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APPLICATION OF INTERVAL VALUED FUZZY SOFT MAX-MIN DECISION MAKING METHOD

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ABSTRACT

In this paper, we study some basic concepts of fuzzy sets, soft sets, fuzzy soft sets, and interval-valued fuzzy soft sets. Secondly, we study the interval-valued fuzzy soft max-min decision-making function and developed the graphical model for the Interval-valued fuzzy soft max-min decision making (IVFSMmDM) method by using Interval-valued fuzzy soft max-min decision-making function. Finally, we used IVFSMmDM for faculty selection in the education department and observed that T_4 is the best teacher for teaching by using hypothetical data.

Contribution/Originality: This paper contributes a new graphical model for IVFSMmDM by using interval-valued fuzzy soft max-min decision-making function and used for faculty selection in the education department.

1. INTRODUCTION

L. A. Zadeh revolutionized the Mathematical approach to answer the problems of uncertainties with the Concept of the fuzzy set (FS) in 1965 [1]. Later this concept leads to many further concepts such as Hesitant FSs, Intuitionistic FSs, Neutrosophic Sets, Linguistic Term Sets, and Hesitant Fuzzy Linguistic term sets, etc. This stream of knowledge flourished when Molodstov formulated the concept of soft set (SS) in 1999 [2]. This idea of SS was channelized by many other Mathematicians who took further actions to establish the foundations. In [3] Maji et al in 2003 defined the ideas such as subset, superset, equality of SSs, union, intersection AND and OR operations of SSs. Pei and Miao also contributed to redefining these ideas in Daowu and Miao [4]. In Ali, et al. [5] some more operations were also defined by Ali, et al. [5]. Cağman and Enginoğlu [6] developed the concept of the soft matrix with different properties and operations. In these days, mathematician plays a vital role in the SS fuzzification. A new perception of the fuzzy soft set (FSS) was introduced in Maji, et al. [7] after the fuzzification of SS with different types and properties. They also reviewed the SS theory which was given by Molodtsov and used this theory for decision making in Roy and Maji [8]. The work on FSS theory was extended and introduced a new concept from FSS which is known as a fuzzy soft aggregation operator and its cardinal set [9]. They introduced the cardinal matrix of the cardinal set also construct a decision algorithm and used this method in decision making successfully. Some limitations were faced in the work of Maji, et al. [7]. which were attensonoid by Chen, et al. [10] by constructing a new definition of parameterization reduction and applied this definition in decision making. Ahmad and Kharal [11] extended the work of Maji, et al. [7]; Roy and Maji [12] with illustrations and counter illustrations and improved their work, also they extended the concept of FSS by defining some new definition such as the union of arbitrary FSS, the intersection of arbitrary FSS and proved De-Morgan inclusion on FSS.

In Majumdar and Samanta [13] the author's generalized the concept of FSS and proposed a new theory of generalized FSS with some properties and used generalized FSS for decision-making problem, also use this theory for the diagnosis of Pneumonia. Dayan, et al. [14]. also used this theory for decision making in Dayan, et al. [14]. In Celik and Yamak [15] fuzzy soft set theory used for medical diagnosis through Sanchez's approach by using fuzzy arithmetic operations. Many researchers extend the concept of studied FS and SS and developed TOPSIS models for fuzzy sets and soft sets and used for decision making and medical diagnoses [16-20]. The work on FSM was extended by Borah, et al. [21] they forwarded the FSM theory with some definitions and properties; they proposed a special type of product of FSMs which is known as T-product of FSMs and used this product in decision-making problem after construct algorithm.

The theory of IVFS was forwarded by Yushi in 2010 and discussed the properties of IVFS [22]. He proposed a new concept on IVFS which is known as a cut set of IVFS with some properties and introduced three decomposition theorems of IVFS. A most important topic of fuzzy algebra is the fuzzy matrix (FM) in this matrix elements are belonging to the unit interval [0, 1]. FM becomes the interval-valued fuzzy matrix (IVFM) when we generalized the FM elements in subintervals of [0, 1]. The membership value of rows and columns is crisp in IVFM, but there are many challenges are faced in real-life problems. To solve those challenges [23] presented the idea of IVFM with the help of interval-valued fuzzy rows and columns and defined some basic definitions with some binary operators. He also defined complement and density of interval-valued fuzzy rows and columns with verification of few important properties. Yang, et al. [24] worked on IVFS and SS and proposed a new theory which is known as interval-valued fuzzy soft set (IVFSS) with different types and properties. They define different operations on IVFSS such as complement, AND-operation and OR-operation on IVFSS with examples also prove the De-Morgan laws, associative laws and distributive laws on IVFSS and successfully used this theory in decision making to show the validity of IVFSS. In Chetia and Das [25] an algorithm developed on IVFSS and used for medical diagnoses by extending Sanchez's approach. The author's studied the fuzzy set theory and used the trapezoidal fuzzy numbers for disease identification by using Sanchez's approach [26]. The work on IVFSS was extended by Shawkat and Abdul [27] they introduced generalized IVFSS with basic operations such as union, intersection, and complement also proved some properties related to these operations. They also proposed AND-operation, OR-operation similarity measure on generalized IVFSS and use this similarity measure in medical diagnoses for decision making. Dayan and Zulqarnain studied the IVFSM and developed some new operations and properties on IVFSM in Dayan and Zulqarnain [28].

Rajarajeswari and Dhanalakshmi worked on IVFSM and proposed new definitions on IVFSM with examples and properties in Rajarajeswari [29]. A decision-making method on IVFSM developed in Zulqarnain and Saeed [30] which is known as the "Interval Valued fuzzy soft max-min decision-making method" and used the developed method for decision making. They introduced some new operations on IVFSM such as arithmetic mean, weighted arithmetic mean, geometric mean, weighted geometric mean, harmonic mean and weighted harmonic mean with some properties of IVFS-matrices in decision making. In Prabhavath and Sarala [31] Sarala and Prabharathi worked on IVFSS and extended Sanchez`s approach for medical diagnoses by using IVFSM with illustrations. They proposed some new definitions on IVFSM such as union and intersection of IVFSM with their examples. They also proved the commutative laws, associative laws and construct an algorithm for medical diagnoses. The author's used

IVFSM for decision making in Zulqarnain and Saeed [32]; Sarala and Prabhavathi [33]. The author's defined some new operations on IVFSM and used for decision making [34].

In this research, we study fuzzy sets, soft sets, fuzzy soft sets, and interval-valued fuzzy soft matrices. We constructed a graphical model for the IVFSMmDM method by using the IVFSMmDM function. Finally, we use this for decision making for the selection of faculty in the educational department.

2. PRELIMINARIES

Definition 1.1[1]

If we identify a set A in X by its membership function μ_A: X → [0, 1]. Then a set A is called an FS. Indeed, A = {(x, μ_A (x)): x ∈ X}. A real number μ_A (x) represents a grade of membership of a fuzzy set A defined over a universe.

Definition 1.2 [2]

A pair (F, A) is called an SS over M if A is any subset of E, and there exists a mapping from A to P (M) is F, P (M) is the parameterized family of subsets of the M but not a set.

Definition 1.3 [6]

Suppose X and E are universe set and set of attributes respectively and A ⊆ E. Let (f_A, E) be a SS over X. Then a subset R_A of X × E, uniquely defined as R_A = {(g, t): t ∈ A, g ∈ f_A (t)}, is called a relation form of the SS (f_A, E).

The characteristic function χ_{R_A} of R_A is defined as χ_{R_A}: X × E → {0, 1} where

$$\chi_{R_A}(g, t) = \begin{cases} 1, & (g,t) \in R_A \\ 0, & (g,t) \notin R_A \end{cases}$$

Now if X $=$ {$g_1, g_2, g_3, \ldots, g_m$} and E $=$ {$t_1, t_2, t_3, \ldots, t_n$} then the SS ($f_A$, E) can be represented by a matrix $[a_{ij}]_{m\times n}$ called an " SM" of SS (f_A, E) over X as follows

$$[a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Where $a_{ij} = \chi_{R_A}(g_i, t_i)$

In other words, an SS is uniquely represented by its corresponding SM. So, we can define a function ρ which maps SS to SM i.e. ρ: $R_A \to [a_{ij}]$ Where $a_{ij} = \chi_{R_A}(g_i, t_i)$.

Definition 1.4 [7]

A pair (F, A) is called FSS over M, and there exists a mapping from A to P (M) is F, P (M) is the collection of fuzzy subsets of M.

Definition 1.5 [35]

A pair (F, A) is called FSS in the fuzzy soft class (M, E). Then (F, A) is represented in a matrix form such as

$A_{m\times n} = [a_{ij}]_{m\times n}$ or A = $[a_{ij}]$ (i = 1→ m), (j = 1 → n)

Where

$$a_{ij} = \begin{cases} \mu_j(b_j) & if\ b_j \in A \\ 0 & if\ b_j \notin A \end{cases}$$

Definition 1.6 [22]

Let A be a set and U be a universal set than IVFS in A over U is defined as

A = {(p, $[\mu_{AL}(p), \mu_{AU}(p)]$): p ∈ U},

Where $\mu_{AL}(p)$ and $\mu_{AU}(p)$ are a fuzzy subset of U and $\mu_{AL}(p) \leq \mu_{AU}(p)$ for all i, j.

Definition 1.7 [22]

Let A = $[\mu_{AL}(p), \mu_{AU}(p)]$ and B = $[\mu_{BL}(p), \mu_{BU}(p)]$ are two IVFS over U, then A is said to be IVF-subset of B if $\mu_{AL}(p) \leq \mu_{BL}(p)$ and $\mu_{AU}(p) \leq \mu_{BU}(p)$ for all p ∈ U., it is represented as A ⊆ B.

Definition 1.8 [24]

A pair (F, A) is called IVFSS over M where F is a mapping such that

F: A→I^M

Where I^M represent the all interval-valued fuzzy subsets (IVFSbS) of M

Definition 1.9 [24]

A pair (F, A) is called IVFSS over M, where F is a mapping such that

F: A→I^M, where I^M represent all IVFSbS of M. Then the IVFSS can be expressed in matrix form as

$A_{m\times n} = [a_{ij}]_{m\times n}$

Or

A= $[a_{ij}]$ (i = 1 → m), (j = 1→ n)

Where

$$a_{ij} = \begin{cases} [\mu_{jL}(b_i), \mu_{jU}(b_i)] & If\ b_i \in A \\ [0,0] & If\ b_i \notin A \end{cases}$$

Where $[\mu_{jL}(b_i), \mu_{jU}(b_i)]$ represents the membership b_i in the IVFS.

Definition 1.10 [34]

Let A= $[a_{ij}]$ and B= $[b_{ij}]$ are two IVFS-matrices of order m×n and n×p respectively than their product defined as

A∗B = $[c_{ik}]_{m\times p}$ = [max min (μ_{AL_j}, μ_{BL_j}), max min (μ_{AU_j}, μ_{BU_j})] for all i, j, k.

Definition 1.11 [34]

Let A = $[a_{ij}]$ and B = $[b_{ij}]$ are two IVFS-matrices of order m×n and n×p respectively than their product defined as

A∗B = $[c_{ik}]_{m\times p}$ = [max (μ_{AL_j}, μ_{BL_j}), max (μ_{AU_j}, μ_{BU_j})] for all i, j, k.

OR

A∗B = $c_{ij} = \sum_{k=1}^{n} (a_{ik} * b_{ik})$, i = 1, 2… m and j = 1, 2… p

A∗B = $[c_{ik}]_{m\times p}$ = [$\sum_{k=1}^{n} (a_{ikL} * b_{ikL})$, $\sum_{k=1}^{n} (a_{ikU} * b_{ikU})$], i = 1, 2… m and k = 1, 2… p for all i, j, k.

Definition 1.12 [34]

A = $[a_{ij}]$ and B = $[a_{ik}]$ are two IVFS-matrices of same order m×n then Or-product is defined as

∨: A×B → $C_{m\times n^2}$, $[a_{ij}]_{m\times n} \vee [b_{ik}]_{m\times n} = [c_{ip}]_{m\times n^2}$ where

c_{ip} = [max (μ_{AL_j}, μ_{BL_j}), max (μ_{AU_j}, μ_{BU_j})] for all i, j, k. such that p = n(j −1) + k.

3. INTERVAL VALUED FUZZY SOFT MAX-MIN DECISION MAKING METHOD

In this section, we present the "Interval Valued Fuzzy Soft Max-min Decision Making (IVFSMmDM)" function and construct a graphical model for decision making by using the IVFSMmDM function in the following Figure 1.

Definition 2.1 [30]

Let $[c_{ip}] \in IVFSM_{m\times n^2}$, I_k = {P: $c_{ip} \neq 0$, (k-1) n < P ≤ kn} for all k ∈ {1,2,3,….,n} and IVFS-max-min decision function define as follows

Mm: $IVFSM_{m\times n^2} \rightarrow IVFSM_{m\times 1}$

Mm: $[c_{ip}] = [d_{i1}] = [\max\{t_{ik}\}]$ where $t_{ik} = \begin{cases} min_{p \in I_k}\{c_{ip}\} & If\ I_k \neq o \\ [0.0, 0.0] & If\ I_k = o \end{cases}$ and

c_{ip} = [min(μ_{AL_j}, μ_{BL_j}), min(μ_{AU_j}, μ_{BU_j})] for all i, j, k and p = n (j - 1) + k is known as IVFSMmDM function.

Definition 2.2 [30]

Let M = $\{m_1, m_2, m_3, \ldots, m_n\}$ be a universal set and max-min $[c_{ip}] = [d_{i1}]$, then optimum IVFS on M is defined as follows

Opt $[d_{i1}]$ (M) = $\{d_{i1}/m_i: m_i \in M, d_{i1} \neq 0\}$

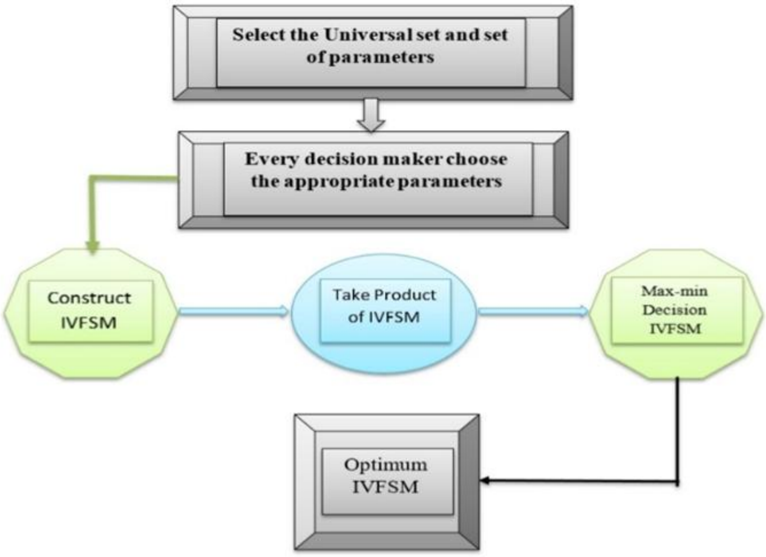

Figure-1. Graphical Model for IVFSMmDM.
Source: Zulqarnain and Saeed [30].

4. APPLICATION IVFSMMDM METHOD FOR FACULTY SELECTION

Let T = $\{T_1, T_2, T_3, T_4\}$ be a set of teachers of Mathematics subject and E = $\{j_1, j_2, j_3, j_4\}$ be a set of parameters for teacher selection represents personality, classroom management skills, subject command, and determination respectively. Higher education commission hires a selection committee that consists of two members to represent as follows C = $\{M_1, M_2\}$.

Step 1

Both committee members choose the parameters for selection of faculty member

$M_1 = \{j_2, j_3, j_4\}$ and $M_2 = \{j_1, j_2, j_4\}$

Step 2

We construct the IVFSMs of both committee members according to their selected parameters.

$$M_1 = [m_{ij}] = \begin{bmatrix} [0.0, 0.0] & [0.8, 0.9] & [0.9, 1.0] & [0.7, 0.8] \\ [0.0, 0.0] & [0.5, 0.6] & [0.3, 0.4] & [0.4, 0.5] \\ [0.0, 0.0] & [0.9, 1.0] & [0.7, 0.8] & [0.8, 0.9] \\ [0.0, 0.0] & [0.7, 0.8] & [0.6, 0.7] & [0.8, 0.9] \end{bmatrix}$$

$$M_2 = [n_{ik}] = \begin{bmatrix} [0.5, 0.6] & [0.9\ 1.0] & [0.0, 0.0] & [0.6, 0.7] \\ [0.8, 0.9] & [0.4, 0.5] & [0.0, 0.0] & [0.8, 0.9] \\ [0.3, 0.4] & [0.6, 0.7] & [0.0, 0.0] & [0.4, 0.5] \\ [0.6, 0.7] & [0.9, 1.0] & [0.0, 0.0] & [0.9, 1.0] \end{bmatrix}$$

Step 3

Take And-product of both developed IVFSMs

$$[m_{ij}] \wedge [n_{ik}] = \begin{bmatrix} [0.0, 0.0] & [0.8, 0.9] & [0.9, 1.0] & [0.7, 0.8] \\ [0.0, 0.0] & [0.5, 0.6] & [0.3, 0.4] & [0.4, 0.5] \\ [0.0, 0.0] & [0.9, 1.0] & [0.7, 0.8] & [0.8, 0.9] \\ [0.0, 0.0] & [0.7, 0.8] & [0.6, 0.7] & [0.8, 0.9] \end{bmatrix} \wedge \begin{bmatrix} [0.5, 0.6] & [0.9\ 1.0] & [0.0, 0.0] & [0.6, 0.7] \\ [0.8, 0.9] & [0.4, 0.5] & [0.0, 0.0] & [0.8, 0.9] \\ [0.3, 0.4] & [0.6, 0.7] & [0.0, 0.0] & [0.4, 0.5] \\ [0.6, 0.7] & [0.9, 1.0] & [0.0, 0.0] & [0.9, 1.0] \end{bmatrix}$$

$$[m_{ij}] \wedge [n_{ik}] = \begin{bmatrix} [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.5,.6] & [.8,.9] & [.0,.0] & [.6,.7] & [.5,.6] & [.9,1] & [.0,.0] & [.6,.7] & [.5,.6] & [.7,.8] & [.0,.0] & [.6,.7] \\ [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.5,.6] & [.4,.5] & [.0,.0] & [.5,.6] & [.3,.4] & [.3,.4] & [.0,.0] & [.3,.4] & [.4,.5] & [.4,.5] & [.0,.0] & [.4,.5] \\ [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.3,.4] & [.6,.7] & [.0,.0] & [.4,.5] & [.3,.4] & [.6,.7] & [.0,.0] & [.4,.5] & [.3,.4] & [.6,.7] & [.0,.0] & [.4,.5] \\ [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.6,.7] & [.7,.8] & [.0,.0] & [.7,.8] & [.6,.7] & [.6,.7] & [.0,.0] & [.6,.7] & [.6,.7] & [.8,.9] & [.0,.0] & [.8,.9] \end{bmatrix}$$

Step 4

Calculate $\mathrm{Mm}\left([m_{ij}] \wedge [n_{ik}]\right) = d_{i1}$ where $i = 1, 2, 3, 4$. First, we find d_{11}, for this

$d_{11} = \max\{t_{1k}\} = \max\{t_{11}, t_{12}, t_{13}, t_{14}\}$, to find d_{11} we need to find t_{1k} for every $k = 1, 2, 3, 4$.

If k = 1 and n = 4, t_{11} is $I_1 = \{P: c_{ip} \neq 0, 0 < P \leq 4\}$, then

$t_{11} = [0.0, 0.0]$

If k = 2 and n = 4, t_{12} is $I_2 = \{P: c_{ip} \neq 0, 4 < P \leq 8\}$, then

$t_{12} = \min\{c_{15}, c_{16}, c_{18}\} = \min\{[0.5, 0.6], [0.8, 0.9], [0.6, 0.7]\} = [0.5, 0.6]$

If k = 3 and n = 4, t_{13} is $I_3 = \{P: c_{ip} \neq 0, 8 < P \leq 12\}$, then

$t_{13} = \min\{c_{19}, c_{110}, c_{112}\} = \min\{[0.5, 0.6], [0.9, 1.0], [0.6, 0.7]\} = [0.5, 0.6]$

If k = 4 and n = 4, t_{14} is $I_4 = \{P: c_{ip} \neq 0, 12 < P \leq 16\}$, then

$t_{14} = \min\{c_{113}, c_{114}, c_{116}\} = \min\{[0.5, 0.6], [0.7, 0.8], [0.6, 0.7]\} = [0.5, 0.6]$

So $d_{11} = \max\{t_{1k}\} = \max\{t_{11}, t_{12}, t_{13}, t_{14}\} = \max\{[0.0, 0.0], [0.5, 0.6], [0.5, 0.6], [0.5, 0.6]\}$ $d_{11} = [0.5, 0.6]$

Similarly, we can find d_{21}, d_{31}, d_{41}

$d_{21} = [0.4, 0.5]$

$d_{31} = [0.3, 0.4]$

$d_{41} = [0.6, 0.7]$

We get IVFSM by using IVFSMmDM as follows

$$\mathrm{Mm}([m_{ij}] \wedge [n_{ik}]) = d_{i1} = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{41} \end{bmatrix} = \begin{bmatrix} [0.5, 0.6] \\ [0.4, 0.5] \\ [0.3, 0.4] \\ [0.6, 0.7] \end{bmatrix}$$

According to the above matrix, we get optimum IVFSM on T.

Opt $\mathrm{Mm}([m_{ij}] \wedge [n_{ik}])$ (T) = {[0.5, 0.6]/ T_1, [0.4, 0.5]/ T_2, [0.3, 0.4]/ T_3, [0.6, 0.7]/ T_4}. So T_4 is a suitable teacher for recruitment according to given parameters.

5. CONCLUSION

We studied some basic concepts of fuzzy sets, soft sets, fuzzy soft sets, and interval-valued fuzzy soft sets in this work with some properties and operations. A graphical model also constructed for IVFSMmDM by using the IVFSMmDM function. Finally, we use the proposed model in decision making for the selection of teachers.

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