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D_X-SCHEMES AND JETS IN CONFORMAL GRAVITY USING INTEGRAL TRANSFORMS

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ABSTRACT

A $\mathcal{D}_X$ – scheme is a scheme equipped with a flat connection over a smooth scheme on a base field. The flat connection equipment is a characterization of this scheme to construct through isomorphisms between commutative algebras and formal moduli problems the conformal images of the space-time that are solutions in conformal field theory. If are considered the $\mathcal{D}_X$ – schemes and their particular tools, the jets, these determine conformal blocks of space-time pieces that are invariant under conformal transformations. These conformal block of space-time pieces determine a homogeneous degree factor that characterizes the solutions in a complex Riemannian model of the space-time of the field equations to certain tensors of the Weyl curvature. Finally, is demonstrated that the algebra belonging to the $\mathcal{D}_X$ – schemes to the mentioned formal moduli problem is the image under a generalized Penrose transform that in the conformal context of many pieces of the space-time, has a structure as objects in commutative rings of $CAlg_k$ each one.

Keywords: Cohomologies, Commutative rings, Conformal blocks, Conformal gravity, DX-schemes, Jets, Spectrum functor.

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1. INTRODUCTION

A cosmological problem existing is to reduce the number of field equations that are resoluble under a same gauge field (*Verma modules*) and can to extend the gauge solutions to other fields using the topological groups symmetries that define their interactions and their actions in the Universe. This extension can be given in the way of geometrical ramifications by a global Langlands correspondence between certain deformed derived categories as the Hecke sheaves category on an adequate moduli stack (physical stacks) and the holomorphic ${}^LG$ – bundles category with a special connection (*Deligne connection*), where this connection is a regular generalization over complex Riemannian manifold with singularities.

The corresponding D – modules may be viewed as sheaves of conformal blocks (or co-invariants) (images under a generalized version of the Penrose transform [1-3] naturally arising in the framework of conformal field theory but wanting to extend this theory to the non-commutative case and the notion of the field singularity.

We are interested in the purely conformal aspect of the field theory (in more concrete, gravity theories that are invariant under conformal transformations in the Riemannian geometry) and their field observable as traces of the

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Weyl curvature tensor[1], as are the Bach tensor or the Eastwood-Dighton tensor as examples. This will permits an understanding of the use of integral transforms to the geometrical analysis inside of a derived category whose sheaves have flat connections extendible to the conformal gravity in the space-time. An geometrical model of these extensions are the flat Metric model of the gravitational waves as solution of a field equation called 4-derivative wave equation which is the scalar 4-derivative wave equation [4-6] (see the figure 1). Here is obtained under the causal structure of the space-time a conformal representation of gravitational waves through electromagnetic gauges. From a point of algebraic view is consider a *formal moduli problem* on the base of CAlg_k, whose objects are obtained as limits of the corresponding jets in a $\mathrm{Aff}_{\mathrm{Spec}}$. In this last, results very useful the jets technique, since helps in the demonstration of invariant conformally of some characteristics of the objects in the $\mathcal{D}_X$ – schemes that can derive of a notorious spectrum deduced of co-cycles obtained for some integral transform and that characterizes functors between derived categories to extensive geometries deduced of the symmetries.

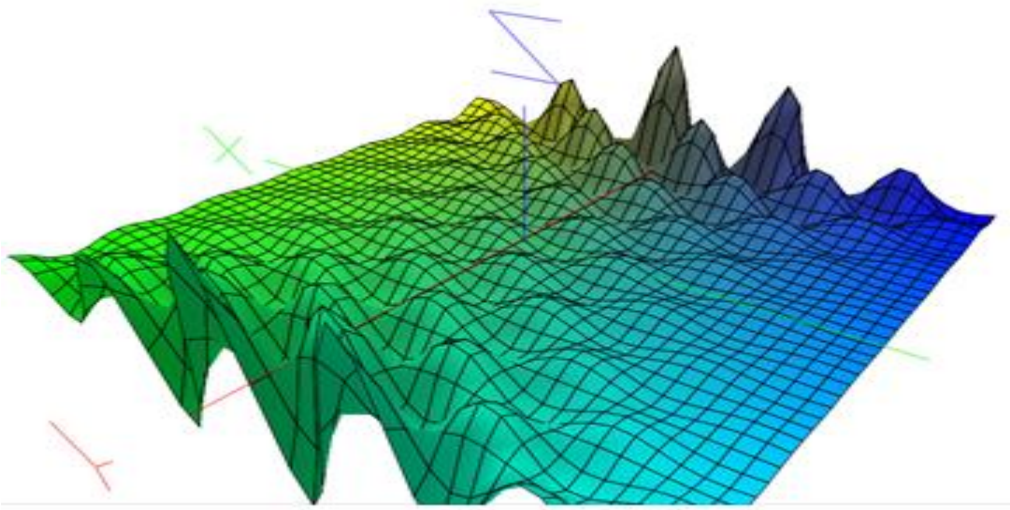

Fig-1. Electromagnetic waves in conformal actions of the group $SU(2,2)$, on a $2-$dimensional flat model of the space-time. The ultra-hyperbolic wave equation is satisfied. In both sides of axis Y, appear the auto-dual Maxwell fields of positive frequency and negative frequency on M, respectively that go being added in each time to each orbit. This corresponds to partial waves expansions in 2-dimensions.

Likewise, from a purely algebraic point of view, the development of the jets technique as *forgetful* functors can establish equivalences between categories of opposite class as $\mathcal{D}_X$ – algebras and affine $\mathcal{D}_X$ – schemes (considering for one side the operators that act over vector bundles and for other side the sheaves of the algebraic modules respectively), considering their relation with these categories as co-limits of ring structures modulo an ideal I, that can be a bi-sided ideal $I^{\pm}$, when we want establish correspondences between two manifolds inside a same context (conformally, holomorphicity, etc) and obtain transformed objects with the same invariance. In the ambit of the commutative algebra [7] these relations can establish equivalences to the construction of the required *formal moduli problem* on the base of CAlg_k, whose objects are mentioned before.

[1] Adding a suitable gravitational term to the standard model action with gravitational coupling, the theory establishes a local conformal Weyl invariance in the unitary gauge of Lie group $SU(2)$, given for the algebra $\mathfrak{su}(2)$. The gauge is fixed such that the Higgs scalar must be constant. This mechanism generates the masses for the vector bosons and matter fields with no physical degrees of freedom for the Higgs fields, which breaks the conformal symmetry of the standard model, where are required ramifications of the field to determine their geometrical representation.

In this research, are used some properties of the jets as the functores $\mathcal{D}_X-\mathrm{sch}\to\mathcal{O}_X-\mathrm{sch}$, and some tools as algebras of conformal blocks cohomologies to establish a commutative scheme of a moduli problem to conformal properties of geometrical invariants. One of them is the curvature measured of conformal gravity in the space-time. These geometrical invariants are obtained as images under a corresponding generalized *Penrose transform* [8] which can derive of the *Verdier duality* in the *cohomological context of the categories* and consigned in the structure of the objects of these categories, that is to say, algebraic modules.

Then the homogeneous bundles and their objects can be extrapolated in *homogeneous polynomials* where these polynomials are direct images of the corresponding jets applied to objects belonging to an $\mathcal{O}_X$ – algebra or $\mathcal{D}_X$ – schemes.

2. CONFORMAL BLOCKS

The functor $k-\mathrm{sch}\to\mathcal{D}_X-\mathrm{sch}$, sending a k - scheme S ,to the "constant" $\mathcal{D}_X-\mathrm{scheme}\ X\times S$ (which has coordinate ring $\mathcal{O}_x\otimes_k\mathcal{O}_S$) has a right adjoint functor:

$$\begin{aligned}&\mathrm{H}_\nabla(X,\cdot):\mathcal{D}_x-\mathrm{sch}\to k-\mathrm{sch},\\&\mathrm{Hom}(S,\mathrm{H}_\nabla(X,\mathcal{Z}))\cong\mathrm{Hom}_{\mathcal{D}_x}(X\times S,\mathcal{Z}),\end{aligned}\tag{1}$$

for any $\mathcal{D}_X$ – scheme $\mathcal{Z}$, and any k - scheme S . Alternatively, we can define this functor for algebras:

$$\begin{aligned}&\mathrm{H}_\nabla(X,\cdot):\mathcal{D}_x-\mathrm{Alg}\to k-\mathrm{Alg},\\&\mathrm{Hom}(\mathrm{H}_\nabla(X,\mathcal{B}),C)\cong\mathrm{Hom}_{D_x}(\mathcal{B},\mathcal{O}_x\otimes_k C),\end{aligned}\tag{2}$$

for any $\mathcal{D}_X$ – algebra $\mathcal{B}$, and any k - algebra C. Obviously, $\mathrm{Spec}\,\mathrm{H}_\nabla(X,\mathcal{B})=\mathrm{H}_\nabla(X,\mathrm{Spec}\mathcal{B})$.

The scheme $\mathrm{H}_\nabla(X,\mathcal{Z})$, is called the scheme of conformal blocks of $\mathcal{Z}$, and it is tautologically the largest constant $\mathcal{D}_X-\mathrm{sub\text{-}scheme}$ of $\mathcal{Z}$.

The conformal blocks can be obtained by the apparatus of the *Penrose transform* through their extension, interpreting the invariances under scheme of "CRings" and $\mathcal{D}_X$ – schemes. Likewise, we can enounce:

2. 1. Theorem (F. Bulnes, S. Fominko)

Let $\mathcal{A},\mathcal{B}\in\mathrm{Alg}_k$, $\mathcal{D}_X$ – algebras (of fact $\mathcal{B}$ is a $\mathcal{O}_X$ – algebra) such that $\mathcal{Y}=\mathrm{Spec}\mathcal{A}$. Let J, the jet defined as the functor $\mathcal{D}_X-\mathrm{sch}\to\mathcal{O}_X-\mathrm{sch}$, where $\mathrm{Spec}\,\mathrm{H}_\nabla(X,\mathcal{B})=\mathrm{H}_\nabla(X,\mathrm{Spec}\mathcal{B})$. Then is had the $\mathcal{D}_X$ – scheme

$$\mathrm{Hom}_{Dx}(J\mathcal{A},\mathcal{B})=\mathrm{Hom}_{\mathrm{Alg}_k}(X,\mathrm{Spec}J\mathcal{A}),\tag{3}$$

Proof. Let X , be smooth and with induced over $\mathbb{Z}$, locally at X. Let I , be the $\mathcal{D}_X$ – ideal generated by $\mathrm{Ker}(\mathrm{Sym}(\mathcal{A}\to\mathcal{A})$, where $\mathcal{A}$, is a $\mathcal{O}_X$ – sub-algebra of $\mathcal{O}_X[1/x]$, generated by the sub-sheaf $x^{-1}I$. But $\mathcal{Y}=\mathrm{Spec}\mathcal{A}$, to $\mathcal{Y}$, any $\mathcal{O}_X$ – scheme, then $J\mathcal{Y}$, is a $\mathcal{D}_X$ – scheme. Specializing $\mathcal{B}=J\mathcal{A}$, we have $\mathrm{Hom}_{\mathcal{D}_x}(J\mathcal{A},\hat{\mathcal{O}}_x\hat{\otimes}C)$ by (2), and the left of (3) is proved. To establish the right side of (3) we need use the relations between morphisms of affine schemes and morphisms of algebras to that the category of schemes can be identified with a full subcategory of Sp [2]. Indeed, for any object $\mathcal{Y}=\mathrm{Spec}\mathcal{A}$, and let $\mathrm{Aff}_{\mathcal{Y}}$, denote the full subcategory of $\mathrm{Sp}_{\mathcal{Y}}$, consisting of objects X, over $\mathcal{Y}$,

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[2] Sp , denote the category of sheaves of sets on Aff .

Aff , denote the category of affine schemes with certain covering topology whose images cover $\mathrm{Aff}_{\mathcal{Y}}$.

where X, is affine. We define $\mathrm{AffRel}_{\mathcal{Y}}$ [3], to be the full subcategory of $\mathrm{Aff}_{\mathcal{Y}}$, consisting of objects X, whose structure map $X \to \mathcal{Y}$, factors through an affine open sub-scheme of $\mathcal{Y}$. The inclusion $\mathrm{AffRel}_{\mathcal{Y}} \to \mathrm{Sp}_{\mathcal{Y}}$, induces an equivalence between $\mathrm{Sp}_{\mathcal{Y}}$, and the category of sheaves of sets on $\mathrm{AffRel}_{\mathcal{Y}}$. $\mathcal{JA}$, is the $\mathcal{D}_X$ – algebra generated by $\mathcal{A}$. By (A. 1) we have for any $\mathcal{O}_X$ – scheme $\mathcal{Y}$, we have $\mathrm{HorSect}(X, \mathcal{JY})$, which is equivalent to the conformal block $\mathrm{H}_{\nabla}(X, \mathcal{JY})$. Then from (2) we have $\mathrm{Spec}\,\mathrm{H}_{\nabla}(X, \mathcal{B}) = \mathrm{H}_{\nabla}(X, \mathrm{Spec}\mathcal{B})$, and considering the property of the jet $\mathcal{J}$, given by $\mathrm{Spec}\mathcal{JA} = \mathcal{J}\mathrm{Spec}(\mathcal{A})$, (mentioned in the *section 2*) we have that the jet carry us to an object in"CRings" defined by an k –.algebra $\mathrm{Spec}R$, where R, is a commutative E_n – ring[4] to $0 \le n < \infty$. Then is followed (3).

2.2. Proposition

For any $\mathcal{D}_X$ – algebra $\mathcal{B}$, we have that every conformal block is the Penrose transform

$$\mathcal{P}: H^0(\mathcal{JA}, \mathcal{O}) \cong H_{\nabla}(X, \mathcal{B}) \quad (4)$$

Proof. For any $\mathcal{D}_X$ – algebra $\mathcal{B}$, we have $\mathrm{Spec}\,\mathrm{H}_{\nabla}(X, \mathcal{B}) = \mathrm{H}_{\nabla}(X, \mathrm{Spec}\mathcal{B})$. But this is a Penrose transform if $\mathrm{H}_{\nabla}(X, \mathcal{B})$, is the twistor image of $H^0(\mathcal{JA}, \mathcal{O})$. Indeed, using the specializing $\mathcal{B} = \mathcal{JA}$, we have in the spectrum:

$\mathrm{Spec}\,\mathrm{H}_{\nabla}(X, \mathcal{B}) = \mathrm{H}_{\nabla}(X, \mathrm{Spec}(\mathcal{JA})) = \mathrm{H}_{\nabla}(X, \mathcal{J}\mathrm{Spec}(\mathcal{A}))$,

At the level of algebras, the functor defined from $\mathcal{D}_X$ – schemes to $\mathcal{O}_X$ – schemes, will be a left adjoint to the *forgetful functor* (A. 3):

$$\mathcal{J}: \mathcal{O}_X - \mathrm{alg} \to \mathcal{D}_x - \mathrm{sch}, \quad (5)$$

$$\mathrm{Hom}_{\mathcal{D}x}(\mathcal{JA}, \mathcal{B}) = \mathrm{Hom}_{\mathcal{O}x}(\mathcal{A}, \mathcal{B}), \quad (6)$$

and we consider $(\phi: \mathcal{JA} \to \mathcal{B}) \to (\phi': \mathcal{A} \to \mathcal{B})$, and $.(\phi': \mathcal{A} \to \mathcal{B}) \leftarrow (\phi\mathcal{JA} \to \mathcal{B})$, [5] then from the assignments $\phi \to \phi'$, and $\phi' \to \phi$, which are well-defined, and are inverses to each other and are natural in $\mathcal{A}$, and $\mathcal{B}$, we have

$\mathcal{J}(\mathrm{Spec}\mathcal{A}) \to \mathcal{B} \to \mathrm{Spec}(\mathcal{A}) \to \mathcal{B}$,

Using the property [9] [6]

$$\mathrm{HorSect}(X, \mathcal{JA}) = \mathcal{J}\mathrm{SecT}(X, \mathcal{B}), \quad (7)$$

But is satisfied that

$$\mathrm{H}_{\nabla}(X, \mathcal{JY}) \cong \mathrm{HorSect}(X, \mathcal{Y}), \quad (8)$$

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[3] $\mathrm{AffRel}_{\mathcal{Y}}$ Category of Affine Relations which is the full subcategory of $\mathrm{Aff}_{\mathcal{Y}}$, consisting of objects X, whose structure map $X \to \mathcal{Y}$, factors through an affine open sub-scheme of $\mathcal{Y}$.

[4] A E_n – ring is a ring of a commutative scheme of rings belonging of an k – algebra, with $0 \le n < \infty$.

[5] ϕ, denotes any map of $\mathcal{D}_X$ – algebras, while ϕ', denotes any map of $\mathcal{O}_X$ – algebras.

[6] **Proposition 2. 1.** For any $\mathcal{O}_X$ – scheme $\mathcal{Y}$, we have: $\mathrm{HorSect}(X, \mathcal{JY}) = \mathcal{J}\mathrm{SecT}(X, \mathcal{Y})$.

where by (7) and (8) we have $\mathrm{H}_{\nabla}(X, \mathcal{J}\mathcal{Y}) \cong \mathcal{J}\mathrm{SecT}(X, \mathcal{B})$, but $\mathcal{J}\mathrm{SecT}(X, \mathcal{B}) \cong \mathcal{J}H(X, \mathcal{B})$, which in the context of the $\mathcal{O}_X$ –algebras is reduced to the space $H^{\bullet}(\mathcal{J}\mathcal{A}, \mathcal{B}) = \mathcal{J}H^{\bullet}(\mathcal{A}, \mathcal{B})$, which by $\phi(d \otimes a) = d\phi'(a)$, which induces a long exact sequence on the d'Rham cohomology:

$$\ldots \rightarrow H^{n-1}_{\mathrm{dR}}(X - x, \mathcal{M}) \xrightarrow{\phi} \mathcal{M}_x \rightarrow H^{n-1}_{\mathrm{dR}}(X, \mathcal{M}) \rightarrow H^{n}_{\mathrm{dR}}(X - x, \mathcal{M}), \tag{9}$$

But the last cohomological group is 0. To see this, recall that the Lichtenbaum's theorem says that the Cĕch cohomological dimension of $X - x$, is at most $n-1$, e.g. $H^n(X - x, \mathcal{F}) = 0$ for any quasi-coherent $\mathcal{D}_X$ –module $\mathcal{F}$ [7]. Then the unique space which is the image under ϕ, of $\mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{F}$, is the cohomological space $H^0_{\mathrm{dR}}(X, \mathcal{M})$,

$\forall \phi(d \otimes a) = d\phi'(a) \in H^0(\mathcal{J}\mathcal{A}, \mathcal{B})$, where applying $\mathrm{H}_{\nabla}(X, \mathrm{Sym}\mathcal{M}) = \mathrm{Sym}\mathrm{H}^{\mathrm{n}}_{\mathrm{dR}}(X, \mathcal{M})$, Fominko [9] we have the required Penrose transform. ■

3. CONFORMAL GRAVITY THROUGH ZEROS OF POLYNOMIALS AND CONFORMAL INVARIANTS $H_{\nabla}(X, \mathcal{B})$.

An application example of the solution classes given by integrals of (4) are given for the solutions to the field equations of the Bach tensor and Eastwood-Dighton tensor[8]

$$B_{ab} = 0, \quad E_{abc} = 0, \tag{10}$$

where the tensors B_{ab}, and E_{abc}, to the *conformal case* can be designed as elements of a $\mathcal{D}_X$ –algebras or $\mathcal{O}_X$ –algebras.

If we consider the scheme commutative moduli we could think in the *forgetful functor* defined in (3), that is to say, the jet $\mathcal{J} : \mathcal{O}_X - \mathrm{alg} \rightarrow \mathcal{D}_x - \mathrm{sch}$, where are verified the relations in the conformal context of the space-time $(\phi : \mathcal{J}\mathcal{A} \rightarrow \mathcal{B}) \rightarrow (\phi' : \mathcal{A} \rightarrow \mathcal{B})$, and.$(\phi' : \mathcal{A} \rightarrow \mathcal{B}) \leftarrow (\phi' : \mathcal{J}\mathcal{A} \rightarrow \mathcal{B})$, to the concrete case of a *conformal factor* between $\mathcal{D}_X$ –algebras and $\mathcal{O}_X$ –algebras.

Indeed, a complex space-time $\mathbb{M}$, satisfying (10) will be called a solution to the conformal gravity equations [4, 6, 10, 11] if $\mathbb{M}$ (as a 4 –dimensional complex Riemannian manifold) implies that $(\mathbb{M}, g)$, is a solution of (10) with algebraically several Weyl curvature implying that exists a conformal factor α, such that $\tilde{g} = \alpha^2 g$,where their meaning of Ricci curvature of $\tilde{\mathcal{R}}$, satisfies $\tilde{\mathcal{R}}_{ab} = \frac{1}{4}\tilde{g}$.

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[7] Let $\mathcal{D}_X$ –module $\mathcal{M}$, a quotient of the form:

$$\mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{F} \rightarrow \mathcal{M},$$

for some quasi-coherent $\mathcal{D}_X$ –module $\mathcal{F}$, and

$$\mathrm{H}^{\mathrm{n}}_{\mathrm{dR}}(X - x, \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{F}) = H^n(X - x, \mathcal{F}) = 0,$$

it also follows that $\mathrm{H}^{\mathrm{n}}_{\mathrm{dR}}(X - x, \mathcal{M}) = 0$.

[8] Here $B_{ab} = (\nabla^c \nabla^d + 1/2\mathcal{R}^{cd})C_{abcd}$, where C_{abcd}, denotes the *Weyl curvature*, while that $E_{abc} = \overline{\Psi}_{A'B'C'D'} \nabla^{DD'} \Psi_{ABCD} - \Psi_{ABCD} \nabla^{DD'} \overline{\Psi}_{A'B'C'D'}$.

To different orders of certain integer l, can be generalized the solutions to

Due that the Bach and Easwood-Dighton equations haves are both symmetric trace-free tensors and are both *conformally invariants*, with conformal weight -2, meaning that under the transformation

$$g \mapsto \tilde{g} = \alpha^2 g, \qquad (11)$$

we have

$$B \mapsto \tilde{B} = \alpha^{-2} B, \qquad (12)$$

and

$$E \mapsto \tilde{E} = \alpha^{-2} E, \qquad (13)$$

Moreover, they are both invariant under bi-holomorphisms meaning that it

$$\phi : \mathbb{M} \to \tilde{\mathbb{M}}, \qquad (14)$$

is a bi-holomorphism then these tensors depends upon the metric in such a manner that

$$B(\phi^*(g)) = \phi^*(B(g)), \qquad (15)$$

where $\phi^*(g) = \mathcal{J}^m\mathcal{A}, \forall m \in \mathbb{N}$ since considering the jet of the metric g [9], in $x \in \mathbb{M}$, we have $B(\mathcal{J}^m\mathcal{A}) \in \mathrm{Spec}(\mathcal{J}^m\mathcal{A})$, spectrum element in a $\mathcal{D}_X$ – sheme. For other side, in the $\mathcal{D}_X$ – algebras, the image $\phi^* B(g) \in \mathcal{J}^m\mathrm{Spec}(\mathcal{A})$, and the similar to[10]

$$E(\phi^*(g)) = \pm\phi^*(E(g)), \qquad (16)$$

The value at $x \in \mathbb{M}$, of these tensors is a holomorphic function in $\mathrm{Spec}(\mathcal{A})$, of the $m-$jet of g, at $x, \forall m \in \mathbb{N}$, that is to say

$$\mathcal{J}^m(\mathrm{Spec}(\mathcal{A})) = \mathrm{Spec}(\mathcal{J}^m\mathcal{A}), \qquad (17)$$

The interesting of this application is the property of the jet of their homogeneous polynomial context. Likewise, if $h(\alpha^2 r, \alpha^3 s, \alpha^4 t, \ldots, \alpha^m u) \in \mathrm{Spec}\mathcal{R}^{\pm}$, then (17) takes the form:

$$h(\alpha^2 r, \alpha^3 s, \alpha^4 t, \ldots, \alpha^m u) = \alpha^{2+l}(h(r, s, t, \ldots, u)). \qquad (18)$$

3. 1. Proposition

If (18) is the spectrum of an $\mathcal{D}_X$ – algebra to each conformal block determined by (4) then the conformal gravity can be given through several representations (their direct sum) $H_{\nabla}(X, \mathcal{B})$, of a gauge conformal group.

Proof. To it is necessary to prove that each conformal block $H_{\nabla}(X, \mathcal{B})$, is a Weyl invariant to the $2+l-$parity of their dimension. Then their product can represent the conformal gravity if is applied the Penrose transform to this, obtaining an image of X, in $SO(n+1,1)$, that is conformal and belonging to commutative rings of CAlg_k. The corresponding $\mathcal{D}_X$ – scheme can be written as Bulnes [2] also see the appendix B:

$$\mathrm{Hom}_{\mathrm{Moduli}_n}(X, \mathrm{Spec}(\mathcal{B})) \cong \mathrm{Hom}_{\mathrm{CAlg(Sp)}}(\mathcal{B}, \mathbb{S}), \qquad (19)$$

[9] Jets of metrics of the form:
$$g_{ab} = \delta_{ab} + r_{(ab)(cd)} x^c x^d + s_{(ab)(cde)} x^c x^d + t_{(ab)(cdef)} x^c x^d x^f + \cdots + u_{(ab)(cde\cdots f)} x^c x^d \cdots x^f.$$

[10] $\pm$, sign depends upon the choice of sign for the associated star operator, $* : \wedge^2 \to \wedge^2$.

since that $\mathrm{Fun}(\mathrm{CAlg}(\mathrm{Sp}),\mathbb{S})$, is the $\mathcal{D}_X$ – scheme of the derived category of the objects $\mathrm{Spec}(\mathcal{B})$.[11] Remember

that the geometrical hypothesis in the functor $\Phi+$ geometrical hypothesis, Bulnes [13] comes established for the geometrical duality of Langlands which says that the derived category of coherent sheaves on a moduli space $\infty-$ category of $\mathcal{D}-$ modules is equivalent to the moduli space $\mathcal{D}-$ modules on the moduli space $\infty-$category of $\mathcal{D}-$ modules of the derived category of coherent sheaves on a moduli space of flatness.

From a point of physical view we want to define a field observable as curvature in a complex Riemannian manifold model of the space-time $\mathbb{M}$, where this space-time model corresponding to their gravity (which no necessary is conformal in whole the space) can be discompose in conformal regions, considering the "sensing" of the gravity field in the space-time singularities as zeros of the polynomials of the homogeneous bundles and whose general integral is the trace of conformal invariants $H_{\nabla}(X,\mathcal{B})$, around of these singularities. The value of the singularity computed as zeros of polynomials in an algebraic context can be re-interpreted by the Penrose transform as varieties belonging to commutative rings of CAlg_k.

Considering the role of $\mathbb{S}$, in the theory of $\infty-$ categories as the analogous to the ordinary category of sets in classical category theory, and considering the *Yoneda embedding* defined by

$j:\mathcal{C}\rightarrow \mathrm{Fun}(\mathcal{C}^{\mathrm{top}},\mathbb{S})$,

where we have in particular to a graded algebra $H^{\bullet}(Bun_G,\mathcal{D}^s)$, obtained from a Yoneda embedding, and generated by one copy of $\mathrm{H}^{\vee}$, over $H^0\cong\mathbb{C}[\mathrm{Op}_{{}^LG}]$, (which is had that on a disk). Then by the Einstein-Weyl structures that are in a Hitchin base [14] we can describe to a finite number of representations $\mathrm{H}^{\vee}$, that appear given through a gauge as

$$\mathbf{H}=H^0(\omega_C)\oplus H^0(\omega_C^{\otimes 2})\oplus\ldots\oplus H^0(\omega_C^{\otimes n}), \qquad (20)$$

and the Hitchin mapping on $T^{\vee}\mathrm{Bun}$, sends a Higgs field (E,θ), to the coefficients of the characteristic polynomial $\det(\lambda \mathrm{id}_E-\theta)$, as densities.

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[11] The Koszul dualities between formal moduli problems and algebras in the categories context are useful tools to determine equivalences of different objects context to design a commutative scheme of the spectrum of rings, when is wanted a formal theory of objects in categories with different characteristics. In particular to objects in $\infty-$ categories and E_n- algebras to construct the functor space $\mathrm{Fun}(\mathcal{D}^{\times},\mathcal{C})$, (of fact, this is a particular case of $\mathrm{Fun}(\mathrm{CAlg}(\mathrm{Sp}),\mathbb{S})$) in deformation theory. From a point of geometrical view, this is defined as a formal E_1- moduli problem of the cycles and co-cycles that live in the spectrum given in the scheme (19). Using integral transforms, we can to see that the kernel of these transforms are in the sheaf $\mathcal{O}_{\mathrm{Op}_{{}^LG}}$, which is studied and discussed in [12] Bulnesse. The obtained results are consequences of the theorem obtained in [2] Bulnes. and are focused in what happen in the "Rings" $\mathrm{CAlg}(\mathrm{Sp})$, when are considered geometrical objects given by the moduli stacks defined in the compactly generated $k-$ linear $\infty-$ category $\mathcal{C}$, to that deformation theory through integral transforms.

The asymptotic behaviors around the space-time singularities is possible if is beyond a post-Newtonian limit predicted by Einstein's relativity. But this due to our study of Langlands ramifications can be explained by integral transforms using the corresponding jet of metrics of the space-time. Philosophically, we want to obtain around of a singular point in a complex Riemannian manifold a decomposition of factor lines bundle (or zeros of homogeneous polynomials) that geometrically define the solutions $\ker(U, \bar{\partial} + \nabla_s)$, [12]to the field equations. These factor lines bundles are of critical level. In this asseveration, was obtained a result where is considered the decomposition of a Hitchin moduli space where one of the factors is a lines bundle of critical level [14]. Finally, all complex space-time $\mathbb{M} = \mathscr{M}_H(G,C)$, can be described through of the moduli stacks proper of the Higgs fields. An example are the realizations to surfaces Σ, where the images of Penrose transform are the blocks $H_\nabla(X,\mathcal{B})$, and energy integrals are the total trace of these blocks (see the figure 2 B)). For example, we can consider, for any $\mathcal{D}_X -$algebra $\mathcal{B}$, the equivalence $\mathrm{H}_\nabla(X,\mathcal{B}) \cong \mathcal{B}_x/(\mathrm{Im}\,\phi)$, where $\mathrm{Im}\,\phi$, denotes the ideal generated by the image of the co-boundary mapping $\phi : H^{n\text{-}1}(X - x,\mathcal{B}) \to \mathcal{B}_x$, which to physical level can result as waves of "energy-densities" that distort or deform the space-time due to the gravity of a black hole (see the figure 2 A)), for example (singularity of the space-time). But this beyond the post-limit of Newtonian gravity planted in relativity. But, it could obey to conformal gravity of the "skies" context[13].

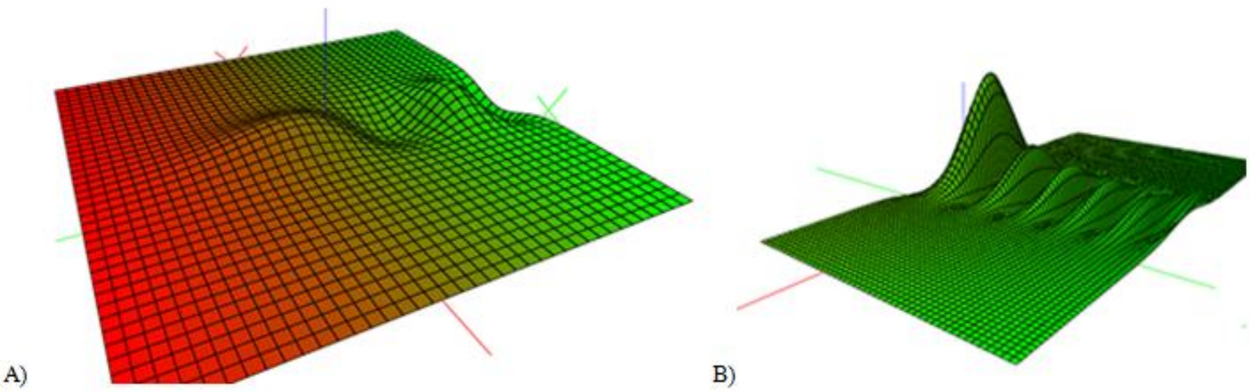

Figure 2. A). Singularity perturbing two 2-dimensional models of the space $\mathbf{H}$. B). The corresponding wave of "energy-densities" is created by the sub-sequent superposition states (conformal blocks) given by (20).

Then from the development of the $m-$jet of g, the Bach and Eastwood-Dighton tensors are vanished in $\mathcal{A}$. Indeed, $\forall h \in \mathrm{Spec}\mathcal{A}$, we have that using $\mathcal{J}(\mathrm{Spec}\mathcal{A}) \to \mathcal{B} \to \mathrm{Spec}(\mathcal{A}) \to \mathcal{B}$, which derives of the $\mathcal{D}_X -$scheme (3) application
$\mathrm{Hom}_{\mathrm{Alg}_k}(X, \mathrm{Spec}\mathcal{J}\mathcal{A}) \to \mathrm{Hom}_{Dx}(\mathcal{J}\mathcal{A}, \mathcal{B})$,

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[12] Image of the Penrose Transform extended to ramifications.

[13] Let $(\mathbb{M}, g)$, be the complex space-time. Each point $x \in \mathbb{M}$, gives rise to a complex sub-manifold $Q_x = q[p^{-1}(x)]$, of the ambi-twistor correspondence [10] LeBrun. which is called the sky of x. (The corresponding ambi-twistor correspondence space of null geodesics of $(\mathbb{M}, g)$, is isomorphic to $\mathbb{P}^1 \times \mathbb{P}^1$).

is had that to the Bondi tensor $B \mapsto \alpha^2 B,$ the corresponding image in the $\mathcal{D}_X$ – algebra $\mathcal{B},$ is $B(h(g)) \in \mathrm{Spec}(\mathcal{J}^m \mathcal{A}),$ which $\forall h \in \mathrm{Spec}\mathcal{A},$ means that $h,$ will be harmonic in Q[14] (in the jet context or space $\mathfrak{J}$[15]) if and only if $\mathcal{J}h = |\alpha|^0 h,$ then the Weyl tensor matrix satisfy to $l+2$ - components that[16]:

$$C^{AB\ldots F} = \underbrace{D_A D_B \ldots D_F}_{m} \tilde{I}, \qquad (21)$$

where $\tilde{I}$, is the corresponding Weyl invariant in the conformal mappings of the tensors of Bondi and Eastwood-Dighton. But each one is a representation in an $\mathcal{D}_X$ – algebra $\mathcal{B},$ as the block $\mathrm{H}_\nabla(X, \mathcal{B})$, where are annulled the first l - elements in said transformation. But for any $\mathcal{D}_X$ – algebra $\mathcal{B}$, we have [9]:

$$\mathrm{H}_\nabla(X, \mathcal{B}) \cong \mathcal{B}_{x_1} \otimes \ldots \otimes \mathcal{B}_{x_m} / (\mathrm{Im}\,\tilde{\phi}), \qquad (22)$$

where the general integral is the trace of conformal invariants $H_\nabla(X, \mathcal{B}),$ around of these singularities. ■

Example 3. 1. We consider a nearest frame to twistor Maxwell theory given by $0 \to \mathbb{C} \to \Omega^\bullet.$ The first and last sheaves that are $\mathcal{O},$ and $\mathcal{O}(-4),$ and the cohomological space of type H^1, of these, gives respectively potential modulo gauge for left-handed and right-handed Maxwell fields $H^1(|P^+|, \Omega^3),$ and $H^1(|P^-|, \mathcal{O}).$ But the constant sheaf $\mathbb{C},$ has relevance in the electromagnetic charge that live in $0 \to \mathbb{C} \to \Omega^\bullet.$ (as been to some elaborate representation space of the conformal group, as for example $SU(2,2)$). In this sense we can consider the Penrose transform framework to obtain two pieces to $H^1(U'', \Omega^1),$ [17] indeed,

$$H^1(|P^+|, \Omega^1) = \{\text{gauge - restricted gauge for right - handed potentials on } U\}, \qquad (23)$$

and the respectively

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[14] $Q = \{e_0 | \mathcal{J}h(g) = |\alpha|^{l+2} h(g) \forall \alpha \in \mathbb{R}_+\}$

[15] $\mathfrak{J} = \{J | Je_0 = |\alpha|^0 e_0 \forall e_0 \in Q\}.$

[16] Here our Weyl invariant $\tilde{I}$, is the mapping from a P — module of jets at e_0, on $\mathbb{R}^{n+2}$, of homogeneous harmonic functions of degree zero (in this case is the cohomology space or conformal block $H_\nabla(X, \mathcal{B})$) to the space of jets at $e_0,$ of functions on Q of homogeneity $q,$ with $\tilde{I}|_{e_0} = I$, such that is satisfied (22).

[17] Remember as has been mentioned we want isomorphism of the form $H^1(U'', \mathcal{O}(V)) = H^1(U', \mu^{-1}\mathcal{O}(V)),$ where considering the natural double fibration, we have $U \subset M,$ which is some (suitable convex) region of the space-time, also $\nu^{-1}(U) = U' \subset \mathbb{F},$ and $\mu(U') = U'' \subset P.$

$$H^1(|P^-|,\Omega^1) = \{\text{left-handed fields on } U\}, \qquad (24)$$

These two pieces are conformal blocks of electromagnetic representations of the space-time [4].

Also the Penrose transform of $H^1(U'',\Omega^2)$, is as (23).

In these identifications, "gauge-restricted" refers to the imposition of the conformally invariant conditions, as for example to field equations [4, 15] $\square^2 f = 0$, and $\square\nabla^{\alpha}\Phi_{\alpha} = 0$, (see the figure 1). Then the Penrose transform of the complex $0 \to \mathbb{C} \to \Omega^{\bullet}$, contains all the spaces of fields that one is interested in the Maxwell theory table [4] where some are gauge to other fields. ■

Appendices

Appendix-A.

Proposition A. 1. For any $\mathcal{O}_X$ – scheme $\mathcal{Y}$, we have:

$$\mathrm{HorSect}(X, \mathcal{J}\mathcal{Y}) = \mathcal{J}\mathrm{SecT}(X, \mathcal{Y}), \qquad (A.1)$$

Proof. We use the called *Jet functor*:

$$\mathcal{J} : \mathcal{O}_X - \mathrm{sch} \to \mathcal{D}_x - \mathrm{sch},$$
$$Hom_{Dx}(\mathcal{Z}, \mathcal{J}\mathcal{Y}) = Hom_{\mathcal{O}_X}(\mathcal{Z}, \mathcal{Y}), \qquad (A.2)$$

and the *forgetful functor*

$$\mathcal{J} : \mathcal{O}_X - \mathrm{alg} \to \mathcal{D}_x - \mathrm{sch},$$
$$\mathrm{Hom}_{\mathcal{D}x}(\mathcal{J}\mathcal{A}, \mathcal{B}) = \mathrm{Hom}_{\mathcal{O}x}(\mathcal{A}, \mathcal{B}), \qquad (A,3)$$

for any $\mathcal{O}_X$ – scheme $\mathcal{Y}$, $\mathcal{D}_X$ – scheme $\mathcal{Z}$, $\mathcal{O}_X$ – algebra $\mathcal{A}$, and $\mathcal{D}_X$ – algebra $\mathcal{B}$. Also we consider $\mathrm{Spec}\mathcal{J}\mathcal{A} = \mathcal{J}\mathrm{Spec}(\mathcal{A})$. Then setting $\mathcal{Z} = X$, the proposition is followed. ■

Appendix B.

Theorem B. 1 (F. Bulnes). If we consider the category $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, \mathrm{Y})$, then a scheme of their spectrum $\mathrm{V}_{\mathrm{critical}}^{\mathrm{Def}}$, where Y, is a Calabi-Yau manifold comes given as:

$$\mathrm{Hom}_{\hat{\mathfrak{g}}}(X, \mathrm{V}_{\mathrm{critical}}^{\mathrm{Def}}) \cong \mathrm{Hom}_{\mathrm{Loc}_{^LG}}(\mathrm{V}_{\mathrm{critical}}, M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, \mathrm{Y})), \qquad (B.1)$$

The equality is demonstrated using the local Serre dualities identities on the global section (*cycles or co-cycles in each case*) of the derived categories $\mathcal{D}_{\kappa_c, I}$.

Proof. [2, 13, 16]. ■

As special case of the scheme given in (B. 1) and considering to $\mathcal{C}$, a derived category we have the scheme [16, 17]:

$$\mathrm{Hom}_{\mathrm{Moduli}_n}(X, \mathrm{Def}(D)) \cong \mathrm{Hom}_{\mathrm{CAlg(Spf)}}(D, \mathcal{C}), \qquad (B.2)$$

which can have applications in deformation theory [14, 16] that is to say, considering the functors in the space $\mathrm{Fun}(\mathcal{D}^{\times}, \mathcal{C})$.

Corollary B. 1. A D_X $-$scheme version of the scheme (B. 1) is given by

$$\mathrm{RHom}_{\mathcal{D}_A}(\mathcal{D},\mathcal{D}^{\times}) = \mathrm{RHom}_{\mathrm{Loc}_{{}^LG}}({}^L\mathrm{Bun}_G, {}^L\Phi^{\mu}(\mathcal{D})) \qquad (B.3)$$

The equality is demonstrated using the local Serre dualities identities on the global section (*cycles or co-cycles in each case*) of the derived categories $\mathcal{D}_{\kappa_c, I}$.

Proof. Of fact, the use of the Penrose transform as $D-$modules transform [18] on the $D_{\mathbb{P}}$ $-$modules do that the $\mathrm{Op}_{{}^LG}$, can be viewed as affine space for the jet [9, 13] $\mathcal{J}: \mathcal{O}_X - \mathrm{alg} \rightarrow \mathcal{D}_x - \mathrm{sch},$ having $\mathcal{J}_X(\Omega_X \otimes T_eG)$ over X. Then there is a D_X $-$scheme isomorphic to $\mathcal{J}_X(\Omega_X \otimes T_eG)$. Then the functor image $R\Gamma_K^H$, given for this case as the functor image $\Gamma(X, \Omega_X \otimes T_eG) \cong \mathcal{D}^{\times}$ [13]. Of it is followed the corollary. ■

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