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FINITE DIFFERENCE METHOD APPLIED TO AN UNSTEADY MAGNETOHYDRODYNAMIC NEWTONIAN FLUID WITH WALL SLIP IN DARCY-FORCHHEIMER POROUS MEDIUM

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ABSTRACT

This study investigated the effects of magnetic field, thermal radiation and wall slip on a Newtonian, incompressible, viscous fluid flowing through a saturated porous medium. Boundary conditions for slip at the wall hold in the fluid. The governing equations are formulated, simplified and non-dimensionalised. The dimensionless equations were solved with the help of Crank Nicolson's finite difference method. This method converges faster and it is unconditionally stable. Numerical results are presented with the aid of graph to account for the fluid parameters affecting the flow on velocity and temperature profiles.

Keywords: Newtonian fluid, Wall slip, Thermal radiation, Magnetic field, Viscosity, Crank nicolson method.

Contribution/ Originality

This study contributes in the existing literature of Newtonian fluids. Series of practical problems involves either Newtonian or non-Newtonian fluids. This study uses new estimation methodology of analyzing the heat transfer properties of Newtonian fluids. This study originates new formula for solving non-linear partial differential equations using Crank Nicolson finite difference method. This study is one of very few studies which have investigated the heat transfer in a Darcy-Forchheimer porous medium. The paper's primary contribution is finding the effects of various thermo physical parameters affecting the flow. This study documents a less well-known Darcy-Forchheimer porous medium by finding the effects of magnetic field, thermal radiation and wall slip on a Newtonian fluid in the presence of viscous dissipation.

1. INTRODUCTION

The problems arising from heat and mass transfer system is of great importance in modeling of fluid flow. Heat transfer problems arise in many industrial and environmental processes like thermal processing, energy utilization and thermal control. In our normal life, we are directly or indirectly exposed to heat transfer problems. In view of this emission, control measure for release of thermal radiation from Newtonian fluid is therefore necessary in the manufacturing industries.

Series of investigations have been carried out on the transfer of heat to or from Newtonian fluid. Effect of slip condition on unsteady magneto hydrodynamic (MHD) oscillatory flow of a viscous fluid in a planer channel was

considered by Mehmood and Ali [1]. Mazumdar and Deka [2] analyzed MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation. A mathematical introduction to incompressible flow was carried out by Sibanda and Makinde [3]. Makinde, et al. [4]. studied the adomian decomposition approach to a boundary layer flow with thermal radiation past a moving vertical porous plate. Yurusoy and Pakdemirili [5] considered the exact solutions of boundary layer equations of a special non-Newtonian fluid over a stretching sheet. The effects of the fluid slippage at the wall for couette flow are considered by Marques, et al. [6] under steady state conditions and only for gases. The closed form solution for steady periodic and transient velocity field under slip condition was studied by Khaled and Vafai [7]. The effect of slip condition on MHD steady flow in a channel with permeable boundaries was examined by Makinde and Osalusi [8]. Sacheti, et al. [9]; Chandran, et al. [10] and Ganesan and Palani [11] all worked on analytical solutions for velocity and temperature using continuous and well-defined conditions at the wall. Samiulhaq, et al. [12] analysed the influence of radiation and porosity on the unsteady magneto hydrodynamic flow past an infinite vertical oscillating plate with uniform heat flux in a porous medium. In their work they concentrated on the relevance of shear stress on the boundary. The unsteady free convection flow of an incompressible viscous fluid near a vertical plate with ramped wall temperature and comparing the results with those of the plate at constant temperature was investigated by Chandran, et al. [13]. He considered some practical problems which may require non-uniform or arbitrary wall conditions. In all the above mentioned research works, the effects of magnetic field, thermal radiation and wall slip in the presence of viscous dissipation is less well-known.

Hence, the aim of this work is to study the effects of magnetic field, thermal radiation and wall slip on a Newtonian, incompressible, viscous fluid flowing through a saturated porous medium in the presence of viscous dissipation. This work extends the work of Mehmood and Ali [1].

2. PROBLEM FORMULATION

Consider a Newtonian, viscous, incompressible, and electrically conducting fluid bounded by two fixed parallel plates and separated apart by distance l in a saturated porous medium. The viscous dissipation effect is present in the medium and a magnetic field of uniform strength B_o is applied perpendicular to the plates. The governing equations are given as follows:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\left(\frac{\partial P^*}{\partial x^*}\right) + \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T^* - T_0^*) - \frac{\sigma B_0^2 u^*}{\rho} - \frac{\mu u^*}{k} - \frac{b u^{*2}}{\rho k} \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} + \frac{\mu}{\rho C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{2}$$

With the following initial and boundary conditions

$$u^* - \lambda \frac{\partial u^*}{\partial y^*} = 0, \qquad T^* = T_0^* \qquad \text{at } y^* = 0$$

$$u^* = 0, \qquad T^* = T_w^* \quad \text{at} \quad y^* = l \tag{3}$$

Where u^* is the velocity of the fluid in the x-direction, t^* is the time, ρ is the density of the fluid, g is the acceleration due to gravity, T^* is the fluid temperature, P^* is the fluid pressure, C_p is the specific heat at constant pressure, β is the coefficient of thermal expansion, k is the porosity parameter, b is the Forchheimer parameter, T_w^* is the temperature of the fluid at $y^* = l$, T_0^* is the temperature of the fluid at $y^* = 0$, q_r is the radiative heat flux, σ is the electrical conductivity, ν is the kinematic viscosity, B_0 is the magnetic field, l is the distance between two plates.

The radiation parameter, according to Mazumdar and Deka [2] is given as

$$\frac{\partial q_r}{\partial y^*} = 4(T^* - T_0^*)L \tag{4}$$

Where $L=\int_0^\infty k\lambda_w \left(\frac{de_{b\lambda}}{dT^*}\right)_w d\lambda$, $k\lambda_w$ is the absorption coefficient and $e_{b\lambda}$ is the plank function.

We define the non-dimensional quantities as follows

$$x = \frac{x^*}{l}, y = \frac{y^*}{l}, u = \frac{u^*}{u_0^*}, t = \frac{t^* u_0^*}{l}, \gamma = \frac{K_c \nu}{u_0^{*2}}, P = \frac{P^*}{\rho u_0^{*2}}, T = \frac{T^* - T_0^*}{T_w^* - T_0^*} \tag{5}$$

By introducing the non-dimensional quantities into equations (1) and (2), the momentum and energy equation becomes

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - Mu + G_r T - Dau - F_s u^2 \tag{6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{PrRe}\frac{\partial^2 T}{\partial y^2} - NT + Ec\left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{7}$$

Where $Re = \frac{u_0^* l}{\nu}, Da = \frac{\nu l}{k u_o^*}, M = \frac{\sigma B_0^2 l}{\rho u_o^*}, Ec = \frac{u_o^{*2}}{C_p(T_1^* - T_0^*)}, Pr = \frac{\nu}{\alpha}, = \frac{4Ll}{\rho c_p u_o^*}, Gr = \frac{g\beta T(T_w^* - T_0^*)l}{u_o^{*2}}, F_s = \frac{bl}{\rho k}, \gamma = \frac{\lambda}{h}$

With the dimensionless boundary conditions

$$u = \gamma\frac{\partial u}{\partial y}, \quad T = 0, \qquad \text{at } y = 0$$

$$u = 0, \quad T = 1, \qquad \text{at } y = 1 \tag{8}$$

Re is the Reynold number, Da is the Darcy number, F_s is the Forchheimer number, Gr is the thermal Grashof number, M is the Magnetic field parameter, γ is the slip parameter, Ec is the Eckert number, T is the dimensionless temperature, N is the thermal radiation parameter and Pr is the Prandtl number.

3. NUMERICAL SOLUTION

These nonlinear partial differential equations (6) and (7) are solved by employing Crank Nicolson finite difference scheme. The Crank-Nicolson Method (CNM) can be thought of as a combination of the forward and backward Euler methods. This method is unconditionally stable for both one and two dimensional applications.

The equations are written in their finite difference equations as follows:

$$\frac{u_i^{k+1} - u_i^k}{\Delta t} = -G_k + \frac{1}{Re}\left[\frac{u_{i-1}^{k+1} - 2u_i^{k+1} + u_{i+1}^{k+1} + u_{i-1}^k - 2u_i^k + u_{i+1}^k}{2(\Delta y)^2}\right] - M\left(\frac{u_i^{k+1} + u_i^k}{2}\right) + G_r\left(\frac{T_i^{k+1} + T_i^K}{2}\right) - Da\left(\frac{u_i^{k+1} + u_i^k}{2}\right) -$$

$$F_s u_j^k\left(\frac{u_i^{k+1} + u_i^k}{2}\right) \tag{9}$$

$$\frac{T_i^{k+1} - T_i^k}{\Delta t} = \frac{1}{prRe}\left[\frac{T_{i-1}^{k+1} - 2T_i^{k+1} + T_{i+1}^{k+1} + T_{i-1}^k - 2T_i^k + T_{i+1}^k}{2(\Delta y)^2}\right] - N\left[\frac{T_i^{k+1} + T_i^k}{2}\right]$$

$$+Ec\left(\frac{u_{i+1}^k - u_{i-1}^k}{2\Delta y}\right)^2 \tag{10}$$

Where G_k (pressure gradient) is constant. By multiplying equations (9) and (10) by Δt, the equation can be arranged so that the temperatures at the next time step ($k+1$) are on the left hand side and previous time step (k) are on the right hand side . Applying these equations to all the nodes, we will obtain a system with tri-diagonal coefficient matrix.

Equation (9) becomes

$$-u_{i-1}^{k+1} \frac{\Delta t}{2Re(\Delta y)^2} + u_i^{k+1}\left[1 + \frac{2\Delta t}{2Re(\Delta y)^2} + \frac{\Delta tDa}{2} + \frac{\Delta tGr}{2} + \frac{\Delta tM}{2} + \frac{\Delta tF_s u_i^k}{2}\right] - u_{i+1}^{k+1}\frac{\Delta t}{2Re(\Delta y)^2} = u_{i-1}^k \frac{\Delta t}{2Re(\Delta y)^2} + u_i^k\left[1 - \right.$$

$$\left. \frac{2\Delta t}{2Re(\Delta y)^2} - \frac{\Delta tDa}{2} - \frac{\Delta tGr}{2} - \frac{\Delta tM}{2} - \frac{\Delta tF_s u_i^k}{2}\right] + u_{i+1}^{k+1}\frac{\Delta t}{2Re(\Delta y)^2} + \frac{\Delta tGr}{2}\left[T_i^{k+1} + T_i^k\right]$$

$$\text{(11)}$$

And equation (10) becomes

$$-T_{i-1}^{k+1}\frac{\Delta t}{2PrRe(\Delta y)^2} + T_i^{k+1}\left[1 + \frac{2\Delta t}{2PrRe(\Delta y)^2} + \frac{\Delta tN}{2}\right] - T_{i+1}^{k+1}\frac{\Delta t}{2PrRe(\Delta y)^2} = T_{i-1}^k \frac{\Delta t}{2PrRe(\Delta y)^2} + T_i^k\left[1 - \frac{2\Delta t}{2PrRe(\Delta y)^2} - \frac{\Delta tN}{2}\right] + $$

$$T_{i+1}^{k+1}\frac{\Delta t}{2PrRe(\Delta y)^2} + \frac{\Delta tEc}{(2\Delta y)^2}\left[u_{i+1}^k - u_{i-1}^k\right]^2 \qquad\qquad\qquad \text{(12)}$$

Subscripts i and k represents grid points along y and t directions respectively. From the initial conditions, the values of u and T are known at all grid points at t=0. The computation of u and T at the next time step (k+1) using the values at previous (k) time step are carried out in this way: The finite difference equation (12) forms a tri-diagonal system of equations where the values of T at every nodal point at next time step length are determined using the known values at previous time. Thomas algorithm was used to solve this tri-diagonal system of equations with the help of MATLAB programming package. In this way, the values of T at every nodal point at this particular time are known. The known values of T at this particular time are used in equation (11). Using the same method, the values of u are computed at that particular time. These steps were repeated several times until a steady state was reached.

4. DISCUSSION OF RESULT

To account for the influence of the fluid parameters on the velocity and temperature field, they have been studied graphical. Default values are: Re=1, M=1, Gr=2, Da=0.1, F_s=0.1, Pr=0.76, N=1, Ec=0.001.

Figures 1 and 2 illustrate the effects of thermal radiation on velocity and temperature profiles. Increase in the values of thermal radiation causes reduction in both the velocity and temperature of the fluid. This experience is evident because, radiation is known to cause emission of heat from the body. On figure 3, increase in the slip parameter increases the wall velocity.

Figures 4 and 5 are plotted for different values of porosity parameter and thermal Grashof number. It is seen from figure 4 that increases in porosity parameter causes reduction on velocity profile while the velocity profile on figure 5 increases due to increase in thermal Grashof number.

Influence of magnetic field is illustrated on figure 6. Due to the presence of Lorentz force, a decrease is seen with increasing the values of magnetic field parameter on the velocity field.

Figures 7 illustrate the effects of increasing Eckert number on velocity profile. Increase in Eckert number as seen on the figure causes a rise in velocity profile because the work of dissipation is to accelerate energy in the fluid motion thereby increasing the buoyancy force.

Figures 8 and 9 shows the influence of Prandtl number on velocity and temperature profiles. Increase in Prandtl number decreases both the velocity and temperature field to decrease.

The steady state for both velocity and temperature is shown on figures 10 and 11. As the time increases, both velocity and temperature of the fluid increases until a steady state is reached when the difference between the values plotted is infinitely small.
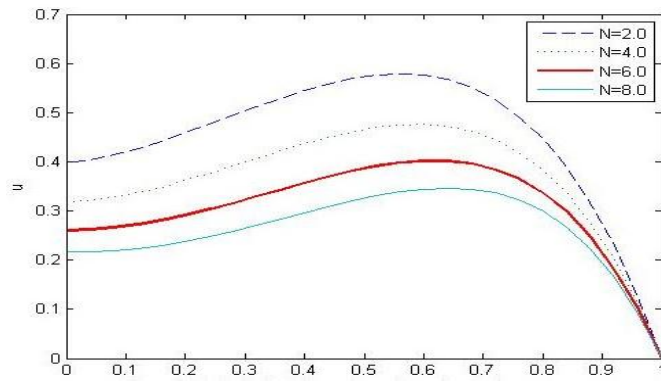
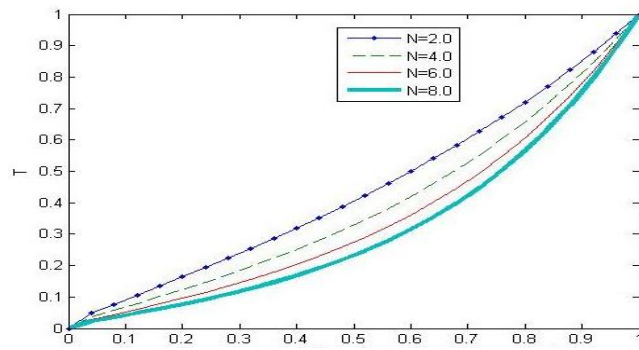**Figure-1.** Velocity field for various thermal radiation parameter

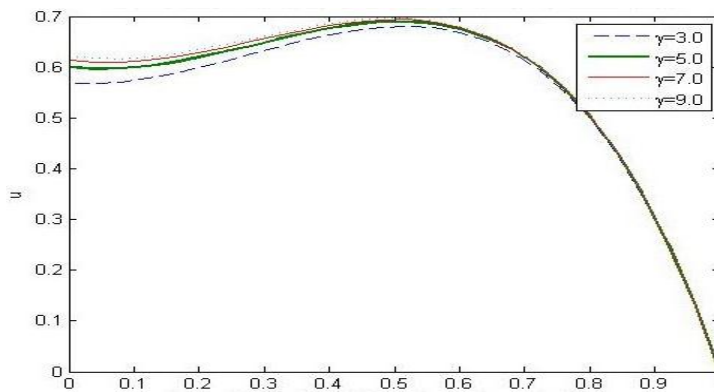**Figure-2.** Temperature field for various values of thermal radiation.

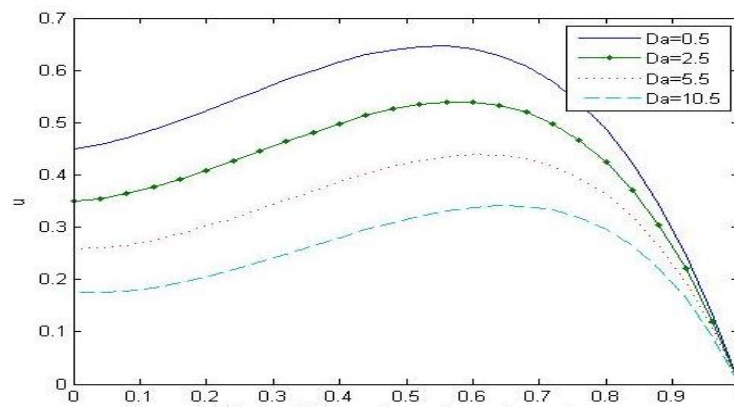**Figure-3.** Velocity field for various values for wall slip

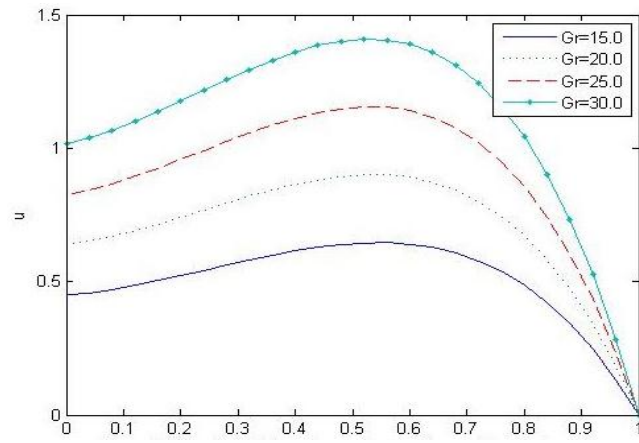**Figure-4.** Velocity for various values of porosity parameter

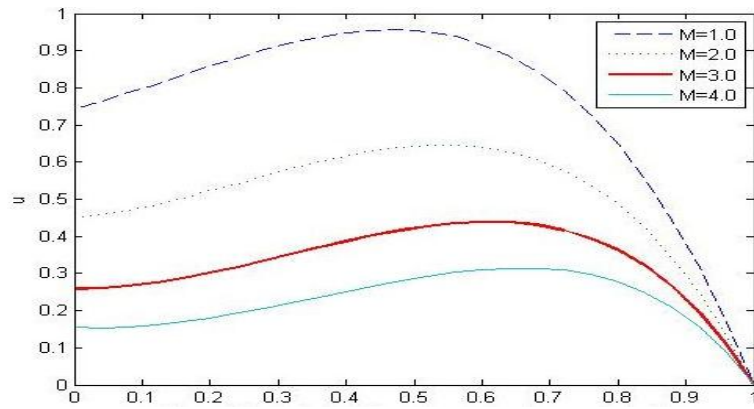**Figure-5.** Velocity field for various values of thermal Grashof number

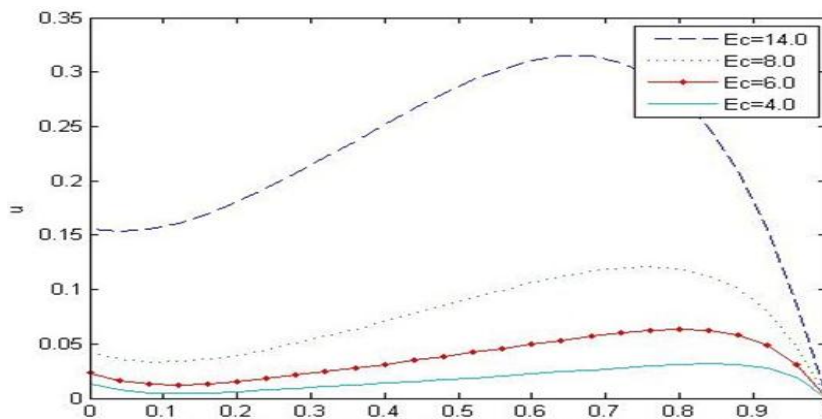**Figure-6.** Velocity field for various values of magnetic field

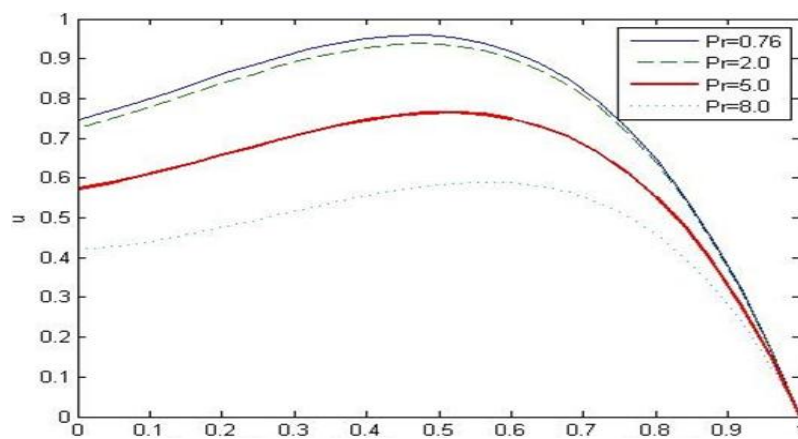**Figure-7.** Velocity field for various values of dissipation function

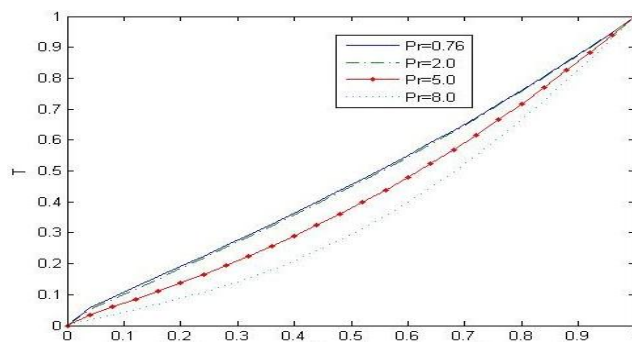**Figure-8.** Velocity field for various values of Prandtl number

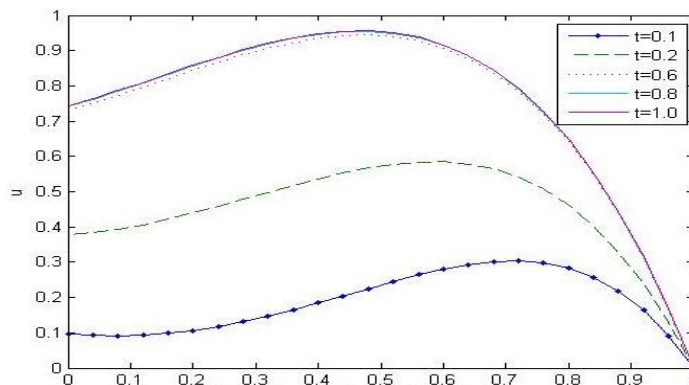**Figure-9.** Temperature field for various values of Prandtl number
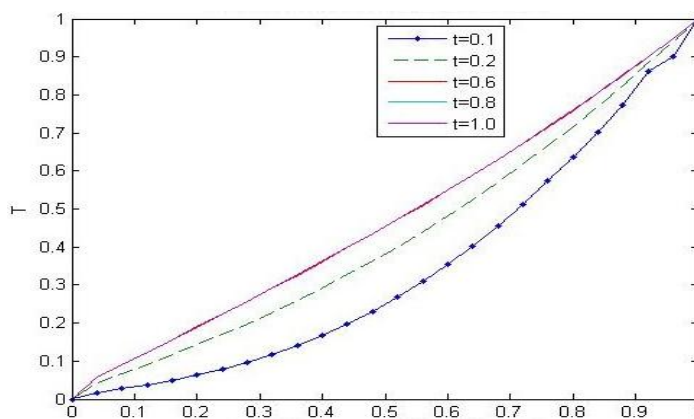

Figure-10. Velocity field at different times

Figure-11. Temperature field at different times

5. CONCLUSION

We consider the effects of magnetic field, thermal radiation and wall slip on a Newtonian, incompressible, viscous fluid flowing through a saturated porous medium with Boundary condition for slip at the wall. In our investigation, we discover that the wall can be strengthened by increasing the wall slip, thermal Grashof number, Eckert number while it is found to be weakened by the effects of increasing thermal radiation parameter, porosity parameter, magnetic field and Prandtl number respectively.

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