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CUBIC DUAL IDEALS IN BCK-ALGEBRAS

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ABSTRACT
In this paper we introduce the concept of cubic set to dual sub algebras and dual ideals in BCK-algebras and investigate some of its properties. The relationship between dual sub algebras and cubic dual sub algebras are given.

Keywords: BCK-Algebra, Fuzzy ideal, Dual subalgebra, Fuzzy dual ideal, Cubic dual subalgebra, Cubic dual ideal.

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Contribution/ Originality

The primary contribution of this paper is application of cubic set to dual ideals in BCK-algebras and investigates some of its properties. This study originates new definition cubic dual ideal in BCK-algebras.

1. INTRODUCTION

The concept of fuzzy set was introduced in 1965 by Zadeh [1] and since then, several researchers have explored the generalization of the notion of fuzzy sets. The study of BCK-algebras was initiated by Imai and Iseki [2] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. The notion of interval-valued fuzzy sets was first introduced by Zadeh [1] as an extension of fuzzy sets. Moreover, Jun, et al. [3] introduced the notion of cubic sets as a generalization of fuzzy set and interval-valued fuzzy set. In Satyanarayana, et al. [4]. Applied the concept of interval-valued intuitionistic fuzzy dual ideals of BF-algebras. In this paper we apply the concept of cubic set to dual ideals in BCK-algebras and investigate some of its properties.

A BCK-algebra is a non-empty set X with a binary operation $*$ and a constant 0 satisfying the following axioms:

(BCK-1) $((x*y)*(x*z))*(z*y)=0$,
(BCK-2) $(x*(x*y))*y=0$,
(BCK-3) $x*x=0$,
(BCK-4) $0*x=0$,
(BCK-5) $x*y=0$ and $y*x=0 \Rightarrow x=y$, for all $x, y, z \in X$

A BCK-algebra can be partially ordered by $x \le y$ if and only if $x*y=0$ this ordering is called BCK-ordering. The following statements are true in a BCK-algebra:

(i) $x*0=x$,

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(ii) $x * y \leq x$,

(iii) $(x * y) * z \leq (x * z) * y$,

(iv) $(x * y) * z \leq (x * z) * (y * z)$,

(v) $x \leq y \Rightarrow x * z \leq y * z$ and $z * y \leq z * x$.

Definition 1.1 Meng [5] a non-empty subset I of a BCK-algebra X is called an ideal, if it satisfies:

(I_1) $0 \in I$

(I_2) $x * y$ and $y \in I \Rightarrow x \in I$, for any $x, y \in X$

If there is an element 1 of X satisfying $x \leq 1$, for all $x \in X$, then the element 1 is called unit of X. A BCK-algebra with unit is called bounded. In a bounded BCK-algebra, we denote $1 * x$ by Nx for brief.

Definition 1.2 Meng [5] a non-empty set D in a BCK-algebra X is said to be dual ideal of X if it satisfies:

(i) $1 \in D$,

(ii) $N(Nx * Ny) \in D$ and $y \in D \Rightarrow x \in D$ for any $x, y \in X$.

Let X be the collection of objects denoted generically by x. Then the fuzzy set A in X is defined as $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A(x)$ is called the membership value of x in A and $0 \leq \mu_A(x) \leq 1$.

Definition 1.3 For fuzzy sets μ and λ of X and $s, t \in [0,1]$. The sets $U(\mu; t) = \{x \in X : \mu(x) \geq t\}$ is called upper t-level cut of μ and

$L(\lambda; s) = \{x \in X : \lambda(x) \leq s\}$ is called lower s-level cut of λ.

Definition 1.4 A fuzzy set $\mu : X \to [0,1]$ is called fuzzy sub- algebra of X, if

$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

Definition 1.5 Meng and Jun [6] afuzzy subset μ of X is said to be fuzzy dual ideal of X, if

(i) $\mu(1) \geq \mu(x)$

(ii) $\mu(x) \geq \min\{\mu(N(Nx * Ny)), \mu(y)\}$ for all $x, y \in X$.

Definition 1.6 Meng and Jun [6] the fuzzy set μ in X is called fuzzy dual sub algebra of X if it satisfies

$\mu(N(Nx * Ny) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

Now we recall the concept of interval-valued fuzzy sets:

By the interval number D we mean an interval $[a^-, a^+]$ where $0 \leq a^- \leq a^+ \leq 1$

For interval numbers $D_1 = [a_1^-, b_1^+]$, $D_2 = [a_2^-, b_2^+]$.

We define

$$Min(D_1, D_2) = D_1 \cap D_2 = min([a_1^-, b_1^+], [a_2^-, b_2^+]) = min[\{a_1^-, a_2^-\}, \{b_1^+, b_2^+\}]$$

$$Max(D_1, D_2) = D_1 \cup D_2 = max([a_1^-, b_1^+], [a_2^-, b_2^+]) = max[\{a_1^-, a_2^-\}, \{b_1^+, b_2^+\}]$$

$$D_1 + D_2 = [a_1^- + a_2^- - a_1^- . a_2^-, b_1^+ + b_2^+ - b_1^+ . b_2^+]$$

And put

$$D_1 \leq D_2 \Leftrightarrow a_1^- \leq a_2^- \text{ and } b_1^+ \leq b_2^+$$

$$D_1 = D_2 \Leftrightarrow a_1^- = a_2^- \text{ and } b_1^+ = b_2^+$$

$$D_1 < D_2 \Leftrightarrow D_1 \leq D_2 \text{ and } D_1 \neq D_2$$

$$mD = m[a_1^-, b_1^+] = [ma_1^-, mb_1^+], \text{ where } 0 \leq m \leq 1.$$

Let X be a given nonempty set. An interval-valued fuzzy set (briefly, i-v fuzzy set) B on X is defined by

$B = \{(x, [\mu_B^-(x), \mu_B^+(x)]) : x \in X\}$,

Where $\mu_B^-(x)$ and $\mu_B^+(x)$ are fuzzy sets of X such that $\mu_B^-(x) \leq \mu_B^+(x)$ for all $x \in X$. Let $\tilde{\mu}_B(x)$ $= [\mu_B^-(x), \mu_B^+(x)]$, then $B = \{(x, \tilde{\mu}_B(x)) : x \in X\}$, Where $\tilde{\mu}_B : X \to D[0,1]$.

The determination of maximum and minimum between two real numbers is very simple but it is not simple for two intervals. Biswas [7] described a method to find max/sup and min/inf between two intervals or set of intervals.

Definition1.7 Biswas [7] consider two set of intervals $D_1, D_2 \in D[0,1]$. If $D_1 = [a_1^-, a_1^+]$ then

$rmin(D_1, D_2) = [min(a_1^-, a_2^-), min(a_1^+, a_2^+)]$ which is denoted by $D_1 \wedge^r D_2$. Thus if $D_i = [a_i^-, a_i^+] \in D[0,1]$

for $1 \leq i \leq n$ then we define $r\sup_i(D_i) = [\sup_i(a_i^-), \sup_i(a_i^+)]$ that is $\vee_i^r D_i = [\vee_i a_i^-, \vee_i a_i^+]$. Now we

call $D_1 \geq D_2$ iff $a_1^- \geq a_2^-$ and $a_1^+ \geq a_2^+$. Similarly the relations $D_1 \leq D_2$ and $D_1 = D_2$ are defined.

Based on (interval-valued) fuzzy sets, Jun et.al.introduced the notion of (internal, external) cubic sets and investigated several properties.

Definition 1.8 Jun, et al. [3] let X be a non-empty set. A cubic set A in X is a structure which is briefly denoted by $A = (\tilde{\mu}_A, \lambda_A)$ where $\tilde{\mu}_A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set in X and λ_A is fuzzy set in X.

Definition 1.9 Jun, et al. [8] a cubic set $A = (X, \tilde{\mu}_A, \lambda_A)$ in X is a cubic fuzzy ideal (C F-ideal) of X, if it satisfies:

(C F1) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$

(C F2) $\tilde{\mu}_A(x) \geq \text{rmin}\{\tilde{\mu}_A(x*y), \tilde{\mu}_A(y)\}$

(C F3) $\lambda_A(x) \leq \max\{\lambda_A(x*y), \lambda_A(y)\}$, for all $x, y \in X$.

2. CUBIC DUAL-IDEALS OF BCK-ALGEBRAS

Let X denotes a BCK-algebra unless otherwise specified. Combined the definitions of fuzzy dual-ideal over a crisp set and the idea of cubic set we define cubic dual-ideal. After that, we give some important consequences of cubic dual sub algebras and cubic dual ideals in BCK-algebras.

Definition 2.1 A cubic fuzzy set $A=\left(X, \tilde{\mu}_A, \lambda_A\right)$ is called cubic fuzzy dual sub algebra of X if:

(i) $\tilde{\mu}_A(N(Nx*Ny)) \geq \text{rmin}\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\}$

(ii) $\lambda_A(N(Nx*Ny)) \leq \max\{\lambda_A(x), \lambda_A(y)\}$ for all $x, y \in X.$

Proposition 2.2 Every cubic fuzzy dual sub algebra $A=\left(X, \tilde{\mu}_A, \lambda_A\right)$ of X satisfies the inequalities

$\tilde{\mu}_A(1) \geq \tilde{\mu}_A(x)$ and $\lambda_A(1) \leq \lambda_A(x)$, for all $x, y \in X.$

Theorem 2.3 If $A=\left(X, \tilde{\mu}_A, \lambda_A\right)$ is cubic fuzzy dual sub algebra of X then the sets

$X_{\tilde{\mu}_A} = \{x \in X / \tilde{\mu}_A(x) = \tilde{\mu}_A(1)\}$ and $X_{\lambda_A} = \{x \in X / \lambda_A(x) = \lambda_A(1)\}$ are dual sub algebras of X.

Proof: Let $x, y \in X_{\tilde{\mu}_A}$. Then $\tilde{\mu}_A(x) = \tilde{\mu}_A(1) = \tilde{\mu}_A(y)$ and so

$\tilde{\mu}_A(N(Nx*Ny)) \geq \text{rmin}\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\} = \text{rmin}\{\tilde{\mu}_A(1), \tilde{\mu}_A(1)\} = \tilde{\mu}_A(1)$

$\Rightarrow \tilde{\mu}_A(N(Nx*Ny)) \geq \tilde{\mu}_A(1)$ but $\tilde{\mu}_A(N(Nx*Ny)) \leq \tilde{\mu}_A(1)$

$\Rightarrow \tilde{\mu}_A(N(Nx*Ny)) = \tilde{\mu}_A(1) \Rightarrow N(Nx*Ny) \in X_{\tilde{\mu}_A}$.

Therefore for all $x, y \in X_{\tilde{\mu}_A} \Rightarrow N(Nx*Ny) \in X_{\tilde{\mu}_A}$.

Let $x, y \in X_{\lambda_A}$. then $\lambda_A(x) = \lambda_A(1) = \lambda_A(y)$ and so

$\lambda_A(N(Nx*Ny)) \leq \max\{\lambda_A(x), \lambda_A(y)\} = \max\{\lambda_A(1), \lambda_A(1)\} = \lambda_A(1)$

$\Rightarrow \lambda_A(N(Nx*Ny)) \leq \lambda_A(1)$ but $\lambda_A(N(Nx*Ny)) \geq \lambda_A(1) \Rightarrow \lambda_A(N(Nx*Ny)) = \lambda_A(1)$

$\Rightarrow N(Nx*Ny) \in X_{\lambda_A}$.

Therefore, $X_{\tilde{\mu}_A}$ and X_{λ_A} are dual subalgebras of X.

Theorem 2.4 Let $A=\left(X,\tilde{\mu}_A,\lambda_A\right)$ is cubic fuzzy dual subalgebra of X.

(i) If there exists $\{x_n\}$ in X such that $\lim_{n\to\infty}\tilde{\mu}_A(x_n)=[1,1]$ then $\tilde{\mu}_A(1)=[1,1]$.

(ii) If there exists $\{x_n\}$ in X such that $\lim_{n\to\infty}\lambda_A(x_n)=0$ then $\lambda_A(1)=0.$

Theorem 2.5 Let A_1 and A_2 be cubic fuzzy dual BCK-sub algebras of X. Then $A_1\cap A_2$ is cubic fuzzy dual BCK-sub algebra of X.

Proof: Let $x,y\in A_1\cap A_2$ then $x,y\in A_1$and $x,y\in A_2$. Since A_1 and A_2 are cubic fuzzy dual BCK-sub algebras of X, we have

$$\tilde{\mu}_{A_1\cap A_2}(N(Nx*Ny))$$

$$=[\mu^-_{A_1\cap A_2}(N(Nx*Ny)),\mu^+_{A_1\cap A_2}(N(Nx*Ny))]$$

$$=[\min\{\mu^-_{A_1}(N(Nx*Ny)),\mu^-_{A_2}(N(Nx*Ny))\},\min\{\mu^+_{A_1}(N(Nx*Ny)),\mu^+_{A_2}(N(Nx*Ny))\}]$$

$$\geq[\min\{\min\{\mu^-_{A_1}(x),\mu^-_{A_1}(y)\},\min\{\mu^-_{A_2}(x),\mu^-_{A_2}(y)\}\},\min\{\min\{\mu^+_{A_1}(x),\mu^+_{A_1}(y)\},\min\{\mu^+_{A_2}(x),\mu^+_{A_2}(y)\}\}]$$

$$\geq[\min\{\min\{\mu^-_{A_1}(x),\mu^-_{A_1}(y)\},\min\{\mu^-_{A_2}(x),\mu^-_{A_2}(y)\}\},\min\{\min\{\mu^+_{A_1}(x),\mu^+_{A_1}(y)\},\min\{\mu^+_{A_2}(x),\mu^+_{A_2}(y)\}\}]$$

$$=[\min\{\min\{\mu^-_{A_1}(x),\mu^-_{A_2}(x)\},\min\{\mu^-_{A_1}(y),\mu^-_{A_2}(y)\}\},\min\{\min\{\mu^+_{A_1}(x),\mu^+_{A_2}(x)\},\min\{\mu^+_{A_1}(y),\mu^+_{A_2}(y)\}\}]$$

$$=[\min\{\min\{\mu^-_{A_1}(x),\mu^-_{A_2}(x)\},\min\{\mu^-_{A_1}(y),\mu^-_{A_2}(y)\}\},\min\{\min\{\mu^+_{A_1}(x),\mu^+_{A_2}(x)\},\min\{\mu^+_{A_1}(y),\mu^+_{A_2}(y)\}\}]$$

$$=[\min\{\mu^-_{A_1\cap A_2}(x),\mu^-_{A_1\cap A_2}(y)\},\min\{\mu^+_{A_1\cap A_2}(x),\mu^+_{A_1\cap A_2}(y)\}]$$

$$=r\min\{\tilde{\mu}_{A_1\cap A_2}(x),\tilde{\mu}_{A_1\cap A_2}(y)\}$$

Therefore, $\tilde{\mu}_{A_1\cap A_2}(N(Nx*Ny))\geq r\min\{\tilde{\mu}_{A_1\cap A_2}(x),\tilde{\mu}_{A_1\cap A_2}(y)\}$, for all $x,y\in X.$

$$\lambda_{A_1\cap A_2}(N(Nx*Ny))=\max\{\lambda_{A_1}(N(Nx*Ny)),\lambda_{A_2}(N(Nx*Ny))\}$$

$$\leq\max\{\max\{\lambda_{A_1}(x),\lambda_{A_1}(y)\},\max\{\lambda_{A_2}(x),\lambda_{A_2}(y)\}\}$$

$$=\max\{\max\{\lambda_{A_1}(x),\lambda_{A_2}(x)\},\max\{\lambda_{A_1}(y),\lambda_{A_2}(y)\}\}$$

$$=\max\{\lambda_{A_1\cap A_2}(x),\lambda_{A_1\cap A_2}(y)\}$$

Therefore, $\lambda_{A_1 \cap A_2}(N(Nx * Ny)) \leq \max\{\lambda_{A_1 \cap A_2}(x), \lambda_{A_1 \cap A_2}(y)\}$, for all $x, y \in X.$

Hence $A_1 \cap A_2$ is cubic fuzzy dual BCK-sub algebra of X.

Corollary 2.6 Let $\{A_i / i \in N\}$ be a family of cubic fuzzy dual BCK-subalgebra of X. Then $\bigcap_{i \in N} A_i$ is also a cubic fuzzy dual BCK-sub algebra of X.

Definition 2.7 A cubic fuzzy set $A=\left(X, \tilde{\mu}_A, \lambda_A\right)$ is called cubic dual ideal of BCK-algebra X if it satisfies the following inequalities:

(C FD1) $\tilde{\mu}_A(1) \geq \tilde{\mu}_A(x)$ and $\lambda_A(1) \leq \lambda_A(x)$

(C FD2) $\tilde{\mu}_A(x) \geq \text{rmin}\{\tilde{\mu}_A(N(Nx * Ny)), \tilde{\mu}_A(y)\}$

(C FD3) $\lambda_A(x) \leq \max\{\lambda_A(N(Nx * Ny)), \lambda_A(y)\}$, for all $x, y \in X$.

Example 2.8 Let $X = \{0, x, y, z\}$ be a BCK-algebra with the following Cayley table

*	0	x	y	z
0	0	0	0	0
x	x	0	0	0
y	y	x	0	0
z	z	y	x	0

We define a cubic set $A=\left(X, \tilde{\mu}_A, \lambda_A\right)$ by $\tilde{\mu}_A(0) = \tilde{\mu}_A(x) = [0.6, 0.7]$, $\tilde{\mu}_A(y) = \tilde{\mu}_A(z) = [0.2, 0.3]$,

$\lambda_A(0) = 0.1$, $\lambda_A(x) = 0.3$, and $\lambda_A(y) = \lambda_A(z) = 0.4.$

By routine calculations we know that $A=\left(X, \tilde{\mu}_A, \lambda_A\right)$ is a cubic dual -ideal of X.

Theorem 2.9: If a cubic set in X is cubic dual sub algebra, then $\tilde{\mu}_A(N(N0 * Nx)) \geq \tilde{\mu}_A(x)$ and $\lambda_A(N(N0 * Nx)) \leq \lambda_A(x)$ for all $x \in X.$

Proof: For all $x \in X,$ we have

(i) $\tilde{\mu}_A(N(Nx * Ny)) \geq \text{rmin}\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\}$

(ii) $\lambda_A(N(Nx * Ny)) \leq \max\{\lambda_A(x), \lambda_A(y)\}$

Put $x = 0, y = x$ in (i) we get

$\tilde{\mu}_A(N(N0 * Nx)) \geq \text{rmin}\{\tilde{\mu}_A(0), \tilde{\mu}_A(x)\}$

$= r\min\{\tilde{\mu}_A(x*x), \tilde{\mu}_A(x)\}$

$= r\min\{r\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(x)\}, \tilde{\mu}_A(x)\}$

$= r\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(x)\}$

$= \tilde{\mu}_A(x)$

Similarly, put $x = 0, y = x$ in (ii) we get

$\lambda_A(N(N0 * Nx)) \le \max\{\lambda_A(0), \lambda_A(x)\}$

$= \max\{\lambda_A(x*x), \lambda_A(x)\}$
$= \max\{\max\{\lambda_A(x), \lambda_A(x)\}, \lambda_A(x)\}$
$= \max\{\lambda_A(x), \lambda_A(x)\}$

$= \lambda_A(x)$

Theorem 2.10 A cubic set $A = (\tilde{\mu}_A, \lambda_A)$ in X is a cubic dual–ideal of X if and only if μ_A^-, μ_A^+ and λ_A are fuzzy dual ideals of X.

Proof: Let μ_A^-, μ_A^+ and λ_A are fuzzy dual ideals of X and $x, y \in X$. Then by definition

$\mu_A^-(1) \ge \mu_A^-(x)$, $\mu_A^+(1) \ge \mu_A^+(x)$,

$\mu_A^-(x) \ge \min\{\mu_A^-(N(Nx * Ny)), \mu_A^-(y)\}$,

$\mu_A^+(x) \ge \min\{\mu_A^+(N(Nx * Ny)), \mu_A^+(y)\}$,

$\lambda_A(x) \le \max\{\lambda_A(N(Nx * Ny)), \lambda_A(y)\}$

Now $\tilde{\mu}_A(x) = [\mu_A^-(x), \mu_A^+(x)]$

$\ge [\min\{\mu_A^-(N(Nx * Ny)), \mu_A^-(y)\}, \min\{\mu_A^+(N(Nx * Ny)), \mu_A^+(y)\}]$

$= r\min\{[\mu_A^-(N(Nx * Ny)), \mu_A^+(N(Nx * Ny))], [\mu_A^-(y), \mu_A^+(y)]\}$

$= r\min\{\tilde{\mu}_A(N(Nx * Ny)), \tilde{\mu}_A(y)\}$

Therefore A is cubic dual-ideal of X.

Conversely assume that A is cubic dual-ideal of X.
For any $x, y \in X$

$[\mu_A^-(x), \mu_A^+(x)] = \tilde{\mu}_A(x) \ge r\min\{\tilde{\mu}_A(N(Nx * Ny)), \tilde{\mu}_A(y)\}$

$= \text{rmin}\{[\mu_A^-(N(Nx*Ny)), \mu_A^+(N(Nx*Ny))], [\mu_A^-(y), \mu_A^+(y)]\}$

$= [\min\{\mu_A^-(N(Nx*Ny)), \mu_A^-(y)\}, \min\{\mu_A^+(N(Nx*Ny)), \mu_A^+(y)\}]$

Thus

$\mu_A^-(x) \geq \min\{\mu_A^-(N(Nx*Ny)), \mu_A^-(y)\}$

$\mu_A^+(x) \geq \min\{\mu_A^+(N(Nx*Ny)), \mu_A^+(y)\}$

$\lambda_A(x) \leq \max\{\lambda_A(N(Nx*Ny)), \lambda_A(y)\}$

Hence μ_A^-, μ_A^+ and λ_A are fuzzy dual-ideals of X.

Theorem 2.11 If $A=(\tilde{\mu}_A, \lambda_A)$ is a cubic dual-ideal of X, then non-empty upper $\tilde{s}$-level cut $U(\tilde{\mu}_A, \tilde{s})$ and the non-empty lower t-level cut $L(\lambda_A, t)$ are dual closed ideals of X for every $\tilde{s} \in D[0,1]$ and $t \in [0,1]$.

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