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DEVELOPMENT, PREFERENCE CHANGE, AND INEQUALITY IN AN INTEGRATED WALRASIAN-GENERAL-EQUILIBRIUM AND NEOCLASSICAL-GROWTH THEORY

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ABSTRACT

This study builds a model to analyze dynamic interdependence between economic growth, structural change, inequality in income and wealth, habit formation and preference change in a heterogeneous-households economy. The analytical framework is based on the Walrasian general equilibrium theory and the neoclassical growth theory. We show that the motion of the economy is described by a set of nonlinear differential equations. Analytically, we focus on transitional processes as well as economic equilibrium. For illustration, we simulate the motion of the economic system with three groups. We identify the existence of a unique stable equilibrium point and carry out comparative dynamic analysis with regard to some parameters. We show how, for instance, habit formation of a group affects the dynamic paths of all the variables of the economic system.

Keywords: Walrasian general equilibrium theory, Neoclassical growth theory, Habit formation and preference, Heterogeneous households, Income and wealth distribution, Economic structure.

JEL: Q41; E21

Contribution/ Originality

An original contribution of this paper is the introduction of habit formation and preference change into the integrated Walrasian-general-equilibrium and neoclassical-growth theory recently developed by Zhang. The study examines dynamic relations between economic growth, economic change, and inequality with wealth accumulation and preference change.

1. INTRODUCTION

One of the most important economic issues is relationship between economic growth and inequality in income and wealth. Nevertheless, only a few theoretical economic models can properly

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explain these dynamic phenomena, even though empirical studies have provided rich observations and statistical explanations about income and wealth inequalities over time and space. The purpose of this study is to build an economic growth model to explain dynamic interdependence between economic growth, structural change, inequality in income and wealth, and habit formation and preference change in a heterogeneous-households economy. The model is built on three main economic theories – the Walrasian general economic theory, the neoclassical growth theory, and the growth theory with habit formation and preference change.

The model is based on the Walrasian general equilibrium theory in the sense that the analytical framework is a general equilibrium model with fixed wealth, habit and preference. The Walrasian general equilibrium theory was initially developed by Walras (1874). The mathematical sophistication of the theory was carried out mainly the 1950s by Arrow, Debreu and others (Arrow and Debreu, 1954; Debreu, 1959; Arrow and Hahn, 1971; Mas-Colell *et al.*, 1995). The basic content of the theory deals with equilibrium of pure economic exchanges with heterogeneous supplies and households. It has become evident that it is very difficult to introduce important dynamic factors, such as endogenous changes of capital, wealth, preference, resource, environment, technology and human capital, into the theory. The necessity of generalizing and extending the Walrasian general theory has long been recognized. But few formal economic models are successful in extending the theory to include, for instance, endogenous wealth. Walras made great efforts but failed in developing a general equilibrium theory with endogenous saving and capital accumulation (Impicciatore *et al.*, 2012). Many economists tried to further develop Walras' capital accumulation within Walras' framework (Montesano, 2008). Nevertheless, no study could have solved the most essential problem of providing microeconomic foundation for wealth accumulation within the Walrasian framework with heterogeneous households till Zhang's recent efforts by using an alternative utility function.

A main contribution of this paper is the introduction of habit formation and preference change into an integrated Walrasian-general-equilibrium and neoclassical-growth theory. We examine dynamic relations between economic growth, economic change, and inequality with wealth accumulation and preference change. Some empirical studies confirm existence of interdependent relations between preference changes and other changes in social and economic conditions (Olsen, 1993; Becker and Mulligan, 1997; Chao *et al.*, 2009). There are also a few studies on growth and preference change in the literature of economic growth theory. Uzawa (1968) proposed a formal modeling of endogenous time preference within the Ramsey growth theory (Lucas and Stokey, 1984; Epstein, 1987). There are other studies on the implications of endogenous time preference for the macroeconomy (Epstein and Hynes, 1983; Shi and Epstein, 1993; Dioikitopoulos and Kalyvitis, 2010) within the traditional Ramsey growth theory. These studies argue that it is necessary to take account of endogeneity of time preference in order to understand process of economic growth and development. Another important approach in describing behavior of household is related to the idea of habit formation or habit persistence. It was introduced to formal economic analysis by Duesenberry (1949). The idea basically implies that individuals tend to get accustomed to a given "standard of living" which they like to keep. Becker (1992) explains: "the habit acquired as a child or young adult generally continue to influence behavior even when the environment changes radically." (Mehra and Prescott, 1985; Campbell and Cochrane, 1999; Boldrin *et al.*, 2001). It should be noted that 'catching up with the Joneses' is often used exchangeable with external habit formation (Abel, 1990). We will apply the habit formation to model dynamics of the propensities to consume consumer goods.

This study follows Uzawa's two sector growth model in describing economic structure (Uzawa, 1961). This study follows an alternative approach in modelling consumer behaviour proposed by Zhang (1993). It should be noted that this study is to integrate two models by Zhang (2012; 2013). Zhang (2012) develops a Solow-type model with heterogeneous households without preference change, while (Zhang, 2013) builds a two-sector model with homogeneous population on the basis of the habit formation and time preference in the Ramsey-type growth theory. The rest of the paper is organized as follows. Section 2 defines the growth model of heterogeneous households with preference dynamics. Section 3 examines dynamic properties of the model and simulates the motion of the economy. Section 4 carries out comparative dynamic analysis with regard to changes in some parameters on the system. Section 5 makes concluding remarks.

2. THE BASIC MODEL

The model is influenced by the Walrasian general equilibrium theory and the neoclassical growth theory. The supply side is the same as the traditional two-sector neoclassical growth model with capital goods and consumer goods sectors (Uzawa, 1961). Uzawa's two-sector model has been generalized and extended in different ways over years (Jensen *et al.*, 2005). Services are classified as consumer goods. We follow the neoclassical growth theory in describing behavior of the production sectors. Capital goods are be used as inputs in the two sectors. Capital depreciates at a fixed rate δ_k, which is independent of the manner of use. Saving is undertaken only by households. Households own assets of the economy and distribute their incomes to consume and save. All the markets perfectly competitive. Factor markets work well and the available factors are fully utilized at every moment. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. The population is classified into J groups and each group has a fixed population, $\bar{N}_j$, ($j = 1, \ldots, J$). In almost all models in the literature of neoclassical growth theory with microeconomic foundation the population is considered homogeneous, i.e., $J = 1$. In the Walrasian general equilibrium theory, the number of types of household is equal to the population. We measure prices in terms of capital goods and choose the price of the commodity to be unit. Let $w_j(t)$ and $r(t)$, respectively, stand for the wage rate of worker of type j and rate of interest. We use $p(t)$ to denote price of consumer goods. The total capital stock $K(t)$ and the total labor supply N are allocated between the two sectors. We use subscript index i and s to stand for capital goods and consumer goods sectors, respectively. We use $N_m(t)$ and $K_m(t)$ to stand for the labor force and capital stocks employed by sector m. The total population $\bar{N}$ and total qualified labor supply N are

$$\bar{N} = \sum_{j=1}^{J} \bar{N}_j, \quad N = \sum_{j=1}^{J} h_j \bar{N}_j, \tag{1}$$

in which h_j is the human capital of group j. We assume human capital constant. As the model is already complicated, we treat human capital constant in this paper. The assumption of labor force being fully employed implies

$$N_i(t) + N_s(t) = N. \qquad (2)$$

2.1. The Capital Goods Sector

Each sector employs two input factors, labor force and capital, in production. Let $F_m(t)$ stand for production function of sector m, $m = i, s$. The production function of the capital goods sector is

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \qquad (3)$$

where A_i, α_i, and β_i are positive parameters. The marginal conditions for the capital goods sector are

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)}. \qquad (4)$$

2.2. The Consumer Goods Sector

The production function of the consumer goods sector is

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \quad \alpha_s + \beta_s = 1, \quad \alpha_s, \beta_s > 0, \qquad (5)$$

where A_s, α_s, and β_s are parameters. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_s p(t) F_s(t)}{K_s(t)}, \quad w(t) = \frac{\beta_s p(t) F_s(t)}{N_s(t)}. \qquad (6)$$

2.3. Consumer Behaviors and Wealth Dynamics

In this study, we use an alternative approach to modeling behavior of household proposed by Zhang (1993). Let $\bar{k}_j(t)$ stand for per capita wealth of group j. We have $\bar{k}_j(t) = \bar{K}_j(t)/\bar{N}_j$, where $\bar{K}_j(t)$ is the total wealth held by group j. Per capita current income from the interest payment $r(t)\bar{k}_j(t)$, and the wage payment $w_j(t) = h_j w(t)$, is $y_j(t) = r(t)\bar{k}_j(t) + w_j(t)$. The per capita disposable income is the sum of the current disposable income and the value of wealth

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \qquad (7)$$

The disposable income is used for saving and consumption. It should be remarked that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$), in the above equation is a flow variable. The representative household of group j distributes the total available budget between saving $s_j(t)$ and consumer goods $c_j(t)$. The budget constraint is

$$p(t) c_j(t) + s_j(t) = \hat{y}_j(t). \qquad (8)$$

At each point in time the household decides the two variables. We specify the utility level as follows

$$U_j(t) = c_j^{\xi_{0j}(t)}(t) s_j^{\lambda_{0j}(t)}(t), \quad \xi_{0j}(t), \lambda_{0j}(t) > 0,$$

where $\xi_{0j}(t)$ is the propensity to consume, and $\lambda_{0j}(t)$ is the propensity to own wealth. The utility function with constant preference was proposed about two decade ago by Zhang (1993). Following the recent studies by Zhang (2013) this study treats preferences of the population as endogenous variables. It is easy to see why these models fail to address basic issues related to, for instance, the role of the propensities on inequality in wealth and income in short term as well as long term. Maximizing the utility subject to (8) yields

$$p(t) c_j(t) = \xi_j(t) \hat{y}_j(t), \quad s_j(t) = \lambda_j(t) \hat{y}_j(t), \qquad (9)$$

where

$$\xi_j(t) \equiv \rho_j(t)\xi_{0j}(t), \quad \lambda_j(t) \equiv \rho_j(t)\lambda_{0j}(t), \quad \rho_j(t) \equiv \frac{1}{\xi_{0j}(t) + \lambda_{0j}(t)}.$$

The definition of $s_j(t)$ implies the change in the household's wealth as follows

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t) = \lambda_j(t)\hat{y}_j(t) - \bar{k}_j(t). \tag{10}$$

This equation simply states that the change in wealth is equal to the saving minus the dissaving.

2.4. Demand For and Supply of the Two Sectors

The balance of demand for and supply of consumer goods satisfies

$$\sum_{j=1}^{J} c_j(t)\bar{N}_j = F_s(t). \tag{11}$$

The depreciation of the total capital stock and the net savings is equal to the output of the capital goods sector. This implies

$$S(t) - K(t) + \delta_k K(t) = F_i(t), \tag{12}$$

where

$$S(t) \equiv \sum_{j=1}^{J} s_j(t)\bar{N}_j, \quad K(t) = \sum_{j=1}^{J} \bar{k}_j(t)\bar{N}_j. \tag{13}$$

2.5. Capital Being Fully Utilized

Full employment of the total capital stock implies

$$K_i(t) + K_s(t) = K(t). \tag{14}$$

2.6. The Time Preference and the Propensity to Hold Wealth

We now follow Zhang (2013) in modeling the change in the propensities to save. Zhang's approach is closely related to the mechanisms of habit formation and preference change the in literature of the neoclassical growth model (e.g.,(Uzawa, 1968; Epstein, 1987; Shi and Epstein, 1993; Chang *et al.*, 2011)). We propose the following propensities to save

$$\lambda_{0j}(t) = \bar{\lambda}_j + \lambda_{wj}\, w_j(t) + \lambda_{kj}\, \bar{k}_j(t), \tag{15}$$

where $\bar{\lambda}_j > 0$, λ_{wj}, and λ_{kj} are parameters. For simplicity, the propensities to save are assumed to be proportional to the wage rate and wealth of the same group.

2.7. The Habit Formation and the Propensity to Consume Consumer Goods

Another important idea in the literature of economic growth with endogenous preference is the so-called habit formation (Deaton and Muellbauer, 1980; Amano and Laubach, 2004; Gómez, 2008; Corrado and Holly, 2011). On the basis of the traditional approach to habit formation as well as the growth models with preference change by Zhang (2013) we specify the habit formation with regard to consumer goods as follows

$$\dot{\hbar}_{cj}(t) = \tilde{\xi}_j\left[c_j(t) - \hbar_{cj}(t)\right]. \tag{16}$$

If the current consumption is higher than the level of the habit stock, then the level of habit stock tends to rise, and vice versa. We consider the propensity to consume a function of the wage income and the habit stock as follows

$$\xi_{0j}(t) = \bar{\xi}_j + \xi_{wj}\, w_j(t) + \xi_{hj}\, \hbar_{cj}(t), \tag{17}$$

where $\bar{\xi}_j > 0$, ξ_{wj} and $\xi_{hj} \ge 0$ are parameters. If $\xi_{wj} = 0$ and $\xi_{hj} = 0$, the propensity is constant. If $\xi_{wj} > (<)\, 0$, a rise in the wage rate enhances (lowers) the propensity to consume. For normal goods we may consider $\xi_{wj} \ge 0$. For inferior goods, the sign may be opposite. Empirical studies show that relations between wage and consumption is complicated (Lusardi, 1996; Storesletten *et al.*, 2004; Lise and Seitz, 2011). In the case of $\xi_{hj} > 0$, a rise in the habit stock causes the propensity to consume to rise, and vice versa. In this initial stage we neglect possibilities that one group's habit and propensity to consume are affected by the other groups' habits and propensities to consume.

The model is completed.

3. THE DYNAMICS AND ITS PROPERTIES

The previous section built a model with any (finite) number of households and dynamic behavior of each type of households is described by two equations for wealth accumulation and habit formation. The following lemma shows that the dimension of the dynamical system is twice as many as the number of household types. We also provide a computational procedure for calculating all the variables at any point of time. First, we introduce a new variable $z(t)$

$$z(t) \equiv \frac{r(t) + \delta_k}{w_j(t)/h_j}.$$

Lemma

The motion of $z(t)$, $\{\bar{k}_j(t)\}$ and $(\hbar_{cj}(t))$ are given by the following $2J$ differential equations

$$\dot{z}(t) = \Lambda_1\left(z(t), \{\bar{k}_j(t)\}, (\hbar_{cj}(t))\right),$$
$$\dot{\bar{k}}_j(t) = \Lambda_j\left(z(t), \{\bar{k}_j(t)\}, (\hbar_{cj}(t))\right), \quad j = 2, \ldots, J,$$
$$\dot{\hbar}_{cj}(t) = \Omega_j\left(z(t), \{\bar{k}_j(t)\}, (\hbar_{cj}(t))\right), \quad j = 1, \ldots, J, \tag{18}$$

which $\Lambda_j(t)$ and $\Omega_j(t)$ are unique functions of $z(t)$, $\{\bar{k}_j(t)\}$ and $(h_j(t))$ defined in the appendix. As shown in the Appendix, for given values of the following $2J$ variables, $z(t)$, $\{\bar{k}_j(t)\}$ and $(\hbar_{cj}(t))$ where $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \cdots, \bar{k}_J(t))$ and $(\hbar_{cj}(t)) \equiv (\hbar_{c1}(t), \cdots, \hbar_{cJ}(t))$ at any point in time, all the other variables are determined as functions of $z(t)$, $\{\bar{k}_j(t)\}$ and $(\hbar_{cj}(t))$ by the following procedure: $r(t)$ and $w_j(t)$ by (A3) $\rightarrow$ $p(t)$ by (A4) $\rightarrow$ $\bar{k}_1(t)$ by (A13) $\rightarrow$ $\xi_{0j}(t)$ by (17) $\rightarrow$ $\lambda_{0j}(t)$ by (14) $\rightarrow$ $\xi_j(t)$ by $\lambda_j(t)$ by the definitions $\rightarrow$ $N_i(t)$ by (A9) $\rightarrow$ $N_s(t)$ by (A7)$\rightarrow$ $w(t) = w_1(t)/h_1$ $\rightarrow$ $\hat{y}_j(t)$ by (A5) $\rightarrow$ $K_s(t)$ and $K_i(t)$ by (A1) $\rightarrow$ $F_i(t)$ and $F_s(t)$ by the definitions $\rightarrow$ $c_j(t)$ and $s_j(t)$ by (9) $\rightarrow K(t) = K_i(t) + K_s(t)$. Moreover all these variable values are unique functions of $\bar{k}_1(t)$ which may have one or two solutions from (A13). The lemma and the computational procedure guarantee that we can plot the motion of the economic system with any number of types of household. As we cannot get explicitly analytical properties of the dynamic system with more than a single type of household, we simulate an economy with three types of household. This implies that we have to deal with six nonlinear differential equations. For illustration, we take on the parameter values as follows

$$A_i = 1.3,\ A_s = 1,\ \alpha_i = 0.34,\ \alpha_s = 0.3,\ \delta_k = 0.05,\ \bar{\xi}_j = 0.25,\ \bar{\xi}_{wj} = 0.01,\ \bar{\xi}_{hj} = 0.02,$$

$$\begin{pmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 30 \\ 60 \end{pmatrix},\ \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0.6 \end{pmatrix},\ \begin{pmatrix} \bar{\lambda}_1 \\ \bar{\lambda}_2 \\ \bar{\lambda}_3 \end{pmatrix} = \begin{pmatrix} 0.65 \\ 0.55 \\ 0.5 \end{pmatrix},\ \begin{pmatrix} \lambda_{w1} \\ \lambda_{w2} \\ \lambda_{w3} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix},\ \begin{pmatrix} \lambda_{k1} \\ \lambda_{k2} \\ \lambda_{k3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix},\ \begin{pmatrix} \tilde{\xi}_1 \\ \tilde{\xi}_2 \\ \tilde{\xi}_3 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}. \tag{19}$$

We specify the initial conditions as follows

$$z(0) = 0.09,\ \bar{k}_2(0) = 4.1,\ \bar{k}_3(0) = 1.9,\ \hbar_{c1}(0) = 2.6,\ \hbar_{c2}(0) = 1.5,\ \hbar_{c3}(0) = 1.$$

The motion of the variables is plotted in Figure 1. It should be remarked that according to (A13) in the appendix, the system may has one or two equations according to the number of meaningful solution(s) of the following equation

$$\bar{k}_1 = \varphi_{1,2}\left(z, \{\bar{k}_j\}, (\hbar_{cj})\right) \equiv -\frac{m_1}{2} \pm \sqrt{\frac{m_1^2}{4} + m_2}.$$

The simulation confirms that the solution

$$\bar{k}_1 = -\frac{m_1}{2} - \sqrt{\frac{m_1^2}{4} + m_2},$$

is economically not meaningful with the initial conditions. Hence, the equation has a unique solution. In Figure 1, the national income is $Y(t) = F_i(t) + p(t)F_s(t)$.

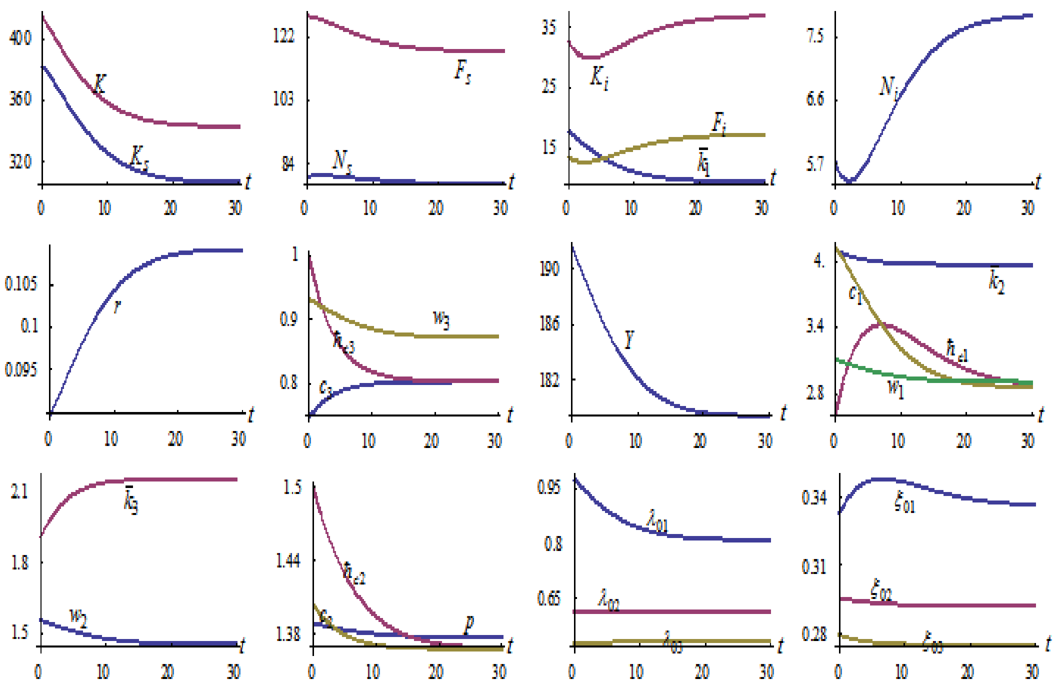

Figure-1. The Motion of the Economic System

The national output and output of the consumer goods sector fall over time from their initial levels. The output level of the capital goods sector falls slightly initially and then rises overtime. As both the output and the price of the consumer goods sector fall, the falling in the value of the consumer goods sector's output is larger than the rising in the value of the capital goods sector's output, the national output falls. The levels of habit stocks of groups 2 and 3 are above the levels of the corresponding consumer goods until they converge in the long term. The level of habit stock of group 1 is initially below and then above the level of the corresponding consumer goods until they converge in the long term. The input levels of the consumer goods sector fall over time. The input levels of the capital goods sector fall initially and then rise till they achieve at the equilibrium levels. As the total wealth falls over time, the rate of interest is enhanced and the wage rates are reduced. The consumption levels of groups 2 and 3 fall over time, and the consumption level of group 1 rises. Group 1's propensity to save falls over time, while the other two groups' We see that the inequality in income and wealth, for instance, between groups 1 and 3 are reduced during the transitional process. The simulation confirms that the system has a unique equilibrium. We list the equilibrium values in (20)

$$Y = 179.55,\ K = 343.35,\ r = 0.109,\ p = 1.38,\ w_1 = 2.91,\ w_2 = 1.45,\ w_3 = 0.87,\ F_i = 17.17,$$
$$F_s = 117.83,\ N_i = 7.80,\ N_s = 78.4,\ K_i = 36.74, K_s = 306.61,\ c_1 = \hbar_{c1} = 2.86,\ c_2 = \hbar_{c2} = 1.37,$$
$$c_3 = \hbar_{c3} = 0.8,\ \bar{k}_1 = 9.51,\ \bar{k}_2 = 3.97,\ \bar{k}_3 = 2.15. \tag{20}$$

It is straightforward to calculate the six eigenvalues as follows

$$\{-0.21 \pm 0,05i,\ -0.19 \pm 0.06i,\ -0.16 \pm 0.08i\}.$$

The real parts of all the eigenvalues have real and negative. The unique equilibrium is locally stable. This guarantees the validity of exercising comparative dynamic analysis.

4. COMPARATIVE DYNAMIC ANALYSIS

We simulated the motion of the national economy under (19). It is significant to examine how the economic system reacts to exogenous changes. As the lemma provides the computational procedure to calibrate the motion of all the variables, it is straightforward to examine effects of change in any parameter on transitory processes as well stationary states of all the variables. We introduce a variable $\overline{\Delta} x_j(t)$ which stands for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value.

4.1. A Rise in the Intercept in Group 3's the Propensity to Save Function

We now study how the economic system reacts as the intercept of group 3's propensity to save function is increased as follows: $\bar{\lambda}_k : 0.5 \Rightarrow 0.55.$ The results are plotted in Figure 2. Group 3's propensity to save is increased and the other two groups' propensities to save are slightly affected. The group's propensity to consume falls initially but is increased in the long term. This occurs because the group's wage rate and wealth are all increased. It should be noted that the habit stock level is changed less violently than the level of consumption. The micro variables of the other two groups are slightly affected. The macroeconomic variables are strongly affected. As group 3 saves more, the national wealth and national output are enhanced. The output level and factor inputs of the capital goods sector are strongly increased. The labor input of the consumer goods sector is slightly affected. The price of consumer goods is slightly increased. The wage rates are increased in association with falling in the rate of interest. By saving more, the gaps in per capita wage, wealth, consumption between group 3 and other two groups are reduced in the long term.

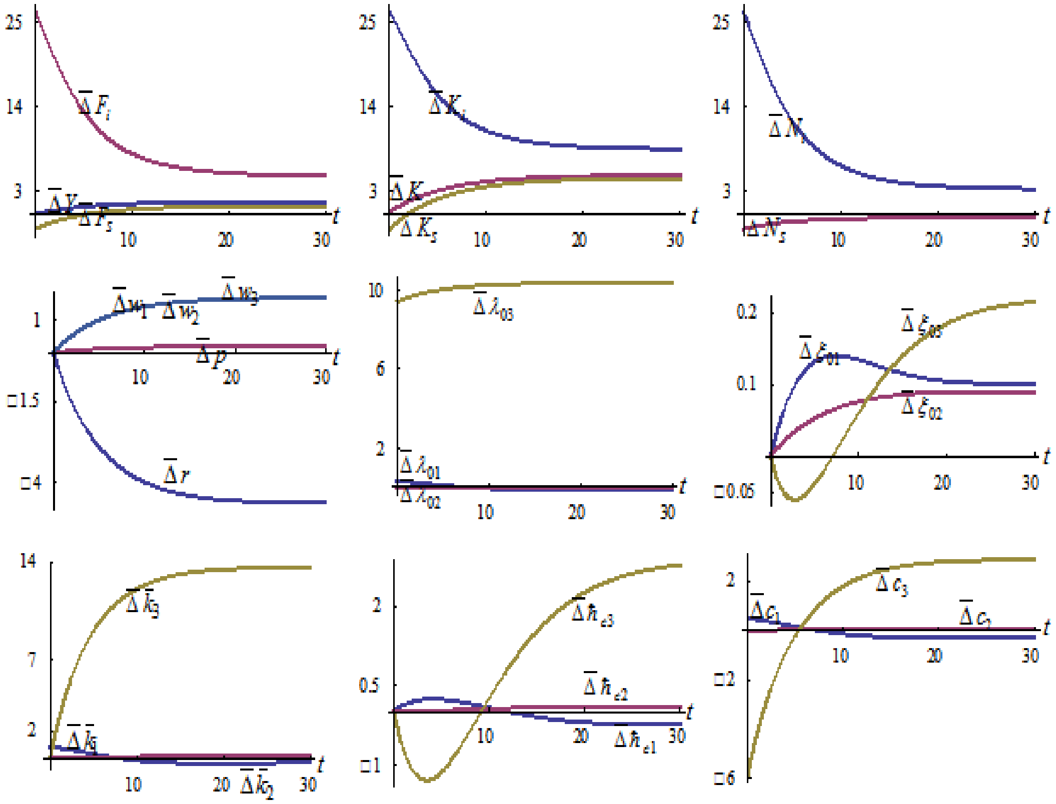

Figure-2. A Rise in the Intercept in Group 3's the Propensity to Save Function

4.2. Group 1 Lowering its Weights in More Distant Values of Consumption Level

We now study what will happen to the economic growth if group 1 lowers its weights in more distant values of the levels of consumption as follows: $\tilde{\xi}_1 : 0.2 \Rightarrow 0.6$. Figure 3 provides the simulation results. The simulation confirms that the change in the parameter has no impact on the long-term equilibrium point of the dynamic system. As the eigenvalues are imaginary, the system oscillates before it is back to its long-term equilibrium point. When $\tilde{\xi}_1$ is increased, it is faster for group 1's habit stock to converge to the current consumption level. Although the disturbance in the speed of adjustment has no effects on the long-term equilibrium point, the transitional paths towards the equilibrium points of the variables are affected. As $\tilde{\xi}_1$ is increased, group 1's habit stock of consumer goods is initially increased, leading to a rise in the group's current consumption level. Group 1's propensities to consume is enhanced and its propensity to save is reduced. Although the other two groups' behavioral variables are slightly affected, the macroeconomic variables oscillate before they come back to their original points.

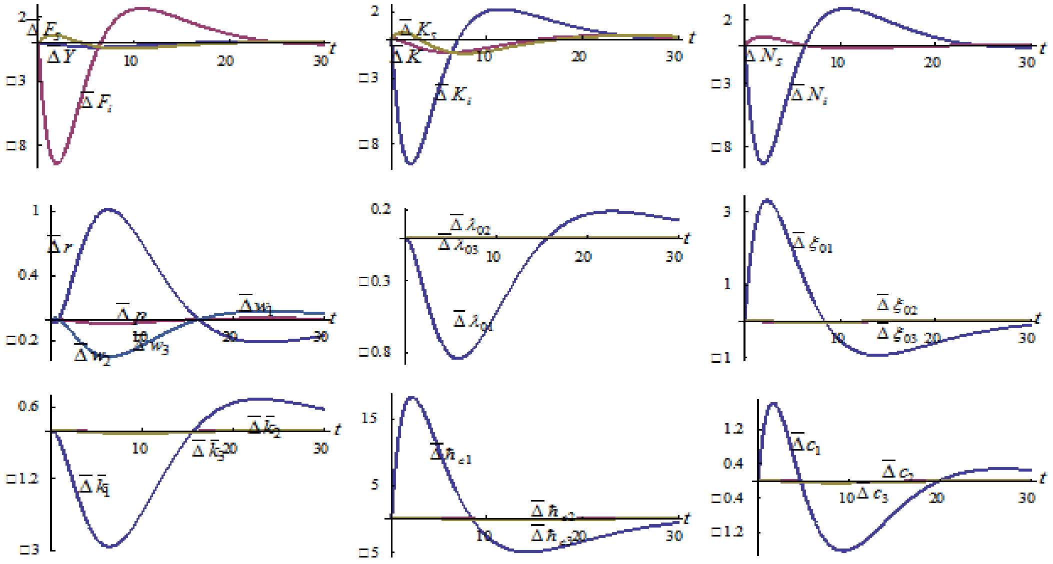

Figure-3. Group 1 Lowering Its Weights in More Distant Values of Consumption Level

4.3. Group 1's Habit Stock Affecting Its Propensity to Consume More Strongly

We allow group 1's effect of the habit stock of consumer goods on the propensity to consume to be changed as follows $\xi_{1h}: 0.02 \Rightarrow 0.03.$ When the habit stock more strongly affects the propensity to consume, group 1's habit stock is initially increased, which also lifts the consumption level. But in the long time group 1's consumption level. The shift in the parameter leads to falling in the propensity to save and rising in the propensity to consume. The group's per capita wealth is reduced, which explains falling in the consumption level in the long term. The other two groups' behavior are slightly affected. The macroeconomic variables are affected. The total wealth, nation output, and output levels and capital inputs of the two sectors are reduced in the long time. Some of the labor force is shifted from the capital goods sector to the consumer goods sector. The inequalities in wealth and income between group 1 and the other two groups are reduced.

4.4. Group 2's Wealth Having Stronger Impact on Its Propensity to Save

We allow group 2's effect of the habit stock of consumer goods on the propensity to consume to be changed as follows $\lambda_{k2}: 0.02 \Rightarrow 0.03.$ When group 2's wealth has a stronger impact on its propensity to save, group 2's propensity to save is increased. The rise in the propensity leads to an increase in group 2's wealth. In the initial stage the rise in the propensity reduces the group's propensity to consume and also lowers the group consumption. However, in the long term both group $2's$ propensity to consume and consumption level are increased. This occurs as a consequence of rises in the wage and wealth are increased. Since group $2's$ wealth is augmented, the national wealth is increased. The rise in the national capital stock results in a rise in the wage rates. As group $1's$ and group $3's$ wage rates are increased, their propensities to consume are increased. Group $1's$ and group $3's$ propensities to save are also affected. The net effects on group 3 are that its wealth and consumption are increased. The net effects on group 1 are that its wealth and consumption are increased initially and are reduced in the long term. Group $2's$ preference has strong impact on economic structure. As the group saves more, the demand of capital goods is much increased. The demand for consumer goods is slightly reduced initially and then is increased. Some of the labor force is shifted to the capital goods sector from the consumer goods sector. The national output is increased. The price of consumer goods rise and the rate of interest falls.

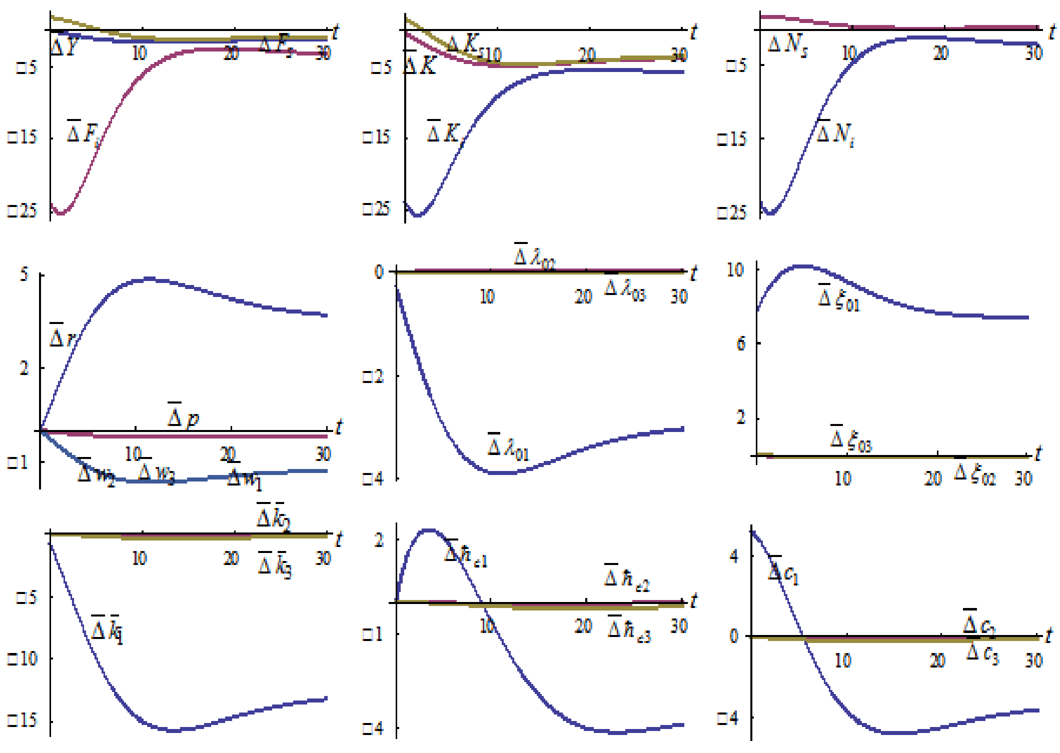

Figure-4. Group $1's$ Habit Stock Affecting Its Propensity to Consume More Strongly

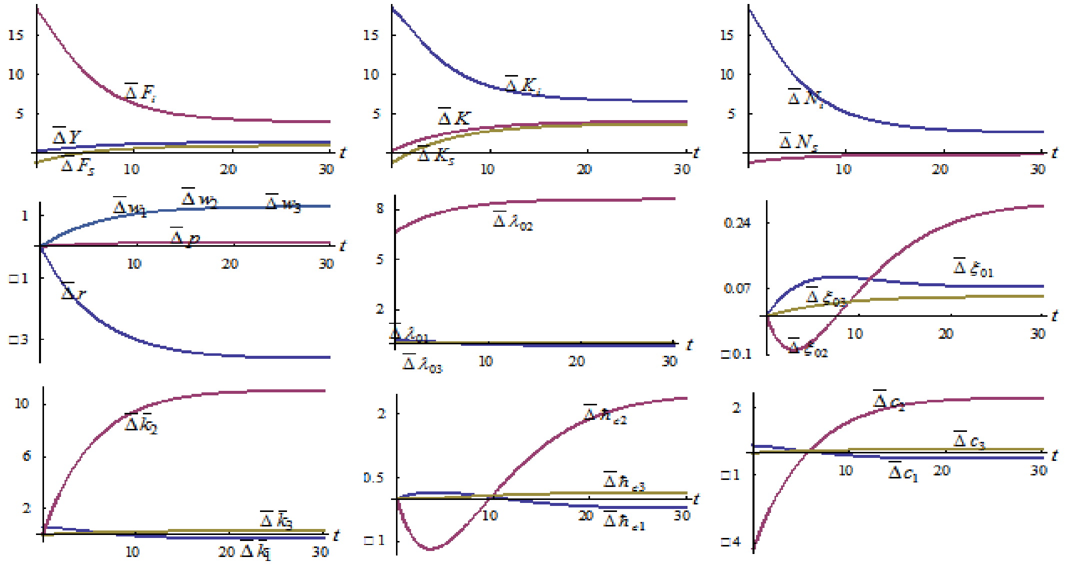

Figure-5. Group 2's Wealth Having Stronger Impact on Its Propensity to Save

5. CONCLUDING REMARKS

This study proposed an economic growth model of heterogeneous households with habit formation and preference change. The heterogeneity in population is essentially different from the heterogeneity in the standard Ramsey growth models with microeconomic foundation. Our analytical framework is built on the basis of a few well-known theories in economics. The economic system consists of one capital goods sector, one consumer goods sector, and any number (of types) of households. The motion is described by a set of nonlinear differential equations. We focused on transitional processes as well as economic equilibrium. We simulated the motion of the economic system with three groups of household.

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Appendix: Proving the lemma

By (4) and (6), we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{N_i}{\bar{\beta}_i K_i} = \frac{N_s}{\bar{\beta}_s K_s}, \tag{A1}$$

where $\bar{\beta}_j \equiv \beta_m / \alpha_m$. From (A1) and (14), we obtain

$$\bar{\beta}_i K_i + \bar{\beta}_s K_s = \frac{N}{z}. \tag{A2}$$

Insert (A1) in (4)

$$r(z) = \alpha_r z^{\beta_i} - \delta_k, \quad w_j(z) = \alpha_w h_j z^{-\alpha_i}, \tag{A3}$$

where $\alpha_r = \alpha_i A_i \bar{\beta}_i^{\beta_i}$, $\alpha_w = \beta_i A_i \bar{\beta}_i^{-\alpha_i}$. From (5) and (6), we have

$$p(z) = \frac{\bar{\beta}_s^{\alpha_s} z^{\alpha_s} w}{\beta_s A_s}, \tag{A4}$$

where $w = w_1 / h_1$. From (A3) and the definitions of $\hat{y}_j$, we have

$$\hat{y}_j = (1 + r)\bar{k}_j + w_j. \tag{A5}$$

Insert $pc_j = \xi_j \hat{y}_j$ in (11)

$$\sum_{j=1}^{J} \xi_j \bar{N}_j \hat{y}_j = pF_s. \tag{A6}$$

Substituting (A5) in (A6) yields

$$N_s = \sum_{j=1}^{J} \tilde{g}_j \bar{k}_j + \tilde{g}, \tag{A7}$$

where we use $pF_s = wN_s / \beta_s$ and

$$\tilde{g}_j(z, \xi_j) \equiv R \beta_s \xi_j \bar{N}_j, \quad R(z) \equiv \frac{1 + r}{w}, \quad \tilde{g}((\xi_j)) \equiv \beta_s \sum_{j=1}^{J} h_j \xi_j \bar{N}_j.$$

From (A1), (14) and $K = \sum_{j=1}^{J} \bar{k}_j \bar{N}_j$, we get

$$\frac{N_i}{\bar{\beta}_i} + \frac{N_s}{\bar{\beta}_s} = z \sum_{j=1}^{J} \bar{k}_j \bar{N}_j. \tag{A8}$$

From (A7) and (A8), we solve

$$N_i = \bar{\beta}_i z \sum_{j=1}^{J} \bar{k}_j \bar{N}_j - \frac{\bar{\beta}_i}{\bar{\beta}_s} \sum_{j=1}^{J} \tilde{g}_j \bar{k}_j - \frac{\bar{\beta}_i \tilde{g}}{\bar{\beta}_s}. \tag{A9}$$

From (A7) and (A9), we have

$$N_i + N_s = \sum_{j=1}^{J} \varphi_j \bar{k}_j + \bar{\beta} \tilde{g}, \tag{A10}$$

where $\varphi_j = \bar{\beta} \tilde{g}_j + z \bar{\beta}_i \bar{N}_j$, $\bar{\beta} \equiv 1 - \bar{\beta} / \bar{\beta}_s$. From (2) and (A10), we have

$$z \bar{\beta}_i \bar{k}_1 + (R \bar{k}_1 + h_1) \bar{\beta} \beta_s \xi_1 = \varphi_h, \tag{A11}$$

where we use the definitions of φ_1 and $\tilde{g}$ and

$$\varphi_h(z, \{\bar{k}_j\}, \{\hbar_{cj}\}) \equiv \left(N - \sum_{j=2}^{J} \varphi_j \bar{k}_j - \bar{\beta} \beta_s \sum_{j=2}^{J} h_j \xi_j \bar{N}_j \right) \frac{1}{\bar{N}_1},$$

where $\{\bar{k}_j\} \equiv (\bar{k}_2, \ldots, \bar{k}_J)$ and $\{\hbar_{cj}\} \equiv (\hbar_{c2}, \ldots, \hbar_{cJ})$. From the definition of ξ_1 we have

$$\xi_1(z, \hbar_{c1}, \bar{k}_1) \equiv \frac{\xi_{01}}{\xi_h + \lambda_{k1} \bar{k}_1}, \tag{A12}$$

in $\xi_h \equiv \xi_{01} + \bar{\lambda}_1 + \lambda_{w1} w_1$. Substituting (A12) into (A11) yields

$$\bar{k}_1^2 + m_1 \bar{k}_1 - m_2 = 0,$$

$$m_1 \equiv \frac{\xi_h z \bar{\beta}_i - \varphi_h \lambda_{k1} + \bar{\beta} R \beta_s \xi_{01}}{\lambda_{k1} z \bar{\beta}_i}, \quad m_2 \equiv \frac{\varphi_h \xi_h - h_1 \bar{\beta} \beta_s \xi_{01}}{\lambda_{k1} z \bar{\beta}_i}.$$

Hence we have two solutions

$$\bar{k}_1 = \varphi_{1,2}(z, \{\bar{k}_j\}, (\hbar_{cj})) \equiv -\frac{m_1}{2} \pm \sqrt{\frac{m_1^2}{4} + m_2}. \tag{A13}$$

where $(\hbar_{cj}) \equiv (\hbar_{c1}, \ldots, \hbar_{cJ})$. From this procedure, (A21), (12) and (14), we have

$$\dot{\bar{k}}_1 = \bar{\Omega}_1(z, \{\bar{k}_j\}, (\hbar_{cj})) \equiv \lambda_1 \hat{y}_1 - \varphi, \tag{A14}$$

$$\dot{\bar{k}}_j = \Lambda_j\left(z, \{\bar{k}_j\}, (\hbar_{cj})\right) \equiv \lambda_j \hat{y}_j - \bar{k}_j, \quad j = 2, \ldots, J,$$
$$\dot{\hbar}_{cj} = \Omega_j\left(z, \{\bar{k}_j\}, (\hbar_{cj})\right) \equiv \tilde{\xi}_j\left(c_j - \hbar_{cj}\right). \tag{A15}$$

Taking derivatives of equation (A13) with respect to t and combining with (A15) implies

$$\dot{\bar{k}}_1 = \frac{\partial \varphi}{\partial z}\dot{z} + \sum_{j=2}^{J}\Lambda_j \frac{\partial \varphi}{\partial \bar{k}_j} + \sum_{j=1}^{J}\Omega_j \frac{\partial \varphi}{\partial \hbar_{cj}}. \tag{A16}$$

Equaling the right-hand sizes of equations (A18) and (A20), we get

$$\dot{z} = \Lambda_1\left(z, \{\bar{k}_j\}, (\hbar_{cj})\right) \equiv \left[\bar{\Omega}_1 - \sum_{j=2}^{J}\Lambda_j \frac{\partial \varphi}{\partial \bar{k}_j} - \sum_{j=1}^{J}\Omega_j \frac{\partial \varphi}{\partial \hbar_{cj}}\right]\left(\frac{\partial \varphi}{\partial z}\right)^{-1}. \tag{A17}$$

In summary, we proved the lemma.

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