_A)^p$)(x, q) = p(a)ν_A(x, q), for every x in R and for some p in P and q in Q.

3. SOME THEOREMS OF Q-INTUITIONISTIC FUZZY SUBSEMIRING:

3.1 Theorem

Intersection of any two Q-intuitionistic fuzzy subsemiring of a semiring R is a Q-intuitionistic fuzzy subsemiring of R.

Proof: Let A and B be any two Q-intuitionistic fuzzy subsemirings of a semiring R and x and y in R and q in Q. Let A = { ((x, q), μ_A(x, q), ν_A(x, q)) / x∈R and q in Q} and B = {((x, q), μ_B(x, q), ν_B(x, q)) / x∈R and q in Q } and also let C = A∩B = { ((x, q), μ_C(x, q), ν_C(x, q)) / x∈R and q in Q}, where min { μ_A(x, q), μ_B(x, q)} = μ_C(x, q) and max { ν_A(x, q), ν_B(x, q) } = ν_C(x, q). Now, μ_C(x+y, q) = min { μ_A(x+y, q), μ_B(x+y, q)} ≥ min{ min {μ_A(x, q), μ_A(y, q)}, min {μ_B(x, q), μ_B(y, q)}}= min{ min{ μ_A(x, q), μ_B(x, q)}, min{μ_A(y, q), μ_B(y, q)}}= min{μ_C(x, q), μ_C(y, q)}. Therefore, μ_C(x+y, q)≥ min {μ_C(x, q), μ_C(y, q)}, forall x and y in R and q in Q. And, μ_C(xy, q) = min { μ_A(xy, q), μ_B(xy, q) }≥ min { min{μ_A(x, q), μ_A(y, q)}, min {μ_B(x, q), μ_B(y, q)}}= min{ min {μ_A(x, q), μ_B(x, q)}, min {μ_A(y, q), μ_B(y, q)}}= min{μ_C(x, q), μ_C(y, q)}. Therefore, μ_C(xy, q) ≥ min{ μ_C(x, q), μ_C(y, q)}, forall x and y in R and q in Q. Now, ν_C(x+y, q)= max {ν_A(x+y, q), ν_B(x+y, q)} ≤ max{max {ν_A(x, q), ν_A(y, q)}, max{ν_B(x, q), ν_B(y, q)}}= max{max{ν_A(x, q),

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$\nu_B(x, q)\}$, $\max\{\nu_A(y, q), \nu_B(y, q)\}\} = \max\{\nu_C(x, q), \nu_C(y, q)\}$. Therefore, $\nu_C(x+y, q) \leq \max\{ \nu_C(x, q), \nu_C(y, q) \}$, forall x and y in R and q in Q. And $\nu_C(xy, q) = \max\{\nu_A(xy, q), \nu_B(xy, q)\} \leq \max\{\max\{\nu_A(x, q), \nu_A(y, q)\}, \max\{\nu_B(x, q), \nu_B(y, q)\}\} = \max\{\max\{\nu_A(x, q), \nu_B(x, q)\}, \max\{\nu_A(y, q), \nu_B(y, q)\}\} = \max\{\nu_C(x, q), \nu_C(y, q)\}$. Therefore, $\nu_C(xy, q) \leq \max\{\nu_C(x, q), \nu_C(y, q)\}$, forall x and y in R and q in Q. Therefore C is a Q-intuitionistic fuzzy subsemiring of R. Hence the intersection of any two Q-intuitionistic fuzzy subsemirings of a semiring R is a Q-intuitionistic fuzzy subsemiring of R.

3.2 Theorem

The intersection of a family of Q-intuitionistic fuzzy subsemirings of a semiring R is a Q-intuitionistic fuzzy subsemiring of R.

Proof: Let $\{V_i : i \in I\}$ be a family of Q-intuitionistic fuzzy subsemirings of a semiring R and let $A = \bigcap_{i \in I} V_i$. Let $x, y \in R$ and $q \in Q$. Then $\mu_A(x+y, q) = \inf_{i \in I} \mu_{V_i}(x+y, q) \geq \inf_{i \in I} \min\{\mu_{V_i}(x, q), \mu_{V_i}(y, q)\} = \min\{\inf_{i \in I} \mu_{V_i}(x, q), \inf_{i \in I} \mu_{V_i}(y, q)\} = \min\{\mu_A(x, q), \mu_A(y, q)\}$. Therefore, $\mu_A(x+y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$, for all $x, y \in R$ and $q \in Q$. And, $\mu_A(xy, q) = \inf_{i \in I} \mu_{V_i}(xy, q) \geq \inf_{i \in I} \min\{\mu_{V_i}(x, q), \mu_{V_i}(y, q)\} = \min\{\inf_{i \in I} \mu_{V_i}(x, q), \inf_{i \in I} \mu_{V_i}(y, q)\} = \min\{\mu_A(x, q), \mu_A(y, q)\}$. Therefore, $\mu_A(xy, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$, forall x and y in R and q in Q. Now, $\nu_A(x+y, q) = \sup_{i \in I} \nu_{V_i}(x+y, q) \leq \sup_{i \in I} \max\{\nu_{V_i}(x, q), \nu_{V_i}(y, q)\} = \max\{\sup_{i \in I} \nu_{V_i}(x, q), \sup_{i \in I} \nu_{V_i}(y, q)\} = \max\{\nu_A(x, q), \nu_A(y, q)\}$. Therefore, $\nu_A(x+y, q) \leq \max\{\nu_A(x, q), \nu_A(y, q)\}$, forall x and y in R and q in Q. And, $\nu_A(xy, q) = \sup_{i \in I} \nu_{V_i}(xy, q) \leq \sup_{i \in I} \max\{\nu_{V_i}(x, q), \nu_{V_i}(y, q)\} = \max\{\sup_{i \in I} \nu_{V_i}(x, q), \sup_{i \in I} \nu_{V_i}(y, q)\} = \max\{\nu_A(x, q), \nu_A(y, q)\}$. Therefore, $\nu_A(xy, q) \leq \max\{\nu_A(x, q), \nu_A(y, q)\}$, forall x and y in R and q in Q. That is, A is a Q-intuitionistic fuzzy subsemiring of a semiring R. Hence, the intersection of a family of Q-intuitionistic fuzzy subsemirings of R is a Q-intuitionistic fuzzy subsemiring of R.

3.3 Theorem

If A and B are any two Q-intuitionistic fuzzy subsemirings of the semirings R_1 and R_2 respectively, then $A \times B$ is a Q-intuitionistic fuzzy subsemiring of $R_1 \times R_2$.

Proof: Let A and B be two Q-intuitionistic fuzzy subsemirings of the semirings R_1 and R_2 respectively. Let $x_1, x_2 \in R_1$, $y_1, y_2 \in R_2$. Then (x_1, y_1) and $(x_2, y_2) \in R_1 \times R_2$. Now, $\mu_{A\times B}[(x_1, y_1)+(x_2, y_2), q] = \mu_{A\times B}((x_1+x_2, y_1+y_2), q) = \min\{\mu_A(x_1+x_2, q), \mu_B(y_1+y_2, q)\} \geq \min\{\min\{\mu_A(x_1, q), \mu_A(x_2, q)\}, \min\{\mu_B(y_1, q), \mu_B(y_2, q)\}\} = \min\{\min\{\mu_A(x_1, q), \mu_B(y_1, q)\}, \min\{\mu_A(x_2, q), \mu_B(y_2, q)\}\} =$

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$\min\{\mu_{A\times B}((x_1, y_1), q), \mu_{A\times B}((x_2, y_2), q)\}$. Therefore, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2), q] \geq \min\{\mu_{A\times B}((x_1, y_1), q), \mu_{A\times B}((x_2, y_2), q)\}$. Also, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2), q] = \mu_{A\times B}((x_1x_2, y_1y_2), q) = \min\{\mu_A(x_1x_2, q), \mu_B(y_1y_2, q)\} \geq \min\{\min\{\mu_A(x_1, q), \mu_A(x_2, q)\}, \min\{\mu_B(y_1, q), \mu_B(y_2, q)\}\} = \min\{\min\{\mu_A(x_1, q), \mu_B(y_1, q)\}, \min\{\mu_A(x_2, q), \mu_B(y_2, q)\}\} = \min\{\mu_{A\times B}((x_1, y_1), q), \mu_{A\times B}((x_2, y_2), q)\}$. Therefore, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2), q] \geq \min\{\mu_{A\times B}((x_1, y_1), q), \mu_{A\times B}((x_2, y_2), q)\}$. Now, $\nu_{A\times B}[(x_1, y_1) + (x_2, y_2), q] = \nu_{A\times B}((x_1+x_2, y_1+y_2), q) = \max\{\nu_A(x_1+x_2, q), \nu_B(y_1+y_2, q)\} \leq \max\{\max\{\nu_A(x_1, q), \nu_A(x_2, q)\}, \max\{\nu_B(y_1, q), \nu_B(y_2, q)\}\} = \max\{\max\{\nu_A(x_1, q), \nu_B(y_1, q)\}, \max\{\nu_A(x_2, q), \nu_B(y_2, q)\}\} = \max\{\nu_{A\times B}((x_1, y_1), q), \nu_{A\times B}((x_2, y_2), q)\}$. Therefore, $\nu_{A\times B}[(x_1, y_1) + (x_2, y_2), q] \leq \max\{\nu_{AxB}((x_1, y_1), q), \nu_{AxB}((x_2, y_2), q)\}$. Also, $\nu_{A\times B}[(x_1, y_1)(x_2, y_2), q] = \nu_{A\times B}((x_1x_2, y_1y_2), q) = \max\{\nu_A(x_1x_2, q), \nu_B(y_1y_2, q)\} \leq \max\{\max\{\nu_A(x_1, q), \nu_A(x_2, q)\}, \max\{\nu_B(y_1, q), \nu_B(y_2, q)\}\} = \max\{\max\{\nu_A(x_1, q), \nu_B(y_1, q)\}, \max\{\nu_A(x_2, q), \nu_B(y_2, q)\}\} = \max\{\nu_{A\times B}((x_1, y_1), q), \nu_{A\times B}((x_2, y_2), q)\}$. Therefore, $\nu_{A\times B}[(x_1, y_1)(x_2, y_2), q] \leq \max\{\nu_{A\times B}((x_1, y_1), q), \nu_{A\times B}((x_2, y_2), q)\}$. Hence $A\times B$ is a Q-intuitionistic fuzzy subsemiring of semiring of $R_1\times R_2$.

3.4 Theorem

Let A be a Q-intuitionistic fuzzy subset of a semiring R and V be the strongest Q-intuitionistic fuzzy relation of R. Then A is a Q-intuitionistic fuzzy subsemiring of R if and only if V is a Q-intuitionistic fuzzy subsemiring of $R\times R$.

Proof: Suppose that A is a Q-intuitionistic fuzzy subsemiring of a semiring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in $R\times R$. We have $\mu_V(x+y, q) = \mu_V[(x_1, x_2)+(y_1, y_2), q] = \mu_V((x_1+y_1, x_2+y_2), q) = \min\{\mu_A(x_1+y_1, q), \mu_A(x_2+y_2, q)\} \geq \min\{\min\{\mu_A(x_1, q), \mu_A(y_1, q)\}, \min\{\mu_A(x_2, q), \mu_A(y_2, q)\}\} = \min\{\min\{\mu_A(x_1, q), \mu_A(x_2, q)\}, \min\{\mu_A(y_1, q), \mu_A(y_2, q)\}\} = \min\{\mu_V((x_1, x_2), q), \mu_V((y_1, y_2), q)\} = \min\{\mu_V(x, q), \mu_V(y, q)\}$. Therefore, $\mu_V(x+y, q) \geq \min\{\mu_V(x, q), \mu_V(y, q)\}$, for all x and y in $R\times R$ and q in Q. And, $\mu_V(xy, q) = \mu_V[(x_1, x_2)(y_1, y_2), q] = \mu_V((x_1y_1, x_2y_2), q) = \min\{\mu_A(x_1y_1, q), \mu_A(x_2y_2, q)\} \geq \min\{\min\{\mu_A(x_1, q), \mu_A(y_1, q)\}, \min\{\mu_A(x_2, q), \mu_A(y_2, q)\}\} = \min\{\min\{\mu_A(x_1, q), \mu_A(x_2, q)\}, \min\{\mu_A(y_1, q), \mu_A(y_2, q)\}\} = \min\{\mu_V((x_1, x_2), q), \mu_V((y_1, y_2), q)\} = \min\{\mu_V(x, q), \mu_V(y, q)\}$. Therefore, $\mu_V(xy, q) \geq \min\{\mu_V(x, q), \mu_V(y, q)\}$, for all x and y in $R\times R$ and q in Q. Wehave, $\nu_V(x+y, q) = \nu_V[(x_1, x_2)+(y_1, y_2), q] = \nu_V((x_1+y_1, x_2+y_2), q) = \max\{\nu_A(x_1+y_1, q), \nu_A(x_2+y_2, q)\} \leq \max\{\max\{\nu_A(x_1, q), \nu_A(y_1, q)\}, \max\{\nu_A(x_2, q), \nu_A(y_2, q)\}\} = \max\{\max\{\nu_A(x_1, q), \nu_A(x_2, q)\}, \max\{\nu_A(y_1, q), \nu_A(y_2, q)\}\} = \max\{\nu_V((x_1, x_2), q), \nu_V((y_1, y_2), q)\} = \max\{\nu_V(x, q), \nu_V(y, q)\}$. Therefore, $\nu_V(x+y, q) \leq \max\{\nu_V(x, q), \nu_V(y, q)\}$, for all x and y in $R\times R$ and q in Q. And, $\nu_V(xy, q) = \nu_V[(x_1, x_2)(y_1, y_2), q] = \nu_V((x_1y_1, x_2y_2), q) = \max\{\nu_A(x_1y_1, q), \nu_A(x_2y_2, q)\} \leq \max\{\max\{\nu_A(x_1, q), \nu_A(y_1, q)\}, \max\{\nu_A(x_2, q), \nu_A(y_2, q)\}\} = \max\{\max\{\nu_A(x_1, q), \nu_A(x_2, q)\}, \max\{\nu_A(y_1, q), \nu_A(y_2, q)\}\} = \max\{\nu_V((x_1, x_2), q), \nu_V((y_1, y_2), q)\} = \max\{\nu_V(x, q), \nu_V(y, q)\}$. Therefore, $\nu_V(xy, q) \leq \max\{\nu_V(x, q), \nu_V(y, q)\}$, for all x and y in $R\times R$ and q in Q. This proves that V is a Q-intuitionistic fuzzy subsemiring of $R\times R$. Conversely assume that V is a Q-intuitionistic fuzzy subsemiring of $R\times R$, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R\times R$, we have $\min\{\mu_A(x_1+y_1, q), \mu_A(x_2+y_2, q)\} = \mu_V((x_1+y_1, x_2+y_2), q) =$

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$\mu_V[(x_1, x_2)+(y_1, y_2), q] = \mu_V(x+y, q) \geq \min\{\mu_V(x, q), \mu_V(y, q)\} = \min\{\mu_V((x_1, x_2), q), \mu_V((y_1, y_2), q)\} = \min\{\min\{\mu_A(x_1, q), \mu_A(x_2, q)\}, \min\{\mu_A(y_1, q), \mu_A(y_2, q)\}\}$. If $\mu_A(x_1+y_1, q) \leq \mu_A(x_2+y_2, q)$, $\mu_A(x_1, q) \leq \mu_A(x_2, q)$, $\mu_A(y_1, q) \leq \mu_A(y_2, q)$, we get, $\mu_A(x_1+y_1, q) \geq \min\{\mu_A(x_1, q), \mu_A(y_1, q)\}$, for all x_1 and y_1 in R and q in Q. And, $\min\{\mu_A(x_1y_1, q), \mu_A(x_2y_2, q)\} = \mu_V((x_1y_1, x_2y_2), q) = \mu_V[(x_1, x_2)(y_1, y_2), q] = \mu_V(xy, q) \geq \min\{\mu_V(x, q), \mu_V(y, q)\} = \min\{\mu_V((x_1, x_2), q), \mu_V((y_1, y_2), q)\} = \min\{\min\{\mu_A(x_1, q), \mu_A(x_2, q)\}, \min\{\mu_A(y_1, q), \mu_A(y_2, q)\}\}$. If $\mu_A(x_1y_1, q) \leq \mu_A(x_2y_2, q)$, $\mu_A(x_1, q) \leq \mu_A(x_2, q)$, $\mu_A(y_1, q) \leq \mu_A(y_2, q)$, we get $\mu_A(x_1y_1, q) \geq \min\{\mu_A(x_1, q), \mu_A(y_1, q)\}$, for all x_1 and y_1 in R and q in Q. We have $\max\{\nu_A(x_1+y_1, q), \nu_A(x_2+y_2, q)\} = \nu_V((x_1+y_1, x_2+y_2), q) = \nu_V[(x_1, x_2)+(y_1, y_2), q] = \nu_V(x+y, q) \leq \max\{\nu_V(x, q), \nu_V(y, q)\} = \max\{\nu_V((x_1, x_2), q), \nu_V((y_1, y_2), q)\} = \max\{\max\{\nu_A(x_1, q), \nu_A(x_2, q)\}, \max\{\nu_A(y_1, q), \nu_A(y_2, q)\}\}$. If $\nu_A(x_1+y_1, q) \geq \nu_A(x_2+y_2, q)$, $\nu_A(x_1, q) \geq \nu_A(x_2, q)$, $\nu_A(y_1, q) \geq \nu_A(y_2, q)$, we get, $\nu_A(x_1+y_1, q) \leq \max\{\nu_A(x_1, q), \nu_A(y_1, q)\}$, for all x_1 and y_1 in R and q in Q. And, $\max\{\nu_A(x_1y_1, q), \nu_A(x_2y_2, q)\} = \nu_V((x_1y_1, x_2y_2), q) = \nu_V[(x_1, x_2)(y_1, y_2), q] = \nu_V(xy, q) \leq \max\{\nu_V(x, q), \nu_V(y, q)\} = \max\{\nu_V((x_1, x_2), q), \nu_V((y_1, y_2), q)\} = \max\{\max\{\nu_A(x_1, q), \nu_A(x_2, q)\}, \max\{\nu_A(y_1, q), \nu_A(y_2, q)\}\}$. If $\nu_A(x_1y_1, q) \geq \nu_A(x_2y_2, q)$, $\nu_A(x_1, q) \geq \nu_A(x_2, q)$, $\nu_A(y_1, q) \geq \nu_A(y_2, q)$, we get $\nu_A(x_1y_1, q) \leq \max\{\nu_A(x_1, q), \nu_A(y_1, q)\}$, for all x_1 and y_1 in R and q in Q. Therefore A is a Q-intuitionistic fuzzy subsemiring of R.

3.5 Theorem

If A is a Q-intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$, then $H = \{x \,/\, x \in R: \mu_A(x, q) = 1, \nu_A(x, q) = 0\}$ is either empty or is a subsemiring of R.

Proof: If the condition is not satisfied by any element, then H is empty. If x and y in H and q in Q, then $\mu_A(x+y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\} = \min\{1, 1\} = 1$. Therefore, $\mu_A(x+y, q) = 1$. And $\mu_A(xy, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\} = \min\{1, 1\} = 1$. Therefore, $\mu_A(xy, q) = 1$. Now, $\nu_A(x+y, q) \leq \max\{\nu_A(x, q), \nu_A(y, q)\} = \max\{0, 0\} = 0$. Therefore, $\nu_A(x+y, q) = 0$. And $\nu_A(xy, q) \leq \max\{\nu_A(x, q), \nu_A(y, q)\} = \max\{0, 0\} = 0$. Therefore, $\nu_A(xy, q) = 0$. We get x+y, xy in H. This implies that H is a subsemiring of R. Hence, H is either empty or is a subsemiring of R.

3.6 Theorem

Let A be a Q-intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$.

(i) If $\mu_A(x+y, q) = 0$, then either $\mu_A(x, q) = 0$ or $\mu_A(y, q) = 0$, for all x and y in R and q in Q.

(ii) If $\mu_A(xy, q) = 0$, then either $\mu_A(x, q) = 0$ or $\mu_A(y, q) = 0$, for all x and y in R and q in Q.

(iii) If $\nu_A(x+y, q) = 1$, then either $\nu_A(x, q) = 1$ or $\nu_A(y, q) = 1$, for all x and y in R and q in Q.

(iv) If $\nu_A(xy, q) = 1$, then either $\nu_A(x, q) = 1$ or $\nu_A(y, q) = 1$, for all x and y in R and q in Q.

Proof: Let x and y be arbitrary elements in R and q in Q. (i) By the definition $\mu_A(x+y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$, we have that $0 \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$. This implies that either $\mu_A(x, q) = 0$ or $\mu_A(y, q) = 0$. (ii) By the definition $\mu_A(xy, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$, we have that $0 \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$. Therefore, either $\mu_A(x, q) = 0$ or $\mu_A(y, q) = 0$. (iii) By the definition $\nu_A(x+y, q) \leq \max\{\nu_A(x, q), \nu_A(y, q)\}$, which implies that $1 \leq \max\{\nu_A(x, q), \nu_A(y, q)\}$. Therefore,

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either $\nu_A(x, q) = 1$ or $\nu_A(y, q) = 1$. (iv) By the definition $\nu_A(xy, q) \leq \max\{\nu_A(x, q), \nu_A(y, q)\}$, which implies that $1 \leq \max\{\nu_A(x, q), \nu_A(y, q)\}$. Therefore, either $\nu_A(x, q) = 1$ or $\nu_A(y, q) = 1$.

3.7 Theorem

If A is a Q-intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$, then $H=\{\langle (x, q), \mu_A(x, q) \rangle$, for all x in R and q in Q $\}$ is either empty or a Q-fuzzy subsemiring of R.

Proof: It is trivial.

3.8 Theorem

If A is a Q-intuitionistic fuzzy subsemiring of a semiring$(R, +, \cdot)$, then $H =\{\langle (x, q), \nu_A(x, q)\rangle\}$ is either empty or an anti Q-fuzzy subsemiring of R.

Proof: It is trivially true.

3.9 Theorem

If A is a Q-intuitionistic fuzzy subsemiring of a semiring$(R, +, \cdot)$, then $\Box A$ is a Q-intuitionistic fuzzy subsemiring of R.

Proof: Let A be a Q-intuitionistic fuzzy subsemiring of a semiring R. Now take $A = \{\langle (x, q), \mu_A(x, q), \nu_A(x, q) \rangle\}$, for all $x \in R$ and $q \in Q$, we take $\Box A = B =\{\langle (x, q), \mu_B(x, q), \nu_B(x, q) \rangle\}$, where $\mu_B(x, q) = \mu_A(x, q)$, $\nu_B(x, q) =1-\mu_A(x, q)$. Clearly, $\mu_B(x+y, q) \geq \min\{\mu_B(x, q), \mu_B(y, q)\}$, for all x and y in R and q in Q and $\mu_B(xy, q) \geq \min\{\mu_B(x, q), \mu_B(y, q)\}$, for all x and y in R and q in Q. Since A is a Q-intuitionistic fuzzy subsemiring of R, we have $\mu_A(x+y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$, for all x and y in R and q in Q, we have that $1-\nu_B(x+y, q) \geq \min\{(1-\nu_B(x, q)), (1-\nu_B(y, q))\}$, which implies that $\nu_B(x+y, q) \leq 1-\min\{(1-\nu_B(x, q)), (1-\nu_B(y, q))\} = \max\{\nu_B(x, q), \nu_B(y, q)\}$. Therefore, $\nu_B(x+y, q) \leq \max\{\nu_B(x, q), \nu_B(y, q)\}$, for all x and y in R and q in Q. And $\mu_A(xy, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$, for all x and y in R and q in Q, which implies that $1-\nu_B(xy, q) \geq \min\{(1-\nu_B(x, q)), (1-\nu_B(y, q))\}$, which implies that $\nu_B(xy, q) \leq 1-\min\{(1-\nu_B(x, q)), (1-\nu_B(y, q))\} = \max\{\nu_B(x, q), \nu_B(y, q)\}$. Therefore, $\nu_B(xy, q) \leq \max\{\nu_B(x, q), \nu_B(y, q)\}$, for all x and y in R and q in Q. Hence $B = \Box A$ is a Q-intuitionistic fuzzy subsemiring of a semiring R.

3.10 Remark: The converse of the above theorem is not true.

It is shown by the following example: Consider the semiring $Z_5 = \{0, 1, 2, 3, 4\}$ with addition modulo 5 and multiplication modulo 5 operations and $Q = \{q\}$. Then $A =\{\langle (0, q), 0.7, 0.2\rangle, \langle (1, q), 0.5, 0.1 \rangle, \langle (2, q), 0.5, 0.4 \rangle, \langle (3, q), 0.5, 0.1 \rangle, \langle (4, q), 0.5, 0.4 \rangle\}$ is not a Q-intuitionistic fuzzy subsemiring of Z_5, but $\Box A =\{\langle (0, q), 0.7, 0.3 \rangle, \langle (1, q), 0.5, 0.5 \rangle, \langle (2, q), 0.5, 0.5 \rangle, \langle (3, q), 0.5, 0.5 \rangle, \langle (4, q), 0.5, 0.5 \rangle\}$ is a Q-intuitionistic fuzzy subsemiring of Z_5.

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3.11 Theorem

If A is a Q-intuitionistic fuzzy subsemiring of a semiring(R, +, ·), then ◊A is a Q-intuitionistic fuzzy subsemiring of R.

Proof: Let A be a Q-intuitionistic fuzzy subsemiring of a semiring R. Take A = { 〈 (x, q), μ_A(x, q), ν_A(x, q) 〉 }, for all x ϵ R and q ϵ Q. Let ◊A = B = { 〈 (x, q), μ_B(x, q), ν_B(x, q) 〉 }, where μ_B(x, q) = 1−ν_A(x, q), ν_B(x, q) = ν_A(x, q). Clearly, ν_B(x+y, q) ≤ max{ν_B(x, q), ν_B(y, q) }, for all x and y in R and q in Q and ν_B(xy, q) ≤ max{ ν_B(x, q), ν_B(y, q) }, for all x and y in R and q in Q. Since A is a Q-intuitionistic fuzzy subsemiring of R, we have ν_A(x+y, q) ≤ max {ν_A(x, q), ν_A(y, q) }, for all x and y in R and q in Q, which implies that 1−μ_B(x+y, q) ≤ max {(1− μ_B(x, q)), (1− μ_B(y, q))}, which implies that μ_B(x+y, q) ≥ 1− max { (1− μ_B(x, q)), (1− μ_B(y, q))} = min { μ_B(x, q), μ_B(y, q) }. Therefore, μ_B(x+y, q) ≥ min{μ_B(x, q), μ_B(y, q) }, for all x and y in R and q in Q. And ν_A(xy, q) ≤ max { ν_A(x, q), ν_A(y, q) }, for all x and y in R and q in Q, which implies that 1−μ_B(xy, q) ≤ max { (1−μ_B(x, q)), (1−μ_B(y, q))} which implies thatμ_B(xy, q) ≥ 1− max {(1−μ_B(x, q)), (1−μ_B(y, q)) }= min { μ_B(x, q), μ_B(y, q) }. Therefore, μ_B(xy, q) ≥ min {μ_B(x, q), μ_B(y, q) }, for all x and y in R and q in Q. Hence B = ◊A is a Q-intuitionistic fuzzy subsemiring of a semiring R.

3.12 Remark: The converse of the above theorem is not true.

It is shown by the following example: Consider the semiring Z_5 = { 0, 1, 2, 3, 4 } with addition modulo 5 and multiplication modulo 5 operations and Q = {q}. Here A = {〈 (0, q), 0.5, 0.1〉, 〈 (1, q), 0.6, 0.4 〉, 〈 (2, q), 0.5, 0.4 〉, 〈 (3, q), 0.6, 0.4 〉, 〈 (4, q), 0.5, 0.4 〉 } is not a Q-intuitionistic fuzzy subsemiring of Z_5, but ◊A = {〈 (0, q), 0.9, 0.1 〉, 〈 (1, q), 0.6, 0.4〉, 〈(2, q), 0.6, 0.4〉, 〈 (3, q), 0.6, 0.4 〉, 〈 (4, q), 0.6, 0.4 〉 } is a Q-intuitionistic fuzzy subsemiring of Z_5. In the following Theorem ◦ is the composition operation of functions:

3.13 Theorem

Let A be a Q-intuitionistic fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then A◦f is a Q-intuitionistic fuzzy subsemiring of R.

Proof: Let x and y be arbitrary elements in R and q in Q and A be a Q-intuitionistic fuzzy subsemiring of a semiring H. Then we have, (μ_A◦f)(x+y, q) = μ_A(f(x + y), q) = μ_A(f(x)+ f(y), q) ≥ min {μ_A(f(x), q), μ_A(f(y), q) } ≥ min {(μ_A◦f)(x, q), (μ_A◦f)(y, q)}, which implies that (μ_A◦f)(x+y, q) ≥ min { (μ_A◦f)(x, q), (μ_A◦f)(y, q) }. And (μ_A◦f)(xy, q) = μ_A(f(xy), q) = μ_A(f(x)f(y), q) ≥ min {μ_A(f(x), q), μ_A(f(y), q) } ≥ min{ (μ_A◦f)(x, q), (μ_A◦f)(y, q) }, which implies that (μ_A◦f)(xy, q) ≥ min {(μ_A◦f)(x, q), (μ_A◦f)(y, q) }. Then we have, (ν_A◦f)(x+y, q) = ν_A(f(x+y), q) = ν_A(f(x)+f(y), q) ≤ max {ν_A(f(x), q), ν_A(f(y), q) }≤ max { (ν_A◦f)(x, q), (ν_A◦f)(y, q) }, which implies that (ν_A◦f)(x+y, q) ≤ max {(ν_A◦f)(x, q), (ν_A◦f)(y, q) }. And (ν_A◦f)(xy, q) = ν_A(f(xy), q) = ν_A(f(x)f(y), q) ≤ max {ν_A(f(x), q), ν_A(f(y), q) }≤ max{ (ν_A◦f)(x, q), (ν_A◦f)(y, q) }, which implies that (ν_A◦f)(xy, q) ≤ max {(ν_A◦f)(x, q), (ν_A◦f)(y, q) }. Therefore (A◦f) is a Q-intuitionistic fuzzy subsemiring of a semiring R.

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3.14 Theorem

Let A be a Q-intuitionistic fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then A∘f is a Q-intuitionistic fuzzy subsemiring of R.

Proof: Let x and y be arbitrary elements in R and q in Q and A be a Q-intuitionistic fuzzy subsemiring of a semiring H. Then we have, $(\mu_A\circ f)(x+y, q) = \mu_A(f(x+y), q) = \mu_A(f(y)+f(x), q) \geq \min\{\mu_A(f(x), q), \mu_A(f(y), q)\} \geq \min\{(\mu_A\circ f)(x, q), (\mu_A\circ f)(y, q)\}$, which implies that $(\mu_A\circ f)(x+y, q) \geq \min\{(\mu_A\circ f)(x, q), (\mu_A\circ f)(y, q)\}$. And $(\mu_A\circ f)(xy, q)= \mu_A(f(xy), q) = \mu_A(f(y)f(x), q) \geq \min\{\mu_A(f(x), q), \mu_A(f(y), q)\} \geq \min\{(\mu_A\circ f)(x, q), (\mu_A\circ f)(y, q)\}$, which implies that $(\mu_A\circ f)(xy, q) \geq \min\{(\mu_A\circ f)(x, q), (\mu_A\circ f)(y, q)\}$. Then we have, $(\nu_A\circ f)(x+y, q) = \nu_A(f(x+y), q) = \nu_A(f(y)+f(x), q) \leq \max\{\nu_A(f(x), q), \nu_A(f(y), q)\} \leq \max\{(\nu_A\circ f)(x, q), (\nu_A\circ f)(y, q)\}$, which implies that $(\nu_A\circ f)(x+y, q) \leq \max\{(\nu_A\circ f)(x, q), (\nu_A\circ f)(y, q)\}$. And $(\nu_A\circ f)(xy, q) = \nu_A(f(xy), q) = \nu_A(f(y)f(x), q) \leq \max\{\nu_A(f(x), q), \nu_A(f(y), q)\} \leq \max\{(\nu_A\circ f)(x, q), (\nu_A\circ f)(y, q)\}$, which implies that $(\nu_A\circ f)(xy, q) \leq \max\{(\nu_A\circ f)(x, q), (\nu_A\circ f)(y, q)\}$. Therefore A∘f is a Q-intuitionistic fuzzy subsemiring of a semiring R.

3.13 Theorem

Let A be a Q-intuitionistic fuzzy subsemiring of a semiring (R, +, .), then the pseudo Q-intuitionistic fuzzy coset $(aA)^p$ is a Q-intuitionistic fuzzy subsemiring of a semiring R, for every a in R.

Proof: Let A be a Q-intuitionistic fuzzy subsemiring of a semiring R. For every x and y in R and q in Q, we have, $((a\mu_A)^p)(x+y, q) = p(a)\mu_A(x+y, q) \geq p(a)\min\{\mu_A(x, q),\mu_A(y, q)\} = \min\{p(a)\mu_A(x, q), p(a)\mu_A(y, q)\}= \min\{((a\mu_A)^p)(x, q), ((a\mu_A)^p)(y, q)\}$. Therefore, $((a\mu_A)^p)(x+y, q) \geq \min\{((a\mu_A)^p)(x, q), ((a\mu_A)^p)(y, q)\}$. Now, $((a\mu_A)^p)(xy, q) = p(a)\mu_A(xy, q) \geq p(a)\min\{\mu_A(x, q), \mu_A(y, q)\}= \min\{p(a)\mu_A(x, q), p(a)\mu_A(y, q)\}= \min\{((a\mu_A)^p)(x, q), ((a\mu_A)^p)(y, q)\}$. Therefore, $((a\mu_A)^p)(xy, q) \geq \min\{((a\mu_A)^p)(x, q), ((a\mu_A)^p)(y, q)\}$. For every x and y in R and q in Q, we have, $((a\nu_A)^p)(x+y, q) = p(a)\nu_A(x+y, q) \leq p(a)\max\{\nu_A(x, q),\nu_A(y, q)\} = \max\{p(a)\nu_A(x, q), p(a)\nu_A(y, q)\}= \max\{((a\nu_A)^p)(x, q), ((a\nu_A)^p)(y, q)\}$. Therefore, $((a\nu_A)^p)(x+y, q) \leq \max\{((a\nu_A)^p)(x, q), ((a\nu_A)^p)(y, q)\}$. Now, $((a\nu_A)^p)(xy, q) = p(a)\nu_A(xy, q) \leq p(a)\max\{\nu_A(x, q),\nu_A(y, q)\} = \max\{p(a)\nu_A(x, q), p(a)\nu_A(y, q)\} = \max\{((a\nu_A)^p)(x, q), ((a\nu_A)^p)(y, q)\}$. Therefore, $((a\nu_A)^p)(xy, q) \leq \max\{((a\nu_A)^p)(x, q), ((a\nu_A)^p)(y, q)\}$. Hence $(aA)^p$ is a Q-intuitionistic fuzzy subsemiring of a semiring R.

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