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ON THE PREDICTION OF THE INFLATION CRISES OF SOUTH AFRICA USING MARKOV-SWITCHING BAYESIAN VECTOR AUTOREGRESSIVE AND LOGISTIC REGRESSION MODELS

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ABSTRACT

The aim of this study is to build an early warning system (EWS) model for inflation rates of South Africa (SA). A logistic regression model (LRM) is used in collaboration with a Markov-switching Bayesian vector autoregressive (MS-BVAR) to produce the estimates. Monte Carlo experimental methods are used to simulate both the inflation rate and repo rate of the SA economy. The procedure simulated 228 observations for the period of January 1999 to December 2017. Preliminary results confirmed the applicability of both models for further analyses. MS (2)-BVAR(1) proved to be the most appropriate model for detecting regime shifts in inflation rates. The results indicate that SA inflation might be in a low inflation regime for the period of 11 years and 4 months. Surprisingly, we discovered that the repo rate is not a good tool to combat the inflation rate in SA i.e a 1% increase in the repo in a month significantly increased inflation rate by about 81%. The findings also confirmed 51% and 53% of in-sample and out-of-sample SA inflation crises forecasts to be correctly classified. This is in accordance with the reported results of Cruz and Mapa (2013) and Makatjane and Xaba (2016). The verdicts of the study are relevant for policy purposes and literature.

Contribution/Originality: This study uses new estimation methodology to quantify the likelihood of future inflation crises in South Africa.

1. INTRODUCTION

The global monetary authorities have targets for smoothing inflation rates including high prices. South Africa (SA) is not an exception in striving for this accomplishment. Volatile inflation rates twist the judgment that consumers make about their living standards, and this is evident from past studies such as Mboweni *et al.* (2008) and Bonga-Bonga and Kabundi (2015) which reported about the importance of price stability. The slow economic growth rates are subdued to markets failure which results from these distortions. Money circulation in the economy for buying goods and services slowly but surely drops off because of high inflation which in the long run harms households specifically those with low incomes. Nevertheless, keeping a low inflation rate and stabilising it supports sustainable economic growth.

After the year 1994, the SA government inherited a persistently high inflation rate which was recorded as above 10% per annum. This high inflation rate caused a regressive effect on lower-income families and older people in society because prices for food and domestic utilities such as water and heating rose at a rapid rate. After the adaption of the inflation targeting by the South African reserve bank (SARB), the SA inflation rate went outside the target interval of 3%-6% in early 2007 where it then increased from 3% to 8.6%. The recurrent upturn in the repo rate in order to curtail the inflation rate has speeded up its trend rather than subduing it. This was a period in which SA had encountered high production levels, a prolonged fall in prices and degrading of business and consumer firm trust indicators. The overarching aim of this study is to establish an early warning system model for inflation in South Africa.

Predictive models that quantify the likelihood of the future occurrence of inflation crises are used to supplement the current toolkit of the SARB in the assessment of inflation environment and the risk of inflation outlook. The current study extends the work done by Cruz and Mapa (2013) and Makatjane and Xaba (2016). We first estimate a Bayesian vector autoregressive (BVAR) model subject to two regime shifts. The choice of the BVAR enhanced Markov switching is based on its ability for establishing the dynamic processes of observed time series Y_t and a discrete state variable S_t for $t = 1,2, \dots, T$. This is being achieved by computing the following joint probability $\Pr(Y_t, S_t)$ and the unravelling information of the conditional probabilities $\Pr(Y_t|S_t)$ and $\Pr(S_t|Y_t)$ (Brandt and Davis, 2014). Furthermore, the estimation of parameters is based on a more complex algorithm known as expectation maximization (EM) as compared to a well-known maximum likelihood algorithm.

The EM algorithm iteratively approximates the maximization of the likelihood function, allowing mixed distributions within the data set. That is, the value of each observation is known, but its distribution is not. Troug and Murray (2015) noted that since the Markov chain (MC) is hidden, therefore, the likelihood function has a recursive nature with an optimal inference in the current period depending on an optimal inference made in the previous period. Under such conditions, the likelihood cannot be maximized using standard techniques. This algorithm comprises two steps that are used to estimate the model parameters through the likelihood function in the presence of latent variables. The first step estimates the latent variables, given the observed data and suggested parameters $\Pr(y_t|Y_{t-1}, \Theta, S_t)$, while the second estimate the parameters, given the latent variables and the observed data $\Pr(y_t|Y_{t-1}, \Theta, k)$. k, in this case, is the number of regimes.

In addition, Droumaguet (2012) also emphasised that Bayesian methods provide densities of the model parameters which solves the problem of the confidence interval, and finally Bayesian shrinkage techniques allow the models to be estimated with higher dimensions and these models would have complex shapes of the likelihood function and hence be more difficult to estimate with classical algorithms. Note that the Markov-Switching Bayesian vector autoregressive (MS-BVAR) model serves as a predecessor for logistic regression (LR) model. The regime classifications which are denoted by the following interval $[0,1]$ for low and high regime respectively are being used to develop a dummy variable for the LR model as the model serves as a warning sign model. This study is the first empirical analysis that employs both the MS-BVAR and LR models to quantify the likelihood of future inflation crises in SA. The rest of the paper is as follow: section 2 presents literature on both MS-BVAR and the logistic regression. This gives a more detailed report review on these models. Section 3 presents data and materials used. Section 4 provides the empirical analysis of the study. And finally, section 5 presents the conclusion of the study.

2. LITERATURE REVIEW

Many time series occasionally exhibit dramatic breaks in their behaviour. Hamilton (2010) has indicated that Markov-switching models (MSM) are quite amenable to the theoretical computation of how these abrupt changes in fundamentals show up, especially in financial time series. To describe the consequence of a dramatic change in the behaviour of a single series, say Y_t, Ang and Bekaert (2002) and Dai *et al.* (2007) have discovered that the behaviour of the past can be described by the first order autoregressive denoted by $AR(1)$. Due to the failure of $AR(1)$ to account for the change in parameters, the current study establishes a larger model encompassing the change in parameters. According to Timmermann (2000) the probability law governing the parameters fully follows the Gaussian innovation σ^2, the coefficient of an autoregressive Φ, the two intercepts C_1 and C_2 and finally the two-state transition probabilities p_{11} and p_{22} that follow homogenous Markov Chain (MC). For further readings on MSM, consult Yang (2000); Moolman (2005); Xaba *et al.* (2016) and Xaba *et al.* (2017).

Abiad (2007) used Markov-Switching vector autoregressive (MS-VAR) model to identify and characterise the crisis period endogenously. In terms of the currency of Asian countries, crises and the prospective contributing factor of exiting tranquil state were tested and a number of variables with significant medians across the panel were found. By using the maximum likelihood methodology, Arias and Erlandsson (2004) in their study of regime switching as an alternative of early warning system of currency crises, found that the method allowed them to extricate smoother transition probabilities than in the standard case, reflecting the need of policymakers to have progress ahead of time in the medium to long term as opposed to the short term. (See also Mariano *et al.* (2002); Abiad (2003) and Brunetti *et al.* (2008)). With the current study, we extend the work of Abiad (2007) by incorporating the Bayesian priors. This also helps in solving the model selection problem which is still an open question in the classical approach and it can be tackled by the comparison of the marginal densities of data computed from the posterior of different model specifications. We then further test for the existence of the long-run nonlinear relationship between the simulated inflation rates and repo rates of SA. (Also see Mariano *et al.* (2002); Abiad (2003) and Brunetti *et al.* (2008)) for Bayesian priors in the model.

To assess the relationship between stock returns and macroeconomic variables in the South African stock market, Moolman (2005) applied an MSM. The results of the author indicated that the degree to which stock returns rely on macroeconomic variables only depends on the business cycle state in SA. Apart from being used to capture cyclical asymmetry in the stock market, the MSM can also be used to identify turning points in the economy and to model economic growth (Xaba *et al.* (2016).

Sims *et al.* (2008) presented the Bayesian methodology for handling general Markov Switching Structural Vector Autoregressive (MS-SVAR) models. For instance, these models were found to be useful in business cycles. Thus they are potentially suited in many cases where SVAR models are traditionally used. Kilian (2006) on the other hand introduced Bayesian impulse response analysis for MS-VAR model. The obtained regimes separated the sample into two periods over the periods of 1986. The structural changes that occurred in time transformed the oil market into the more competitive market as highlighted within the regime dynamics.

Krolzig (2000) on the other hand focused on the predictability of MS-VAR processes as the property of a stochastic process in relation to an information set. He derived an optimal predictor, and showed that its properties depend on (i) the significance of regime movements, (ii) the timelessness of the regime-generating process, (iii) the asymmetry of the regime generating process and (iv) the interaction with the AR elements. The outcomes obtained a permit to infer parametric conditions under which the optimal predictor shrinks to a linear forecast prediction rule.

In developing an early warning system (EWS) for inflation crises, there are three methodologies that are emphasized. These are the bottom-up methodology, the aggregate methodology and the macroeconomic methodology. In first, the odds of inflation crisis are addressed and the systemic volatility is being activated and signed if the odds become significant. For the second method, the model is applied to data other than individual data. In the third method, the focal point is centred on building a relationship between economic variables with the view that various macroeconomic variables are required to affect the financial system and reflect their own condition.

Davis and Karim (2008) used a multivariate logit model in their comparative study of an early warning system with the aim of relating the likelihood of occurrence or non-occurrence of a crisis to a vector of n explanatory variables. The probability that the dummy variable takes a value of one (crisis occurs) at a point in time was given by the value of the logistic cumulative distribution evaluated for the data and parameters at that point in time. Their results showed that the logit model they estimated may be the best model for globally detecting the banking crisis.

Moreover, Gunsel (2005) designated that the financial ratios and failure of banks in North Cyprus have some sort of relationship which has been linked to a multivariate logit model. The main point of using a multivariate logit model is to compute the probability of bank failure as a vector of explanatory variables. Basically, financial ratios are designed to measure information for the six categories in which the natural potential risk within the financial institutions is emphasized. In spite of the potentiality to predict the sign of the crisis, binary time series models have been considered for this purpose. Further, than that, financial indicators are mostly used as bank indicators (Gunsel, 2005).

Due to the small samples and the need to keep the degrees of freedom, Kolari *et al.* (2002) added to the work of EWS by estimating the stepwise logistic regression in order to identify the subset of the covariates that are needed in the model through their power to discriminate. The predefined significance level was set at 10% and the impact of this was that few variables were chosen in the model, hence the need to increase the significance level to 30 which now was used as a threshold to add variables in the model. The main problem that caused the lack of significance of the variables in entering the model is due to the fact that the error term in the regression model followed a cumulative distribution which does not accurately estimate a logit function.

3. DATA AND MATERIALS USED

This section presents the discussion of the data and methods used in the study. Both the preliminary and primary data analyses methods are discussed.

3.1. Data Description

The study uses the Monte-Carlo simulated inflation and repo rates from the period of January 1999 to December 2017. This data generation process follows a nonlinear VAR mechanism of order 1 to 2 as recommended by Smith *et al.* (2013). The process generates 50+T data points for both inflation rate and repo rate with the starting values for both series set to zero and S_0=1. In order to attenuate the effect of the initial values, the last $T = 1$ is in the Monte Carlo replication. The sample size is chosen to keep power close to 0.80 (Dell *et al.*, 2002). According to these authors, 0.80 is a generally conventional value for sufficient power. Muthén and Muthén (2002) also suggested 0.8 as the best threshold for sample size and model power determination.

3.2. Maximum Lag Length Selection

Lag length is the most substantial method in bivariate and multivariate time series analysis. In this study, while estimating the VAR and KSS_NADF models with p autoregressive parameters, the optimal lag length must be selected. Scott and Hatemi (2008) emphasised that there are three most used information criteria namely the Akaike information criterion (AIC), Schwarz Bayesian information criterion (SBC) and Hanna Quinn (HQ) criterion. These are used in this study to decide on the maximum lag length to include in the analysis. Furthermore, the model with the smallest AIC, SBC and HQ is the one selected among the competing models. The SBC measure is estimated as follows:

$$\mathrm{SBC} = -2[\ln\hat{\varphi} + k * \ln(n)], \tag{1}$$

Where n is the sample size, k is the number of estimated parameters, $\hat{\varphi}$ is the likelihood function of the estimated model. That is $\hat{\varphi} = p(x|\hat{\theta}, M)$, x is the observed data and θ is, the estimated parameters of the model. Furthermore, AIC is computed by:

$$\mathrm{AIC} = n\log\left(\frac{RSS}{n}\right) + 2k \tag{2}$$

n is the sample size as in (6), RSS is the residual sum of squared from the fitted model and 2k is the variance. Nevertheless, the Hannan Quinn (HQ) is not asymptotically efficient. According to Hjort (2008) the value of HQ is small even for a very large n. The procedure is computed by:

$$Q = -2\log\tilde{L}_n(\theta_n) + 2ck_0\log\log n, c > 1 \tag{3}$$

Where, k_0 is the number of estimated parameters of the model, n is the sample size and $\tilde{L}_n(\theta_n)$ is the likelihood function.

3.3. Nonlinear Unit Root Test

In order to address the unit root in nonlinear time series, we employ Kapetanios, Shin- Shell Nonlinear Augmented Dickey-Fuller (KSS-NADF). Kapetanios *et al.* (2003) unveiled that the KSS is a modified Augmented Dickey-Fuller (ADF) test which is grounded in the following nonlinear model:

$$Y_t = \beta Y_{t-1} + \gamma Y_{t-1}\left[1 - e^{-\vartheta Y_{t-k}^{s}}\right] + \varepsilon_t, \tag{4}$$

Where its parametrization yields:

$$\Delta Y_t = \varphi Y_{t-1} + \gamma Y_{t-1}\left[1 - e^{-\vartheta Y_{t-k}^{s}}\right] + \varepsilon_t \tag{5}$$

$\varphi = \beta - 1,\ \gamma,\ \vartheta$ are the parameters to be estimated and ε_t is the error term. To have a reduced constant model, we set $\varphi = 0$ and the delay parameter $k = 1$ giving rise to a modified (4) as:

$$\Delta Y_t = \gamma Y_{t-1}\left[1 - e^{-\delta Y_{t-k}^{3}}\right] + \varepsilon_t \tag{6}$$

In that case, we test the following linear stationary hypothesis:

$H_0: \rho = 0$
$H_1: \rho > 0$

By the application of the first order Taylor series expansion, (5) is estimated as a nonlinear specification for testing the nonlinear stationary series which is augmented by the first order lag differencing to account for a deemed serial correlation in the error term as:

$$\Delta y_t = \delta y_{t-1}^3 + \sum_{k=1}^{q} \rho \Delta y_{t-k} + \varepsilon_t, \tag{7}$$

Where the coefficient for testing the presence of the unit root is δ, and q is the number of augmentations that can be specified using lag length selection criteria discussed in section 3.2. Hence, KSS-NADF unit root test statistic is presented as:

$$\tau_{NL} = \hat{\delta} \big/ se(\hat{\delta}). \tag{8}$$

Here, the hypothesis to be tested is

$H_0: \delta = 0$
$H_1: \delta < 0.$

The decision is ruled to reject the null hypothesis if the calculated probability value or if the calculated value of τ_{NL} exceeds the observed probability value or observed critical value.

3.4. Structural Break Test

To test the possibility of structural change within the simulated series, the CUSUM test is established. Firstly we estimate the Bayesian autoregressive model of order P denoted by $BAR(P)$and further, assume that the estimated parameters of a $BAR(P)$ are stable over time i.e, coefficients Φ_{ij} and σ_ε^2 are constant and these coefficients are obtained from the matrix developed by Brown *et al.* (1975):

$$\hat{\Phi} = (Y_{t-1}^T Y_{t-1})^T Y_{t-1}^T Y_t + \varepsilon_t \tag{9}$$

Where Y_t is the response variable and $Y_{t-1} = \left(1\ Y_{t-1}\ Y_{t-2} \dots Y_{t-p}\right)^T$ with $\varepsilon_t \sim i.i.d(\mu = 0, \sigma_\varepsilon^2 = 1)$. The hypothesis is $H_0: \sigma_{ij} = 0$, $H_1: \sigma_{ij} \neq 0$ and the recursive residuals are estimated by:

$$w_t = \frac{Y_t - \hat{\Phi}_{t-1} Y_t}{\sqrt{\hat{\sigma}^2 \left[1 - Y_t^T \left(Y_t^T Y_t\right)^{-1} Y_t\right]}} \tag{10}$$

As a result, the CUSUM test statistic is given by:

$$CUSUMSQ_t = \frac{\sum_{j=k+1}^{t} \hat{w}_j^2}{\sum_{j=k+1}^{n} \hat{w}_j^2} \tag{11}$$

The decision rule is, reject the null hypothesis if the observed probability is less than the expected probability. Nevertheless, (11) can be plotted against time with the confidence bands which Montgomery (2007) defined by:

$$\begin{aligned} C_i^+ &= \max[0, x_i - (\mu + k) + C_{i-1}^+] \\ C_i^- &= \max[0, (\mu - k) - x_i + C_{i-1}^-] \end{aligned} \tag{12}$$

The starting values are $C_0^+ = C_0^- = 0$. And k is the reference value and it is usually chosen halfway between the target μ_0 and out of the control value of the mean μ_1.

3.5. Markov Switching Bayesian Vector Autoregressive

In order to execute the first stage of the analysis, we first construct a Markov Switching model (MSM) by enhancing it with the Bayesian Vector Autoregressive (BVAR). Troug and Murray (2015) clearly pointed out that the MSM is highly suited for describing data which exhibit discrete dynamic patterns over different periods of time.

According to Krolzig (2013) MS-BVAR can be considered as a generalization of the basic finite order of BVAR model of order p. Consider the p^{th} order autoregressive for the $K-\text{dimensional}$ time series vector, $Y_t = (y_{1t}, y_{2t}, \dots, y_{kt})^T, t = 1,2,\dots,T$ then,

$$Y_t = \Phi_{c1} + \Phi_{11} Y_{t-1} + \Phi_2 Y_{t-2} + \cdots + \Phi_p Y_{1t-p} + \varepsilon_t;\ \varepsilon_t \sim i.i.d.\, N(\mu = 0, \sigma_\varepsilon^2 = 1), \tag{13}$$

Where, $t = 1, 2,3, \dots, T$. Each equation has $M = NxP + P$ regressors because the coefficient matrix is of the form $M \times N$, with, $\Phi = \left[\Phi_c \ \ \Phi_1 \ \dots \Phi_p\right]$. For the reason that $\varepsilon_t \sim i.i.d(\mu = 0\ \sigma_\varepsilon^2 = 1)$, then (13) is branded as an intercept form of a stable Gaussian $VAR(p)$ model. When reparameterizing by adjusting the mean from the VAR model, Krolzig (2013) simplified the model to:

$$Y = X\Phi + \epsilon \tag{14}$$

$Y = [y_1, \dots, y_T]^T$, $X = [x_1, \dots, x_T]^T$ and, $\epsilon = [\varepsilon_1, \dots, \varepsilon_T]^T$ respectively are the $T \times N$, and $T \times M$ matrices. Following Carriero *et al.* (2015) procedure, we use the following Normal-Inverted Wishart (N-IW) prior conjugate:

$$\Phi | \sigma_\varepsilon^2 \sim N(\Phi_0, \sigma_\varepsilon^2 \otimes \Omega_0), \sigma_\varepsilon^2 \sim IW(S_0, \upsilon_0). \tag{15}$$

Likewise, the conditional posterior distribution of this model is the N-IW as Greenberg (2012) has shown:

$$\Phi|\sigma_\varepsilon^2, Y \sim N(\bar{\Phi}, \sigma_\varepsilon^2 \otimes \bar{\Omega}), \sigma_\varepsilon^2|Y \sim IW(\bar{S}, \bar{\upsilon}.) \quad (16)$$

The general thought behind this class of models is that, parameters of the principal data making the procedure of the observed time series vector Y_t depend on the unnoticeable regime variable S_t, which addresses the probability of being in an alternate condition. Since the MSM has a particular trademark that the undetectable acknowledgement of the regime is administered by a discrete state Markov stochastic procedure, then the transition probabilities are characterized by:

$$P_{ij} = pr(S_{t+1} = j|S_t = i), \sum_{j=1}^{q} P_{ij} = 1 \ \forall_{ij} \in \{1,2,\dots,q\}. \quad (17)$$

More precisely, it is expected that S_t takes after an irreducible ergodic q state Markov process with the following transition matrix:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1q} \\ \vdots & & \ddots & \vdots \\ p_{11} & p_{12} & \cdots & p_{1q} \end{bmatrix}, \quad (18)$$

Where $p_i q = 1 - p_{i1} - \cdots - p_{i,q-1}$ for $i = 1,2,3,\dots,q$ are estimated from the $Ms(k) - BVAR(p)$ model. The underlying $Ms(k) - BVAR(p)$ model is then given by:

$$Y_t - \mu(S_t) = \Phi_1(S_t)(Y_{t-1} - \mu(S_{t-1})) + \cdots + \Phi_p(S_t)\left(Y_{t-p} - \mu(S_{t-p})\right) + \varepsilon_t, \quad (19)$$

Where, $\varepsilon_t \sim i.i.d\ N(0, \sum S_t)$ and $\mu(S_t), \Phi_1 S_t, \dots, \Phi_p(S_t), \sum S_t$ are parameter shift functions describing the dependence of the parameters. The expected duration is computed as $\frac{1}{p_{ij}}$ for the process to stay in i[th] regime. Model parameters are estimated using the EM algorithm. This is an iterative technique for discovering maximum likelihood estimates of parameters in statistical models, where the model depends upon imperceptibly latent variables.

3.6. Forecasting Performance of $MS(k) - BVAR(p)$

The forecasting exercise is performed in Pseudo real-time; $i.e$ the information which is not accessible is never utilized at the time the forecast is made. According to Carriero *et al.* (2015) forecasting performance is checked to discover the best performing model and utilised the three error measurements; to be specific; mean square error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). In any case, the present study tails these methods of Carriero *et al.* (2015) and Makatjane and Moroke (2016) for forecasting performance of the

MS (k)-BVAR (p) and use the root mean square error (RMSE), MAPE and MAE. Given the time series Y_t and the estimated $\widehat{Y}_t$, (Makatjane and Moroke, 2016) defined the MAE and MAPE as:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left[Y_t - \widehat{Y}_t\right]^2 \qquad (20)$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|Y_t - \widehat{Y}_t\right| * 100 \qquad (21)$$

where n is the sample size, Y_t is the original time series data and finally $\widehat{Y}_t$ is the forecasted time series. Carriero *et al.* (2015) specified RMSE as:

$$RMSE = \sqrt{\sum_{t=1}^{n}\left[Y_t - \widehat{Y}_t\right]^2} \qquad (22)$$

3.7. Logistic Regression Model

The logistic model is engaged as part of this study to determine an EWS for the inflation rate. This method is utilized as a development from the $Ms(k) - BVAR(p)$ model. The regimes are regarded as the binary response variable in an LRM indicated by Cruz and Mapa (2013). The practicality of the inflation crisis is being assessed through the probabilities that are extracted from the LRM.

We follow the discussion of Tong (2012) in which he emphasized that the binary series is premeditated with the values of 0 or 1 signifying the low inflation and high inflation regimes respectively in this study. These regimes classification result from the estimated $Ms(k) - BVAR(p)$ model. The past covariates, say $t = 1,2,3 \dots, T$, that originate from a set of series are clearly expressed by $W_{t-1} = \left(W_{(t-1)1}, \dots, W_{(t-1)p}\right)$ with their equivalent P dimensional vector that denotes the process as W_t. The mean series which is being expressed by $\mu_t = E[Y_t|S_{t-1}]$, is assumed to be the conditional expectation of the response given earlier values. Therefore, the conditional likelihood estimation considered in this study is presented by:

$$P_\beta(Y_t = 1|S_{t-1}), \qquad (23)$$

Where, β is a p-dimensional vector and S_{t-1} indicates the observed components to the researcher at the time $t - 1$ of the time series and the covariates information. Declaring that $P_\beta(Y_t = 1|S_{t-1})$ produces precise likelihood estimates, a suitable inverse link $h \equiv F$ is chosen and engaged in such a way that it maps both real tine and the interval $[0,1]$. Finally, designating the probability of success F_{t-1} as Π_t, the model is then established as:

$$\Pi_t(\beta) = \mu_t(\beta) = P_\beta(Y_t = 1|S_{t-1}) = F(\beta^{`} Z_{t-1}), \quad (24)$$

Here, F is continuous and severely an increasing function, that returns values ranging between 0 and 1. The speculation of the LRM is as follows:

$$\Pi_t(\beta) = P_\beta(Y_t = 1|S_{t-1}) = \frac{1}{1+e^{\left(-\beta^{`} Z_{t-1}\right)}}, \quad (25)$$

β, present the P −dimension covariates process of W_{t-1} while the inverse link is well-defined by $F \equiv \Phi$, and Φ forms the standard normal distribution specified by a cumulative function (Nyberg, 2010).

3.8. Computation of Marginal Effects

The marginal effects are computed in this study to quantify the likelihood of the occurrence of inflation crises. Marginal effects are simply partial derivatives of the event probability with respect to the predictor of interest. According to Norton *et al.* (2004) the conditional mean of the response variable is given by:

$$\begin{aligned} E[y|x_1, x_2, X] = \Phi(\beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + X\beta) \\ = \Phi(u) \end{aligned} \quad (26)$$

Where, Φ is the standard normal cumulative distribution and u denotes the index $\beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + X\beta$. Supposing that x_1 and x_2 are continuous, then the marginal effect of just the interaction term $x_1 x_2$ is:

$$\frac{\partial \Phi(u)}{\partial x_1 x_2} = \beta_{12} \Phi'(u) \quad (27)$$

4. EMPIRICAL ANALYSIS

This section provides and discusses the preliminary and primary analyses of results.

4.1. Exploratory Data Analysis Results

Figure 1 presents the graphical presentation of a monthly simulated inflation rate and repo rate of South Africa for the period of January 1999 to December 2017. The year 1999 marks the period before the South African Reserve Bank (SARB) adopted the inflation targeting framework policy. Through a visual inspection, both the series seem to possess the same characteristics as they are moving towards one direction on the same wavelength. The movement of the series from the year 2000 suggests the possibility of Cointegration prior to imposing transformation on the data. The up and down spikes are an indication of inflation volatility and this is expected for macroeconomic time series data. The co-movement and irregular patterns serve as a strong motivation to use these variables in multivariate MS (k)-BVAR (p) and LRM. To explore the properties of the simulated series, descriptive statistics are computed and the results are reported in Table 1.

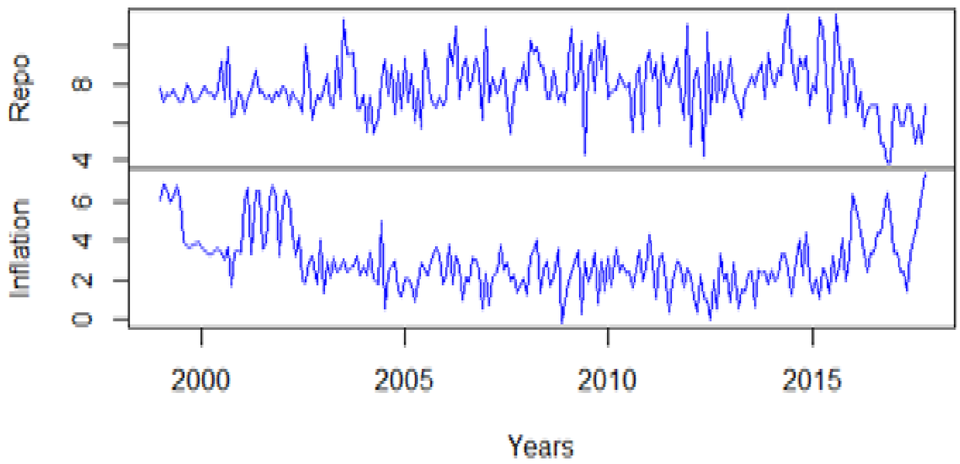

Figure-1. Simulated South Africa's Inflation and Repo Rate

Table-1. Exploratory Data Analysis

Statistic	Inflation Rate	Repo Rate
Mean	6.039	7.039
Median	6.035	7.035
Jarque-Bera	1.699(0.3978)	0.647(0.705)

NB: The numbers in () reports the JB probability values for both inflation and repo rate

The mean and the median of the inflation rate are reported at 6.039% and 6.035% respectively. This implies that on a monthly basis, the SA inflation has an average of 6% while the repo rate revolves around 7%. It can also be noted that the inflation rates are a bit lower than the repo rate. Van der Merwe and Mollentze (2010) advocate for high repo rates and advised from the monetary transmission mechanism that setting this sector high decreases the domestic assets prices which will consequently cause the inflation rate to decline. The current estimates of these variables according to the reports by global rates. com, which publicised that the SA repo rate stands at around 6.750% and inflation rates are reported to be 4.858% in September 2017. This suggests that simulated repo rates are significantly higher than the actual. This is not surprising as, in fact, these small differences are expected from the simulated and actual time series data sets.

4.2. Maximum Lag Length Selection

In selecting an optimal lag length of the parsimonious lag order of the $\mathrm{MS(k)} - \mathrm{BVAR(p)}$ model and KSS-NADF(p) test, AIC, SBC, and HQ are used and results are tabulated in Table 2. As reported in Table 2, SBC and HQ select the optimal lag length as 1. The AIC has the smallest value at the lag length of 2 contradicting the suggestions by the SBC and the HQ criteria. However, theory suggests the optimal lag selected by the SBC should be considered and this should be a final decision (Tsay (2010). Moreover, following the discussion of Scott and Hatemi (2008), for each simulated models with large samples, SBC and HQ are found to be the best performers to select the optimal lag length. The current study adopted the suggestion by these authors; hence we select the optimal lag length of 1.

Table-2. Lag-length selection

	Information criterions		
Number of lags	**AIC**	**SBC**	**HQ**
1	0.693	0.791**	0.733**
2	0.689**	0.853	0.756
3	0.713	0.943	0.806
4	0.731	1.025	0.850
5	0.767	1.127	0.913

**signifies a significant lag

4.3. Nonlinear Unit Root Tests Results

Having established that the simulated inflation rate and repo rate are from a normal distribution as per the JB test in Table 1, KSS-NADF test is established. This test is used in the study as an endorsement for the presence of nonlinear unit root and table 3 gives a summary report of the results. Alluding to the results in Table 3, the null hypothesis of the linear unit root is rejected for both inflation rate and repo rate at all levels of significance, supporting the findings in Figure 1. The calculated probability values of the KSS-NADF test are all significant, that is, they are all less than 1%, 5%, and 10% significance levels. This suggests that the simulated data is asymmetric and it is nonlinear in nature.

Table-3. KSS-NADF unit root test

Variable	lag length	Coefficient	Std. Error	t-Statistic	Prob.	KSS
Inflation	1	-0.50185	0.06035	-8.316	0.0000	-4.1013
Repo	1	-0.52636	0.05889	-8.937	0.0000	-4.1818

Critical values of the KSS-NLADF test with constant and trend at the 10%, 5% and 1% significant levels are -3.13, -3.40 and -3.93, respectively

4.4. Structural Change Test

There is abundant evidence that both simulated series are nonlinear in nature as they possess nonlinear unit root. We therefore further test the presence of structural change as the estimated linear $BAR(1)$ is unstable. To achieve this objective, we use the recursive residuals of a linear $BAR(1)$ are extracted and plotted against as presented in Figure 2. The plots show that the recursive residuals of the two estimated $BAR(1)$ models for inflation rate and repo rate are moving outside the bounds suggesting that both the series are not stable. This further entails structural change. These findings are in support of Figure 1 and one could infer that both series tend to rise outside the specified target of 3%-6% of inflation and the fixed value of 7% for repo rate reflecting the perfect truth that the two series are volatile.

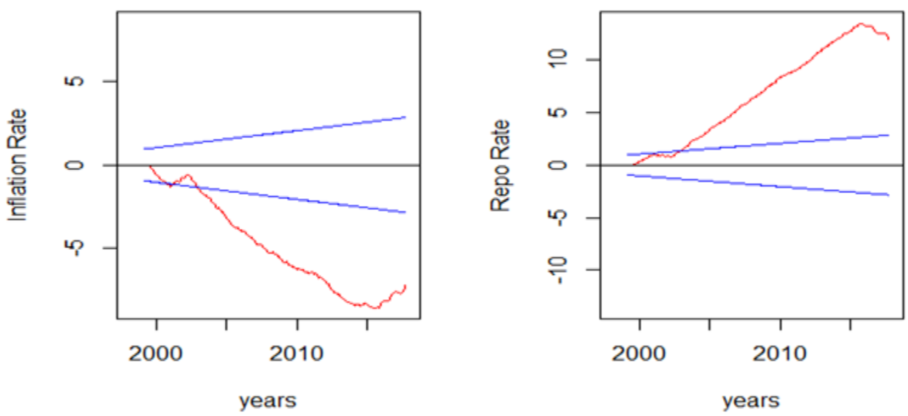

Figure-2. CUSUM of squares for inflation rate and repo rate

4.5. Estimation of $MS(2) - BVAR(1)$

The parameter estimates presented herein are obtained from $MS(2) - BVAR(1)$ with two regimes. The estimated coefficients for capturing the impact of repo rate movement $(\Phi_{21} \text{ and } \Phi_{22})$ on the inflation rate are significant in all the cases as reported in Table 4. The implication here is that the fluctuations in the SA repo rate do have strong effects on the dynamics of the inflation rate. The transition probability matrix presented as $\begin{matrix} p(S_t = 0|S_{t-1} = 0) = 0.981 \\ p(S_t = 1|S_{t-1} = 1) = 0.995 \end{matrix}$ suggests that the probability of the inflation rate in low regime being lower than that of high inflation regime with about 1.4%. This implicitly articulates that when the inflation series is in Regime 0 (Low inflation regime), the probability that it switches to the high inflation regime, denoted as regime 1, is $P(S_t = 1|S_{t-1} = 0) = 0.019$ which is higher than that of regime 1. These findings suggest that in-sample inflation rate of SA is lower for 8 years and 6 months starting from January 1999 to December 2017. The period for anticipated high inflation rates is 11 years and 4 months. The filtered probabilities for inflation rate are plotted and reported in Figure 3 are in support of those presented in Table 4.

Table-4. $MS(2) - BVAR(1)$ Parameter estimates for inflation

Regime 1			
Parameter	**Coefficient**	**Standard error**	**Significance**
C_1	2.474	0.690	0.011
Φ_{11}	0.433	0.021	0.004
Φ_{21}	-0.725	0.259	0.011
σ_1	1.359	0.099	0.010
Regime 2			
C_2	0.668	0.479	0.001
Φ_{12}	0.128	0.033	0.013
Φ_{22}	-0.651	0.112	0.008
σ_2	0.495	0.010	0.002
Transition Probabilities			
Regime1 [Low Inflation]		**Regime2 [High Inflation]**	
0.981		0.019	
0.005		0.995	

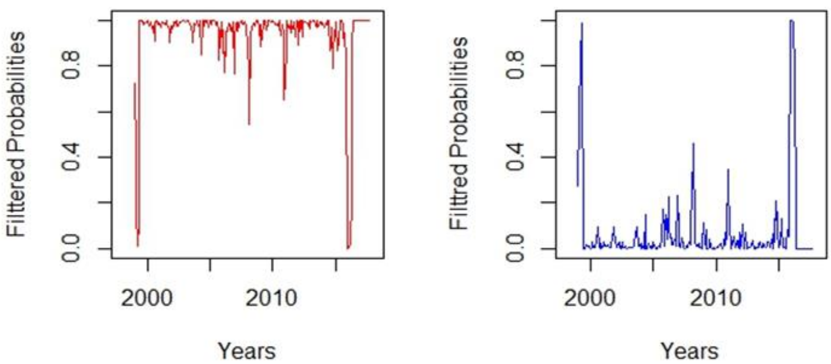

Figure-3. Filtered Probabilities for low and high inflation rate

4.6. Forecasting Performance of $\mathbf{MS(2)-BVAR(1)}$

In order to come up with EWS for the coming few years, the out-of-sample forecasts are extracted then used to re-estimate the $\mathrm{MS(2)-BVAR(1)}$ model to correctly classify the inflation regimes accordingly. An out-of-sample test has the ability to control either the possibility of over-fitting or over-parameterization problems and gives a more powerful framework to evaluate the performance of the estimated model. To check the in-sample and out-of-sample forecast performance of the $\mathrm{MS(2)-BVAR(1)}$ RMSE; MAE and MAPE are estimated respectively.

As indicated in Table 5, the in-sample forecasts are much better than the out-of-sample forecasts. By individually looking at the metrics, the RMSE has increased by 12.84% from the in-sample forecasts to the out-of-sample forecasts. Formerly, MAPE has increased by 54.3% while MAE increased by 6.31%. Because both the RMSE and MAE increment is less than 50%, then the study concludes that for both in-sample and out-of-sample forecast, the estimated $\mathrm{MS(2)-BVAR(1)}$ had to mimic the actual simulated data.

Table-5. Forecasting performance of $\mathbf{MS(2)-BVAR(1)}$

sample	RMSE	MAPE	MAE
In-sample	0.0080	0.0253	0.0068
Out-of-sample	0.1328	0.5687	0.0700

4.7. Logistic Regression Results

In building SA's early warning system for inflation rates, the LRM framework is followed. The reported probabilities from the logistic model give a sensible appraisal about the achievability of forecasting SA's inflation crisis. Through the grouping of the regimes from the $\mathrm{MS(2)-BVAR(1)}$, we create a response variable for the LRM to become binary with a scene of high inflation coded 1 while that of low inflation is named 0, and covariate to the model to be estimated is the repo rate. The results of an estimated EWS logit model are summarized in Table 6.

The coefficient sign of the repo rate as an indicator of the inflation crisis is found not to be consistent with the expectations from the monetary theory but are in support of Van der Merwe and Mollentze (2010) who suggested high repo rates bring down inflation rates. The positive repo rate is directly proportional to the inflation rate; hence inference of a surge in the value of the repo rate increases the likelihood that the country enters a high inflation period. These results are in conjunction with the ones reported by the $\mathrm{MS(2)-BVAR(1)}$ in Table 4. Leshoro (2014) also reported that a shock in the repo rate increases the inflation rate. The results also mark that the percent of crisis correctly called (PCCC) is 51%, while 49% are incorrectly classified crises.

In order to quantify the possibility of the occurrence of an incidence of high inflation, the marginal effects coefficient of the repo rate is reported as 0.817 which is directly proportional to the inflation rate as long-established by the logistic model in Table 6. The inference that one can make here is that a 1% increase in repo rate in the past month, increases the probability of SA to enter into an inflation crisis by approximately 81% per month. These findings are similar to those reported by Bonga-Bonga and Kabundi (2015). Their study revealed that a positive shock in repo rate increases inflation rate for more than 18 months. This gives no evidence of the likelihood of the inflation rate to decrease after a positive monetary shock. Mboweni *et al.* (2008) and Gupta and Komen (2009) found similar results.

Table-6. Estimated logit model

Variable	Coefficient	Std. Error	z-statistic	Prob
constant	-27.6876	5.692925	-4.86	0.00
Repo rate	3.733378	0.7706061	4.84	0.00
Probabilities for each Regime				
	Regime 1		**Regime 2**	
Probability	0.49		0.51	

4.8. Evaluation of the Performance of the Logit-Based EWS Model

The performance assessment results of the Logit based EWS model are given as a summary in Table 8 and Table 9. We further adopt the following scenarios as presented by Kaminsky *et al.* (1998) and Cruz and Mapa (2013):

Table-7. Probabilities of correct and incorrect crisis prediction

	High Inflation	**Low Inflation**
Signal Issued	$P(1,1)$ Correct call of crisis[A]	$P(1,2)$ Type II Error or Wrong Signal [B]
No Signal Issued	$P(2,1) = 1 - P(1,1)$ Type I error or Missing Signal [C]	$P(2,2) = 1 - P(1,2)$ Correct call for non-event

Kaminsky *et al.* (1998) and Cruz and Mapa (2013)

In Table 7, event A epitomizes the occasion when the model indicates a crisis when a high inflation event indeed occurs. Event B refers to an event when a signal issued by the model is not followed by the occurrence of high inflation, i.e., wrong signal. It is also possible that the model does not signal a crisis (low estimated probability) but a crisis, in fact, occurs, i.e., missing signal, event C. Finally, Event D indicates a situation in which the model does not predict a crisis and no crisis occurs. In this paper, a threshold value of 0.5 is used to indicate whether the probabilities can already be interpreted as crisis signals. To evaluate the EWS model performance, we make use of the following performance criteria recommended by Kaminsky *et al.* (1998).

a) Percent of crisis correctly called (PCCC): $\frac{A}{B+C}$

b) Percent of non-crisis correctly called (PNCCC): $\frac{D}{B+D}$

c) Percent of observations correctly called (POCC): $\frac{A+D}{A+B+C+D}$

d) Adjusted noise-to-signal ratio (ANSR): $\frac{B}{B+D} / \frac{A}{A+C}$

e) The probability of an event of high inflation given a signal (PRGS): $\frac{A}{B+A}$

f) The probability of an event of high inflation given no signal (PRGNS): $\frac{C}{D+C}$

g) Percent of false alarms to total alarms (PFA): $\frac{B}{B+A}$

Table-8. Forecasts of the EWS Model*

In-sample	High Inflation			Low inflation		Total
	High Inflation	59.67	51%	56.61		116.28
Predicted	Low inflation	57.33		54.39	49%	111.72
	Total	117		111		228
Out-of-sample	High Inflation			Low inflation		Total
	High Inflation	16.96	53%	14.37		31.33
Predicted	Low inflation	15.04		13.63	47%	28.67
	Total	32		28		60

*The first set of numbers represents counts while figures in parentheses represent percentages of correctly predicted observations with respect to the two inflation regimes.

The results in Table 8 infer that the model has some EWS potential. In view of the in-sample estimates, the model has the capacity to accurately predict 51% of high inflation and 49% of low inflation occasions. As it is, the proportion of high inflation rates given a signal is relatively high at 51% in the in-sample while the out of sample is at 54% The proportion of false alarms to total alarms shown in Table 9 is relatively low at 49% for the in-sample and 46% in out-of-sample.

Table-9. Performance of the EWS

	In-Sample	Out-of-sample
PCCC	51	53
PNCCC	49	47
POCC	50	51
ANSR	100	97
PRGS	51	54
PRGNS	51	52
PFA	49	46

Results in Tables 9 indicate that there is 53% chance of high inflation rates in the next five years and this probability is about 2% higher than the current situation. The implication here is that the country could expect more increases in the future. This inference is made based on the in-sample estimates presented. In addition, the proportion of the expected false alarm (PFA) to the total alarm is premeditated as 46% which is lower than that of the in-sample by 3%. The out-of-sample percent of the non-crisis correctly called is 47%, which is 2% less than that of the in-sample. One could conclude that, after all, inflation rates could be lower in future.

5. DISCUSSION AND CONCLUSION

In this study, the main focus is to estimate the $\mathrm{MS(k)-BVAR(p)}$ model and to implement the estimated regimes in an LRM. This is done with the expectation that the model might give the results that quantify the probability of an inflation crisis in South Africa. Monte Carlo simulation method is used to generate the series on inflation and repo rates for January 1999 to December 2017 on a monthly basis. The two-time series data are selected on the assumption that they are cointegrated.

This study is innovative in a sense that no similar study in the context of SA has used the $\mathrm{MS(k)-BVAR(p)}$ model together with the LRM in predicting the possibility of an inflation crisis. The estimation of the two models has delivered an enhanced understanding of the prediction and classification of inflation crises together with the relationship between the selected variables. In particular, the study is unique in terms of uniting econometric multivariate methods in predicting inflation crises in SA. Jointly, the results can be useful in guiding policymakers in identifying episodes of high inflation rates in SA and safeguard the inflation crisis well ahead of time.

In order to classify inflation crises optimally on the basis of the repo rate, the LRM is estimated. The two regime probabilities from the $\mathrm{MS(2)-BVAR(1)}$ model are incorporated in the logistic model as the binary dependent variable. The low inflation regime is denoted as zero and high inflation denoted as one. The study here addresses the events of high and low inflation accordingly and found that for the in-sampling, the model has indicated that the probability of high inflation is 57%. This infers that South Africa has 51% chance that it will be in inflation crises. The out-of-sample reported only 53% of high inflation in the next five years.

In reckoning the likelihood of the inflation rate crises, the study established the marginal effects test and the conclusion made is that, as long as the repo rate increases, the inflation rate is likely to increase by 81%. From this positively reported sign of the repo rate, one can conclude that fragmented results on the marginal interaction effects in terms of the sign have been reported. In addition, the same results are reported by Cruz and Mapa (2013).

For future Studies, scholars may also use the same developed EWS model in the Cointegration approach where they would focus on the co-movement of the variables in the long run and short run. In this, the case vector error corrected or vector autoregressive moving average enhanced Markov Switching model may be used. The results of the two models may be compared with the current one. Other data generating processes other than Monte Carlo simulation may be used.

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