



### AN INVENTORY MODEL WITH LOST SALE IS TIME DEPENDENT

Ying-Chieh Chen<sup>1</sup> — Po-Yu Chen<sup>2</sup>

<sup>1</sup>Department of Accounting, Ming Chuan University, Taipei, Taiwan

<sup>2</sup>Department of Advertising, Ming Chuan University, Taipei, Taiwan

#### ABSTRACT

*In this paper, we shall construct an inventory model with partially backlogged shortage and Poisson demand. In the shortage period, we assume that lost sale is dependent on the length of waiting time. Under these assumptions, we find the optimal planning and shortage periods such that the expected profit per unit time is maximized. Also, we can estimate the optimal expected backordered quantity and the expected order quantity.*

**Keywords:** Inventory, Partially backordered, Poisson demand, EOQ, Lost sales, Withdrawal rate.

#### Contribution/ Originality

In the past works, researchers usually developed the Markov model to represent the on-hand inventory level when demand is random. Here, we intend to propose the concept of expected order quantity and expected backordered quantity. And finally, we extend the traditional EOQ model and give a new interpretation of economic order quantity. On the other hand, we introduce the withdrawal rate to estimate the proportion of lost sales and determine the optimal backordered quantity of goods and the optimal length of shortage time.

#### 1. INTRODUCTION

In the past works, there are two types of assumptions are considered in the inventory problems: (1) shortage is not permitted and (2) shortage is fully/partially backlogged. The basic EOQ problem is the typical one of the first type. On the other hand, many researchers focus on the second type problem, e.g. Park (1983) presents an inventory model for situations in which, during the stockout period, a fraction  $b$  of the demand is backordered and the remaining fraction  $1 - b$  is lost. By defining a time-proportional backorder cost and a fixed penalty cost per unit lost, a unimodal objective function representing the average annual cost of operating the inventory system is obtained. Ouyang *et al.* (2007) also proposed a periodic review inventory model with partial lost-sales to effectively increase investment and to reduce the lost-sales rate. Rosenberg (1979) reformulate the cost equation for the lot-size model with partial backlogging and the formulation is in terms of fictitious demand rate. Chiang (2006) proposed a dynamic programming model for periodic-review systems in which a replenishment cycle consists of a

number of small periods (each of identical but arbitrary length) and holding and shortage costs are charged based on the ending inventory of small periods.

In the recent years, some researchers discussed the compound Poisson type demand. For example, [Bijvank and Johansen \(2012\)](#) develop new models allowing constant lead times of any length when demand is compound Poisson. [Chen and Chen \(2010\)](#) consider an inventory problem based on the assumption that lost sales depend on the waiting time when the whole period is stockout. [Axsäter \(2007\)](#) considers a single-echelon inventory system with a warehouse facing compound Poisson customer demand. [Thangam and Uthayakumar \(2008\)](#) consider a two-level supply chain with a number of identical, independent 'retailers' at the lower echelon and a single supplier at the upper echelon controlled by continuous review inventory policy (R,Q), Each retailer experiences Poisson demand with constant transportation times. [Bijvank and Vis \(2011\)](#) also classify the lost-sales inventory models in the literature based on the characteristics of the inventory systems and review the proposed replenishment policies.

The situation of shortage occurs frequently in the real world. When the customer faces the occurrence of stockout, he may leave and visit another store. It is important to the decision makers, because of lost sales will reduce the profit. Therefore, how to estimate the proportion of customers will wait for the goods is important. We think the length of waiting time is one of the major factors that the customer will decide to wait or leave. In other words, how to decide the suitable length of stockout period will maximize the profit is the major concern of the decision maker.

Therefore, in this article, we shall construct the inventory model based on two assumptions: (1) the customer demand follows a Poisson distribution and (2) the lost sales in the shortage period depend on the length of waiting time, i.e, the longer the waiting time the lesser the waiting willingness of the customer. Under these assumptions, we shall find the optimal (planned) stockout and non-stockout periods such that the profit per unit time is maximized.

## 2. MODEL FORMULATION

To formulate the mathematical model the following notation and assumptions are used throughout the article.

1.  $k$ : The setup cost.
2.  $z$ : The unit price.
3.  $c$ : The unit cost of goods.
4.  $h$ : The unit holding cost per unit time.
5.  $[0, t]$ : The planning period.
6.  $t$ : The length of planning period, where  $t$  is a decision variable.
7.  $[x]^+$ :  $[x]^+ = \text{Max}\{0, x\}$
8.  $[0, \epsilon]$ : The time interval we plan to be unshortage, where  $\epsilon$  is a decision variable.

Here, we assume the lead time will be constant and, without loss of generality, we let it be equal to zero.

In  $[0, \epsilon]$ , there are two random phenomena should be considered:

A.  $N_\epsilon$ : The total demand in the time interval  $[0, \epsilon]$ .

We assume that  $N_\epsilon$  has a Poisson distribution with parameter  $\lambda\epsilon$ . Although the total number of customers are usually less than or equal to the total demand, there is no price discount in this model. Thus, the objective functions will be the same, whether the total number of customers are less than or equal to the total demand. Therefore, for the sake of convenience, we assume that the total demand will be equal to the total number of customers.

B. X: The time that the customer comes to purchase the goods in the time interval  $[0, \epsilon]$ . We say that it is the arrival time of the customer. Here, we also assume that  $X_i$  is the purchasing time of the  $i^{th}$  customers, where  $i=1, 2, \dots$ . We also assume that  $X_1, X_2, \dots$ , are independent and identically distributed with p.d.f.  $f_X(x)=1/\epsilon$

9.  $[\epsilon, t]$ : The time interval we plan to be shortage, where  $\epsilon \in [0, t]$ .

In  $[\epsilon, t]$ , there are three random phenomena should be considered:

A.  $N_{t-\epsilon}$ : The total demand in the time interval  $[\epsilon, t]$ ;  $N_{t-\epsilon}$  has a Poisson distribution with parameter  $\lambda(t - \epsilon)$ .

B. Y: The arrival time of the customer who comes to purchase the goods in the time interval  $[\epsilon, t]$ .

Since the period where stock-outs occur, so when the customer comes at time  $y, y \in [\epsilon, t]$ , then he must wait until  $t$  to get the goods. The waiting willingness of the customer depends on the length of waiting time. The larger the waiting time is, the lesser the waiting willingness or possibility. When  $t$  is given and the customers arrive at time  $y$ , the proportion of them will wait until  $t$  can be obtained by the historical data. In other words, the empirical distribution of the customer's waiting willingness can be obtained by the past experiences. Suppose that it follows a probability distribution and its corresponding random variable is as follows:

C. W: The time that the customer is willing to wait.

Both Y and W are random variables with joint p.d.f.  $f_{Y,W}(y,w)$ , where  $\epsilon \leq y \leq t$  and  $w \geq 0$ . And, the marginal p.d.f. of Y and W are  $f_Y(y)$  and  $f_W(w)$ , respectively. Here we shall assume that Y and W are independent and Y is uniformly distributed on  $[\epsilon, t]$ , i.e.,  $f_Y(y) = 1/t - \epsilon, \epsilon \leq y \leq t$ , and  $\mu = E[W] = \int_0^\infty w f_W(w)dw$ .

- In the selling period  $[0, \epsilon]$ :

1. Expected total holding cost E.H.:

Since  $X_i$  is the purchasing time of the  $i^{th}$  customers, it means that the length of holding time for the  $i^{th}$  unit of goods is  $X_i$ . Hence, the total length of holding time for  $N_\epsilon$  units of goods is then

$$S=X_1+X_2+\dots+X_{N_\epsilon}.$$

The distribution of  $S$  is the so-called compound Poisson distribution with parameter  $\lambda\epsilon$ . Since the holding cost per unit time is  $h$ , therefore the total holding cost is  $h \cdot S$ , and the expected total holding cost is given by

$$E.H. = E[h \cdot S] = hE[E[S|N_\epsilon]] = h\lambda\epsilon \cdot E[X] = \frac{h\lambda\epsilon^2}{2}. \tag{1}$$

2. Expected total purchasing cost E.P.:

Since the total demand is  $N_\epsilon$ , so the total purchasing cost is  $c \cdot N_\epsilon$ , and the expected total purchasing cost is

$$E.P. = \sum_{n_\epsilon=0}^{\infty} c \cdot n_\epsilon P(N_\epsilon = n_\epsilon) = cE[N_\epsilon] = c\lambda\epsilon. \tag{2}$$

3. Expected total revenue E.R.:

Since the total demand is  $N_\epsilon$ , so the total revenue is  $v \cdot N_\epsilon$ , and the expected total revenue is

$$E.R. = \sum_{n_\epsilon=0}^{\infty} v \cdot n_\epsilon P(N_\epsilon = n_\epsilon) = vE[N_\epsilon] = v\lambda\epsilon. \tag{3}$$

• In the stockout period  $[\epsilon, t]$ :

If the customer arrives at time  $Y = y$ , then the length of waiting time is  $t - y$ . The probability that the customer will decide to wait is then given by

$$P(W > t - y | y) = \int_{t-y}^{\infty} f_{W|Y}(w|y)dw,$$

which is the conditional probability that the customer is willing to wait to purchase the goods, given that he arrives at time  $y$ .

Thus, the unconditional probability that the customer arrives in the time interval  $[\epsilon, t]$  and will wait to purchase the goods is

$$\theta = \int_{\epsilon}^t \int_{t-y}^{\infty} f_{W|Y}(w|y) f_Y(y) dw dy$$

Since  $Y$  and  $W$  are independent, and  $f_Y(y) = \frac{1}{t-\epsilon}$ , so

$$\theta = \int_{\epsilon}^t \int_{t-y}^{\infty} f_W(w) \frac{1}{t-\epsilon} dw dy$$

Since the total number of customers arrive in  $[\epsilon, t]$  is  $N_{t-\epsilon}$ , so the proportion of customers will wait until  $t$  is then  $\theta N_{t-\epsilon}$ . Furthermore, we know that  $N_{t-\epsilon}$  is Poisson distribution with parameter  $\lambda(t - \epsilon)$  and the unit profit is  $(v - c)$ , so the expected total profit is  $(v - c) \theta \lambda(t - \epsilon)$ , i.e.,

$$(v - c) \lambda(t - \epsilon) \int_{\epsilon}^t \int_{t-y}^{\infty} f_W(w) \frac{1}{t-\epsilon} dw dy \tag{4}$$

From (1), (2), (3), (4) and setup cost  $k$ , the expected total profit is

$$(v - c)\lambda\epsilon - \frac{h\lambda\epsilon^2}{2} + (v - c)\lambda(t - \epsilon) \int_{\epsilon}^t \int_{t-y}^{\infty} f_W(w) \frac{1}{t-\epsilon} dw dy - k$$

Therefore, the expected profit per unit time is then

$$\frac{(v-c)\lambda\epsilon}{t} - \frac{h\lambda\epsilon^2}{2t} + \frac{(v-c)\lambda}{t} \int_{\epsilon}^t \int_{t-y}^{\infty} f_W(w) dw dy - \frac{k}{t} \tag{5}$$

### 3. OPTIMAL SOLUTION

Next, we shall find  $\epsilon$  and  $t$ ,  $0 \leq \epsilon \leq t$ , such that expected profit per unit time is maximized per unit time as follows:

$$\text{Max}_{0 \leq \epsilon \leq t} A(\epsilon, t) \tag{6}$$

Where

$$A(\epsilon, t) = \frac{(v-c)\lambda\epsilon}{t} - \frac{h\lambda\epsilon^2}{2t} + \frac{(v-c)\lambda}{t} \int_{\epsilon}^t \left[ \int_{t-y}^{\infty} f_W(w) dw \right] dy - \frac{k}{t}$$

Assume that the optimal solution of (6) exists and let  $(\epsilon^*, t^*)$  be its optimal solution.

To find the maximum of function A, we first note that

$$\int_{\epsilon}^t \left[ \int_{t-y}^{\infty} f_W(w) dw \right] dy = y \left[ \int_{t-y}^{\infty} f_W(w) dw \right] \Big|_{\epsilon}^t - \int_{\epsilon}^t y f_W(t-y) dy \tag{7}$$

Let  $z = t-y$  in the second term of the right-hand side, then

$$\int_{\epsilon}^t \left[ \int_{t-y}^{\infty} f_W(w) dw \right] dy = (t - \epsilon) \left[ \int_{t-\epsilon}^{\infty} f_W(w) dw \right] + \int_0^{t-\epsilon} z f_W(z) dz \tag{8}$$

And using the Leibnitz's rule

$$\begin{aligned} \frac{D}{dt} \int_{\epsilon}^t \left[ \int_{t-y}^{\infty} f_W(w) dw \right] dy &= 1 - \int_{\epsilon}^t f_W(t-y) dy \\ &= 1 - \int_0^{t-\epsilon} f_W(z) dz = \int_{t-\epsilon}^{\infty} f_W(w) dw \end{aligned} \tag{9}$$

Then, by (8) and (9), the partial derivatives of  $A(\epsilon, t)$  are

$$\frac{\partial A(\epsilon, t)}{\partial \epsilon} = \frac{\lambda}{t} \left[ (v - c) \int_0^{t-\epsilon} f_W(w) dw - h\epsilon \right]; \quad \frac{\partial^2 A}{\partial \epsilon^2} = \frac{-\lambda}{t} [h + (v - c) f_W(t - \epsilon)] < 0 \tag{10}$$

$$\begin{aligned} \frac{\partial A(\epsilon, t)}{\partial t} &= \frac{h\lambda\epsilon^2}{2t^2} - \frac{(v-c)\lambda\epsilon}{t^2} \int_0^{t-\epsilon} f_W(w) dw - \frac{(v-c)\lambda}{t^2} \int_0^{t-\epsilon} z f_W(z) dz + \frac{k}{t^2}, \text{ by (10)} \\ &= -\frac{\lambda\epsilon}{t^2} \left[ \frac{t}{\lambda} \frac{\partial A(\epsilon, t)}{\partial \epsilon} + \frac{h\epsilon}{2} \right] - \frac{(v-c)\lambda}{t^2} \int_0^{t-\epsilon} z f_W(z) dz + \frac{k}{t^2} \\ &= -\frac{\epsilon}{t} \frac{\partial A(\epsilon, t)}{\partial \epsilon} + \frac{1}{t^2} g(\epsilon, t) \end{aligned} \tag{11}$$

where  $g$  is defined by

$$g(\epsilon, t) = \frac{-\lambda h \epsilon^2}{2} - (v - c) \lambda \int_0^{t-\epsilon} z f_W(z) dz + k$$

If  $\epsilon \geq \frac{v-c}{h}$ , then by (10) we have  $\frac{\partial A(\epsilon, t)}{\partial \epsilon} < 0, \forall t \geq \epsilon$ , and hence  $(\epsilon, t), t \geq \epsilon$ , is not an optimal solution of (6). Therefore, we can rewrite (6) as follows:

$$\text{Max}_{0 \leq \epsilon \leq t} A(\epsilon, t) = \text{Max}_{0 \leq \epsilon \leq t \ \& \ \epsilon < \frac{v-c}{h}} A(\epsilon, t) \tag{12}$$

From (12), we may assume  $\epsilon < \frac{v-c}{h}$  in the following statement. From (10), we have  $\frac{\partial A(\epsilon, t)}{\partial \epsilon} \Big|_{\epsilon=0^+} >$

0, and  $\frac{\partial A(\epsilon, t)}{\partial \epsilon} \Big|_{\epsilon=t^-} < 0$ , and hence for each  $t$  there exists unique value, denoted by  $\bar{\epsilon}(t)$ , satisfying

$$\frac{\partial A(\bar{\epsilon}(t), t)}{\partial \epsilon} = 0 \text{ and } \text{Max}_{0 \leq \epsilon \leq t} A(\epsilon, t) = A(\bar{\epsilon}(t), t) \tag{13}$$

From (10) and (13), we may get  $\bar{\epsilon}'(t)$  as follows:

$$(v - c) f_W(t - \bar{\epsilon}(t)) (1 - \bar{\epsilon}'(t)) - h(\bar{\epsilon}'(t)) = 0,$$

i.e.,

$$\bar{\epsilon}'(t) = \frac{(v-c) f_W(t - \bar{\epsilon}(t))}{h + (v-c) f_W(t - \bar{\epsilon}(t))} \in (0, 1), \forall t \tag{14}$$

Let  $\bar{t} = \bar{t}(\epsilon)$  be the inverse function of  $\bar{\epsilon} = \bar{\epsilon}(t)$ , then by (14), we have

$$\begin{aligned} \bar{t}'(\epsilon) &= \frac{1}{\bar{\epsilon}'(t)} = \frac{h+(v-c)f_W(t-\bar{\epsilon}(t))}{(v-c)f_W(t-\bar{\epsilon}(t))} \\ &= 1 + \frac{h}{(v-c)f_W(t-\bar{\epsilon}(t))} > 0 \end{aligned} \tag{15}$$

This means that the vertical distance  $[\bar{t}(\epsilon)-\epsilon]$  from the point  $(\epsilon, \bar{t}(\epsilon))$  to line  $t \equiv \epsilon$ , is strict increasing on  $\epsilon$ . (16)

Since by (14),  $\frac{d}{dt}(t - \bar{\epsilon}(t)) > 0$  and, by assumption,  $f_W(w)$  is a decreasing function of  $w$ , we have

$$\bar{t}''(\epsilon) > 0. \tag{17}$$

Therefore, the graph of the function  $t = \bar{t}(\epsilon)$  can be shown as figure 1.

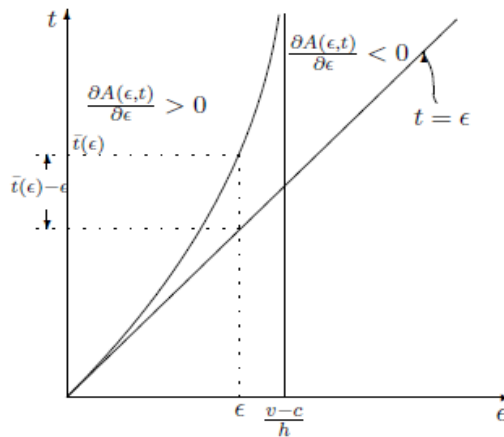


Figure 1. the graph of the function  $t = \bar{t}(\epsilon)$

By (12) and (13), we have:

$$\text{Max}_{0 \leq \epsilon \leq t} A(\epsilon, t) = \text{Max}_{t \geq 0} A(\bar{\epsilon}(t), t) = \text{Max}_{0 \leq \epsilon \leq \frac{v-c}{h}} A(\epsilon, \bar{t}(\epsilon)) \tag{18}$$

Together with (10), (11) and (13), it yields

$$\begin{aligned} \frac{d}{d\epsilon} A(\epsilon, \bar{t}(\epsilon)) &= \frac{\partial A(\epsilon, \bar{t}(\epsilon))}{\partial \epsilon} + \frac{\partial A(\epsilon, \bar{t}(\epsilon))}{\partial t} \cdot \bar{t}'(\epsilon) \\ &= \frac{1}{\bar{t}^2(\epsilon)} \left[ \frac{-\lambda h \epsilon^2}{2} - (v-c) \lambda \int_0^{\bar{t}(\epsilon)-\epsilon} z f_W(z) dz + k \right] \bar{t}'(\epsilon) = \frac{1}{\bar{t}^2(\epsilon)} g(\epsilon, \bar{t}(\epsilon)) \bar{t}'(\epsilon) \end{aligned} \tag{19}$$

And hence,

$$0 = \frac{d}{d\epsilon} A(\epsilon^*, \bar{t}(\epsilon^*)) = \frac{1}{\bar{t}^2(\epsilon^*)} g(\epsilon^*, \bar{t}(\epsilon^*)) \bar{t}'(\epsilon^*) \tag{20}$$

Given  $\epsilon$ , it is valid that the function  $g$  appearing in (19):

$$g(\epsilon, t) = \frac{-\lambda h \epsilon^2}{2} - (v-c) \lambda \int_0^{t-\epsilon} z f_W(z) dz + k \tag{21}$$

is strict decreasing of  $t$ , with the greatest lower bound:

$$\lim_{t \rightarrow \infty} \left[ \frac{-\lambda h \epsilon^2}{2} - (v - c) \lambda \int_0^{t-\epsilon} z f_W(z) dz + k \right] = \frac{-\lambda h \epsilon^2}{2} - (v - c) \lambda \mu + k \quad (22)$$

and the least upper bound:

$$\lim_{t \rightarrow \epsilon^+} \left[ \frac{-\lambda h \epsilon^2}{2} - (v - c) \lambda \int_0^{t-\epsilon} z f_W(z) dz + k \right] = \frac{-\lambda h \epsilon^2}{2} + k \quad (23)$$

From (21) and (22), we have the following properties:

Given  $\epsilon$ , the inequality  $\sqrt{\frac{2}{h\lambda} [k - (v - c) \lambda \mu]^+} \leq \epsilon \leq \sqrt{\frac{2k}{h\lambda}}$ , holds if and only if there exists unique value, denoted by  $\hat{t}(\epsilon)$ , satisfying  $g(\epsilon, \hat{t}(\epsilon))=0$ , i.e.,

$$0 = g(\epsilon, \hat{t}(\epsilon)) = \frac{-\lambda h \epsilon^2}{2} - (v - c) \lambda \int_0^{\hat{t}(\epsilon)-\epsilon} z f_W(z) dz + k, \quad 0 \leq \epsilon \leq \hat{t}(\epsilon) \quad (24)$$

Note that (12), (20), (22) and (23) yield that

$$\epsilon^* \in \left[ \sqrt{\frac{2}{h\lambda} [k - (v - c) \lambda \mu]^+}, \frac{v-c}{h} \right] \quad (25)$$

Differentiating (24) with respect to  $\epsilon$ , it yields

$$-\lambda h \epsilon - (v - c) \lambda [\hat{t}(\epsilon) - \epsilon] f_W(\hat{t}(\epsilon) - \epsilon) [\hat{t}'(\epsilon) - 1] = 0,$$

and hence

$$\hat{t}'(\epsilon) - 1 = \frac{-h\epsilon}{(v-c)[\hat{t}(\epsilon) - \epsilon] f_W(\hat{t}(\epsilon) - \epsilon)} < 0. \quad (26)$$

This means that the vertical distance  $[\hat{t}(\epsilon) - \epsilon]$  from the point  $(\epsilon, \hat{t}(\epsilon))$  to the line  $t \equiv \epsilon$ , is strict decreasing on  $\epsilon$ . (27)

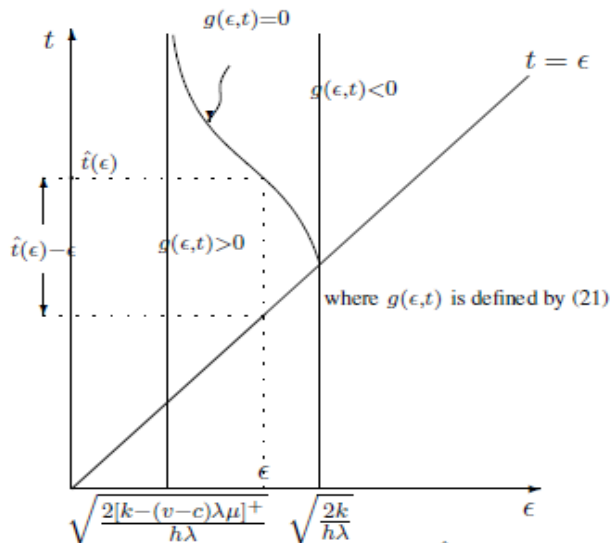


Figure 2. The graph of  $t = \hat{t}(\epsilon)$

From (11) and (27), it is valid that the curve  $t = \hat{t}(\epsilon)$  and the curve  $t = \bar{t}(\epsilon)$  intersects at one and only one point, denoted by  $B$ , is shown in Figure 3.

Together with Figure 1 and Figure 2, it yields that:

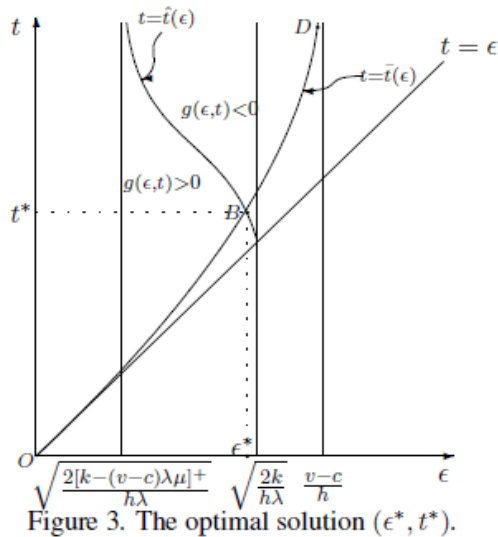


Figure 3. The optimal solution  $(\epsilon^*, t^*)$ .

Using (18) and the following properties:

1. if  $(\epsilon, \bar{t}(\epsilon)) \in \widehat{BD}$  (c.f. Figure 3), then by (15) and (19)

$$\frac{dA(\epsilon, \bar{t}(\epsilon))}{d\epsilon} = \frac{\bar{t}'(\epsilon)}{[\bar{t}(\epsilon)]^2} g(\epsilon, \bar{t}(\epsilon)) < 0,$$

2. if  $(\epsilon, \bar{t}(\epsilon)) \in \widehat{OB}$  (c.f. Figure 3), then by (15) and (19)

$$\frac{dA(\epsilon, \bar{t}(\epsilon))}{d\epsilon} = \frac{\bar{t}'(\epsilon)}{[\bar{t}(\epsilon)]^2} g(\epsilon, \bar{t}(\epsilon)) > 0,$$

it can be shown that the coordinate value of  $B$  as shown in Figure 3 is indeed the optimal solution of (6).

### 3. CONCLUSION

As mentioned above,  $(\epsilon^*, t^*)$  is the optimal solution of (6). This means that the total demand is  $N_{\epsilon^*}$  and total backordered quantity of goods is  $N_{t^*-\epsilon^*}$ . Since  $N_{\epsilon^*}$  and  $N_{t^*-\epsilon^*}$  are Poisson distributions with parameters  $\lambda$  and  $\lambda(t^*-\epsilon^*)$ , respectively. We also see that the probability that the customer is willing to wait is  $\theta$ . It is also a good estimate of the fraction of the demand is backordered.

Thus, the proportion of the customers will wait until  $t$  is then  $\theta N_{t^*-\epsilon^*}$ . Then we can find the expected order quantity of goods and the expected backordered quantity as follows:

$$q_o^* = E[N_{\epsilon^*}] = \lambda \epsilon^* \tag{28}$$

and

$$q_b^* = E[\theta N_{t^*-\epsilon^*}] = \theta \lambda (t^* - \epsilon^*) = \lambda \int_{\epsilon^*}^{t^*} S_W(t^* - y) dy. \tag{29}$$

This means that  $q_o^*$  and  $q_b^*$  derived from (28) and (29) will maximize the expected profit function in (6).

We can interpret the meaning of  $q_o^*$  and  $q_b^*$  as follows:

If we order  $q_o^*$  at time 0 and reorder at  $t^*$ , then  $q_o^*$  may use up less than  $\epsilon^*$  because of the demand is random. And, in this case, the backordered quantity of goods will increase. Conversely,



if  $q_0^*$  use up greater than  $\epsilon^*$ , then the backordered quantity of goods will decrease. But, in the long run, the expected backordered quantity of goods will be equal to  $q_b^*$  as in (29).

Furthermore, to estimate  $\theta$ , we have to know the distribution of  $W$ . For this purpose, we first define the withdrawal rate  $r_w$  as follows:

$$r_w = \frac{f_W(w)}{S_W(w)}$$

where  $S_W(w) = P(W \geq w)$ .

In the lifetime study, this is the so-called force of mortality or failure rate (Hogg and Tanis, 1993), i.e., the instantaneous rate of death when a person is at age  $w$ . Here, we call it the force of withdrawal or withdrawal rate. Usually, the withdrawal rate is an increasing function of  $w$ . In other words, instantaneous rate of withdrawal is increasing when waiting time increases. There are some distributions that satisfy the condition, such as the Gopertz distribution. However, in the short term study, it is also reasonable to assume that the withdrawal rate is constant. In this situation, the distribution of  $W$  that we can choose is then the exponential function, i.e.,  $f_W(w) = \frac{1}{\beta} e^{-\frac{w}{\beta}}$ ,  $w \geq 0$ . Here, the only parameter of the exponential distribution,  $\beta$ , can be interpreted as the mean time that the customer is willing to wait.

Finally, we see that the special case of this study is  $\epsilon = t$ . And, in this case, the expected profit function is then reduced to

$$A(\epsilon) = (v - c) \lambda - \frac{h\lambda\epsilon}{2} - \frac{k}{\epsilon}$$

It is easy to see that

$$A'(\epsilon) = -\frac{h\lambda}{2} + \frac{k}{\epsilon^2} = 0$$

and

$$\epsilon^* = \sqrt{\frac{2k}{h\lambda}}$$

and  $A''(\epsilon) = -\frac{2k}{\epsilon^3} < 0$ . Hence,  $A(\epsilon)$  has a maximum at  $\epsilon^*$ .

Furthermore, the total quantities of goods demand in the time interval  $[0, \epsilon^*]$  is  $N_{\epsilon^*}$ , as mentioned above,  $N_{\epsilon^*}$  has a Poisson distribution with parameter  $\lambda\epsilon^*$ , i.e.,

$$E[N_{\epsilon^*}] = \lambda\epsilon^* = \sqrt{\frac{2k\lambda}{h}}$$

which is the case of (Chen, 2003).

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