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OPTIMAL SOLUTION OF BALANCED AND UNBALANCED FUZZY TRANSPORTATION PROBLEM USING HEXAGONAL FUZZY NUMBERS

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ABSTRACT

A fuzzy transportation problem (FTP) includes cost, supply and demand of transportation problems. Its numbers are fuzzy numbers. Fuzzy transportation problem works to reduce transportation cost of some commodities through a capacitate network. Present research paper points out a technique with an alpha cut, optimal solution for solving transportation problem. We suggest a technique to find the fuzzy optimal solution on scales of transportation problem and propose a new hexagonal representation of fuzzy numbers. In general, the comparison of balanced fuzzy transportation problem (BFTP) and unbalanced fuzzy transportation problem (UFTP) shows that the optimal transportation cost of UFTP is less than BFTP.

Keywords: Fuzzy transportation problem, Hexagonal fuzzy numbers, α- Optimal solution, Robust ranking method.

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Contribution/ Originality

This study contributes in networking for transportation. This study uses new estimation methodology. This study originates optimal solution and comparison between BFTP and UFTP. This study investigated comparatively. This paper contributes the first logical analysis that is minimizes or maximizes objective solution is an optimal solution in hexagonal fuzzy numbers.

1. INTRODUCTION

Transportation is the need of globalization. All nations are come close not only by culture, tradition and custom but demand and supply of goods and material to one another. Transportation problem of material and goods has come forward as a major difficulty. Transportation of demanded goods and material in given period with original freshness are challengeable. Transportation problem is used in operation research. In mathematics, transportation theory is useful to give an optimal solution. Main objective of transportation problem is to maintaining shipping schedule.

The objective of this study is to fulfill demand of goods and its destinations in minimum transportation cost. To produce things by machine on a large scale at different plants to different warehouses is deals with transportation problem. Gaspard [1] invented the optimal transportation problem that deals with the relocation of materials in highly economical way. Gaspard has defined the problem of optimal transportation. Hitchcock has developed basic transportation problem. Hitchcock problems are concerned with some transportation problem [2].

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The fuzzy transportation problem is considers as special kind of fuzzy linear programming problems. Operations on fuzzy numbers are important to calculate FTP. Pandian and Natrajan [3] explained a new advanced algorithm for finding a fuzzy optimal solution of FTP. A feasible solution that minimizes or maximizes an objective solution is an optimal solution. It means the best solution out of all the values. Thamaraiselvi and Santhi [4] have used Vogel's approximation method to attain a fuzzy basic feasible solution by zero point method. Fuzzy set theory was presented by Zadeh [5]. The theory provided a mathematical approach for solving inaccurate problems. The objective of the FTP is to minimize shipping schedule. Anandhi [6] provides an applicable optimal solution which helps to the decision maker using pentagonal fuzzy number and some basic arithmetic operations.

The ranking method is to rank the fuzzy objective values. There are objective functions to rank best alternative. On the basis of this idea the Robust ranking method with the help of alpha solution has been adopted to transform FTP. Chandrasekaran [7] concludes that the solution of FTP can be obtained by robust ranking method effectively. In his work we can see the fuzzy demand and supply are all in the form of hexagonal fuzzy number and alpha cut optimal solution. All transportation problems can be shifted into get an optimal solution by ranking procedure using zero suffix method. Chandrasekaran [8] obtained an optimal solution for a FTP using heptagon fuzzy numbers.

Life is full of uncertainties and has lack of evidence, judgments, accuracy etc. In this globalised era, we can see throughout competitive market, the organizations have pressure to find the better ways to create value to customers become stronger. Transportation models provide a challenging and powerful work to meet this problem. They ensure the efficient movement with timely availability of raw materials and finished goods. It is useful in many fields like Engineering, Medical, Logistics, statistics etc. Timothy [9] used in Engineering Applications with fuzzy logic.

2. PRELIMINARIES

2.1. Fuzzy Number

A fuzzy set A of the real line R with membership function μA(x): R [0,1] is called fuzzy number if a) A must be normal and convex fuzzy set; b) The support of A, must be bounded; c) α.A must be closed interval for every α in [0,1].

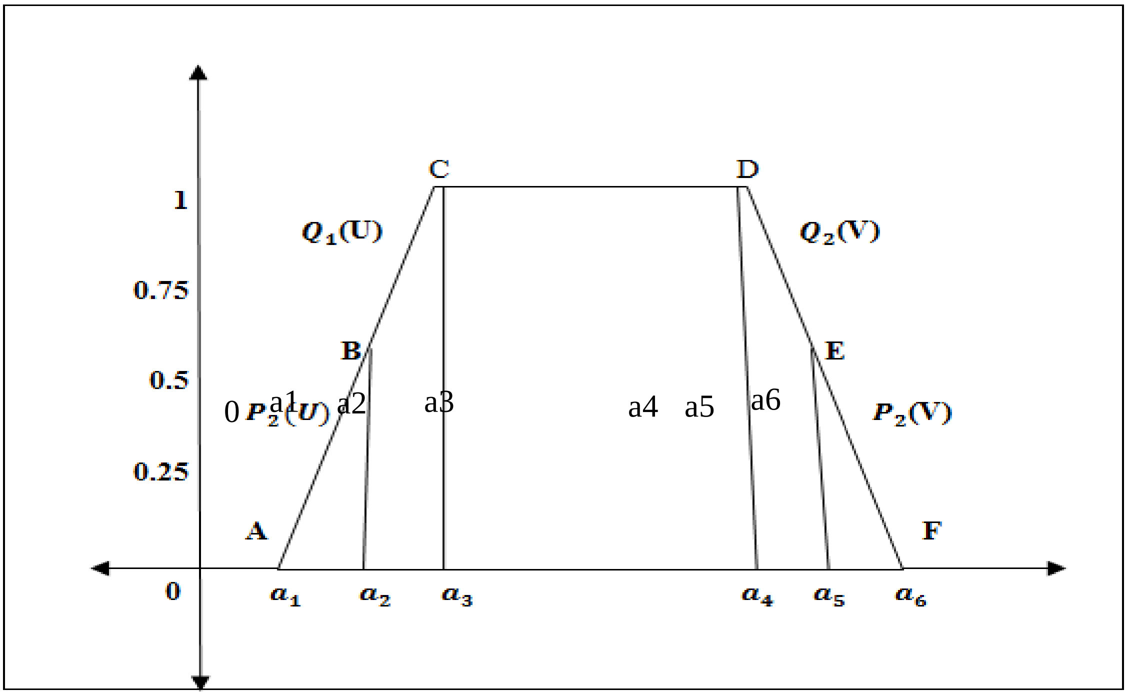
2.2. Robust Ranking Technique

Robust Ranking Technique satisfies the following properties: a) compensation b) linearity c) additively. It provides results which are consist human intuition. If A is a fuzzy number then the robust ranking defined by R (A) $=\int_0^1 (0.5)(a_{h\alpha}^L , a_{h\alpha}^U)\, d\alpha$ where $(a_{h\alpha}^L ,\ a_{h\alpha}^U)$ is the α level cut of the fuzzy number A_H.

2.3. Hexagon Fuzzy Number

The fuzzy number H is a hexagonal fuzzy A_H is a hexagonal fuzzy number denoted A_H (a, b, c, d, e, f; 1) and its membership function $\mu A_H(X)$ is given below:

$$\mu AH\ (X) = \begin{cases} \frac{y-a}{b-a}, & a \le y \le b \\ 1, & b \le y \le c \\ \frac{d-y}{d-c}, & a \le y \le d \\ 0, & otherwise \\ \frac{y-c}{d-c}, & c \le y \le d \\ 1, & d \le y \le e \\ \frac{f-y}{f-c}, & e \le y \le f \\ 0, & otherewise \end{cases}$$



There are different types of numbers which are use and developed by some methods. Khadhirvel and Balamurugan [10] used trapezoidal fuzzy numbers and proposed a new algorithm. Thangaraj and Priyadharsini [11] proposed an intuitionist fuzzy numbers. Poonam [12] used triangular numbers with alpha cut.

3. BALANCED FUZZY TRANSPORTATION PROBLEM (BFTP)

The fuzzy transportation problems in which a decision maker is uncertain about the precise values of transportation cost, availability and demand can be formulated as follows:

Minimize z $=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i\, , i = 1,2,3 \ldots m$

$\sum_{i=1}^{m} x_{ij} = b_j$, j = 1, 2, 3…n

$\sum_{j=1}^{n} x_{ij} = a_i\, , i = 1,2,3 \ldots m$ and $\sum_{i=1}^{m} x_{ij} = b_j$, j = 1, 2, 3…n and $x_{ij} \geq 0$. If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

The above formulation shows that the FTP is balanced fuzzy transportation problem otherwise on contrary it is unbalanced fuzzy transportation problem.

4. UNBALANCED FUZZY TRANSPORTATION PROBLEM (UFTP) CHANGE IN TO BALANCED FUZZY TRANSPORTATION PROBLEM (BFTP) AS FOLLOWS:

An UFTP is converted in to a BFTP by introducing a dummy origin or a dummy destination which will provide for the too much availability or the requirement cost of transporting a unit from this dummy origin (or dummy destination) to any place is taken to be zero. After converting the unbalanced problem into a balanced problem we adopt the usual procedure for solving a balanced transportation problem. Poonam [13] proposed unbalanced FTP with robust ranking technique. Deepika [14] studied fully unbalanced transportation problem in which the total availability is more than the total demand.

Numerical example 1:

Let us consider a fuzzy transportation problem with rows representing 6 persons A,B,C,D,E,F columns and representing 6 jobs like job1, job2, job3, job4, job5, job6.The cost matrix $[a_{ij}]$ is given whose elements are hexagonal fuzzy numbers. The problem is to find the optimal transportation cost so that the total cost of job become minimum.

Table-1. Unbalanced Hexagonal Fuzzy Transportation Problem

	Destination1	Destination2	Destination3	Destination4	Supply
O1	(14,16,18; 12,16,20)	(0,1,2; -1,1,3)	(7,8,9; 6,8,10)	(11,13,15; 10,13,16)	(2,4,6; 1,4,7)
O2	(8,11,14; 7,11,15)	(3,4,5; 2,4,6)	(5,7,9; 4,7,10)	(8,10,12; 6,10,14)	(5,6,7; 4,6,8)
O3	(6,8,10; 5,8,11)	(13,15,17; 12,15,18)	(7,9,11; 6,9,12)	(1,2,3; 0,2,4)	(7,8,9; 5,8,11)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

Source: Hexagonal fuzzy numbers

The above FTP is an unbalanced hexagon fuzzy transportation problem. Then convert the unbalanced problem in to balanced problem introducing dummy origin as follows:

Table-2. Balanced Fuzzy Transportation Problem

	Destination1	Destination2	Destination3	Destination4	Supply
O1	(14,16,18; 12,16,20)	(0,1,2; -1,1,3)	(7,8,9; 6,8,10)	(11,13,15; 10,13,16)	(2,4,6; 1,4,7)
O2	(8,11,14; 7,11,15)	(3,4,5; 2,4,6)	(5,7,9; 4,7,10)	(8,10,12; 6,10,14)	(5,6,7; 4,6,8)
O3	(6,8,10; 5,8,11)	(13,15,17; 12,15,18)	(7,9,11; 6,9,12)	(1,2,3; 0,2,4)	(7,8,9; 5,8,11)
O4	(0,0,0; 0,0,0)	(0,0,0; 0,0,0)	(0,0,0; 0,0,0)	(0,0,0; 0,0,0)	(8,10,12; 6,10,14)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

Source: Hexagonal fuzzy numbers with dummy variables

Min z = R (14, 16, 18; 12, 16, 20)x_{11} + R (0, 1, 2; -1, 1, 3)x_{12} + R (7, 8, 9; 6, 8, 10)x_{13} +
R (11, 13, 15; 10, 13, 16)x_{14} + R (8, 11, 14; 7, 11, 15)x_{21} + R (3, 4, 5; 2, 4, 6)x_{22} +
R (5, 7, 9; 4, 7, 10)x_{23} + R (8, 10, 12; 6, 10, 14)x_{24} + R (6, 8, 10; 5, 8, 11)x_{31} +
R (13, 15, 17; 12, 15, 18)x_{32} + R (7, 9, 11; 6, 9, 12)x_{33} + R (1, 2, 3; 0, 2, 4)x_{34}

R (A) = $\int_0^1 (0.5)(a_{h\alpha}^L,\ a_{h\alpha}^U) d_\alpha$,

where = {(b-a) α + a, d- (d-c) α} + {(d-c) α +c, f- (f-e) α}d_α

R (14, 16, 18; 12, 16, 20) = $\int_0^1 (0.5)$ (2_α+ 14, 12+6_α, -6_α+ 18, 20- 4_α) d_α= 31.5

Similarly

R (0, 1, 2; -1, 1, 3) = 2.75, R (7, 8, 9; 6, 8, 10) = 15.75, R (11, 13, 15; 10, 13, 16) = 25.75, R (8, 11, 14; 7, 11, 15) = 21.75, R (3, 4, 5; 2, 4, 6) = 7.75, R (5, 7, 9; 4, 7, 10) = 13.75, R (8,10, 12;6,10, 14) = 19.5, R (6, 8, 10; 5, 8, 11) = 15.75, R (13, 15, 17; 12, 15, 18) = 29.75, R (7, 9, 11; 6 , 9, 12) = 17.75, R (1, 2, 3; 0, 2, 4) = 3.75

Rank of all supply:

R (2, 4, 6; 1, 4, 7) = 7.75, R (5, 6, 7; 4, 6, 8) = 11.75, R (7, 8, 9; 5, 8, 11) = 15.5, R (8, 10, 12; 6, 10, 14) = 19.5

Rank of all demand:

R (3, 4, 5; 2, 4, 6) = 7.75, R (3, 5, 7; 1, 5, 9) = 9.5, R (10, 12, 14; 8, 12, 16) = 23.5, R (6, 7, 8; 5, 7, 9) = 13.75

Table-3. Using VAM Method

	D1	D2	D3	D4	Supply
O1	31.5	2.75	15.75	25.75	7.75
O2	21.75	7.75	13.75	19.5	11.75
O3	15.75	29.75	17.75	3.75	15.5
O4	0	0	0	0	19.5
Demand	7.75	9.5	23.5	13.75	

Source: By Calculation

Now using Vogel's approximation method (VAM) we have,

Table-4. Using VAM and Robust Ranking Method

	D1	D2	D3	D4	Supply
O1	31.5	2.75 [7.75]	15.75	25.75	7.75
O2	21.75 [6]	7.75 [1.75]	13.75 [4]	19.5	11.75
O3	15.75 [1.75]	29.75	17.75	3.75 [13.75]	15.5
O4	0	0	0 [19.5]	0	19.5
Demand	7.75	9.5	23.5	13.75	

Source: By Calculation

And the fuzzy transportation cost is,

= (7.75) (2.75) + (6) (21.75) + (1.75) (7.75) + (4) (13.75) + (1.75) (15.75) + (13.75) (3.75) + (19.5) (0)

= 21.31 + 130.5 + 13.56 + 55 + 27. 56 + 51.56 + 0

= 299.49

Numerical Example 2:Let us consider a fuzzy transportation problem with rows representing 6 persons A,B,C,D,E,F columns and representing 6 jobs like job1, job2, job3, job4, job5, job6.The cost matrix [a_{ij}]is given whose elements are hexagonal fuzzy numbers. The problem is to find the optimal transportation cost so that the total cost of job become minimum.

Table-5. Balanced Fuzzy Transportation Problem

	D1	D2	D3	D4	Supply
O1	(3,7,11; 15,19,24)	(13,18,23; 28,33,40)	(6,13,20; 28,36,45)	(15,20,25; 31,38,45)	(7,9,11; 13,16,20)
O2	(16,19,24; 29,34,39)	(3,5,7; 9,10,12)	(5,7,10; 13,17,21)	(20,23,26; 30,35,40)	(6,8,11; 14,19,25)
O3	(11,14,17; 21,25,30)	(7,9,11; 14,18,22)	(2,3,4; 6,7,9)	(5,7,8; 11,14,17)	(9,11,13; 15,18,20)
Demand	(3,4,5; 6,8,10)	(3,5,7; 9,12,15)	(6,7,9; 11,13,16)	(10,12,14; 16,20,24)	

Source: Balanced Hexagonal fuzzy Numbers

Now using Robust Ranking Technique and Vogel's Approximation method we have,

	D1	D2	D3	D4	Supply
O1	26.25 11.75	51.5	49.0 10.25	57.5 3	25
O2	53.5	15.5 16.75	24 10.25	57.5	27
O3	39	26.5	10.25	20.25 28.5	28.5
Demand	11.75	16.75	20.5	31.5	

And the fuzzy transportation cost is,

= (26.25) (11.75) + (49.0) (10.25) + (57.5) (3) + (15.5) (16.75) + (24) (10.25) + (20.25) (28.5)

= 2065.8

5. CONCLUSION

In this way we have solved FTP by applying hexagonal fuzzy numbers. The comparison of UFTP and BFTP give the new optimal transportation cost on hexagon fuzzy numbers are employed to get the fuzzy optimal solutions. Further by using Robust ranking method we have obtained fuzzy transportation cost. In this work UFTP and BFTP have been comparing but these both have their own important too. Through the comparison we have found that UFTP get less optimal cost than BFTP in general. The numerical examples are solved to illustrate the transportation problems concerned in real life situations.

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