International Journal of Education and Practice
2022 Vol. 10, No. 2, pp. 69-83.
ISSN(e): 2310-3868
ISSN(p): 2311-6897
DOI: 10.18488/61.v10i2.2942

A COMPARATIVE STUDY OF THE PROBABILITY AND STATISTICS CURRICULA IN THE HIGH SCHOOL MATHEMATICS TEXTBOOKS OF VIETNAM AND GERMANY

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Article History
Received: 5 January 2022
Revised: 17 February 2022
Accepted: 2 March 2022
Published: 21 March 2022

Keywords
Comparative study
High school mathematics
Mathematics textbooks
Probability content
Statistics content
Textbook analysis.

ABSTRACT

An international comparative study of mathematics textbooks is essential in identifying trends in international mathematics education, developing textbooks and improving the quality of teaching and learning in mathematics. This study analyses and compares the contents of the probability and statistics curricula in the high school textbooks of Vietnam and Germany. It highlights the similarities and differences in the contents and the way they are presented, consolidated and deepened in textbooks of the two countries. The probability and statistics curricula in Vietnamese textbooks are much less than that of German textbooks. The Vietnamese textbooks focus on the mathematical content whereas the German textbooks focus on applications of the content in real life. Vietnamese textbooks tend to force students to accept probability and statistics knowledge while German textbooks encourage students to explore knowledge. Because of this, contexts relating to the contents in German textbooks are more detailed and more natural. German textbooks mainly use an inductive approach to present the contents whereas Vietnamese textbooks often use a deductive approach. In addition, German textbooks unlike those of Vietnam, always take advantage of realistic situations in exploring the contents, encourage co-operative learning and promote statistical reasoning by implementing several project works and provide an opportunity for practical experience and experiment by using the simulations of probability and statistics situations. The comparison shows the strengths of probability and statistics contents in German textbooks that Vietnamese textbooks can learn for the development of textbooks in future.

Contribution/Originality: This study shows the similarities and differences in the probability and statistics contents and the way they are presented in the high school textbooks of Vietnam and Germany. This comparison contributes to explore the trend in international mathematics education, developing textbooks and improving the quality of teaching and learning mathematics.

1. INTRODUCTION

Textbooks play an important role in the teaching process because they are the main basis to decide the teaching content and methods (Alajmi, 2012; Fan & Zhu, 2007; Fan, Zhu, & Miao, 2013; Park & Leung, 2006; Vicente, Sánchez, & Verschaffel, 2020; Wang, Barmby, & Bolden, 2017). They are effective curriculum material which not only link education policy and teaching practice but also connect teacher' teaching with students' learning (Wang & Lu, 2018). The analysis of textbooks can provide reasons for differences in student achievement (Hong & Choi, 2014; Zhu & Fan, 2006).

International comparative studies of mathematics textbooks have currently increased, since comparing textbooks of different countries is one way to identify trends in international mathematics education and to improve teaching and learning in mathematics. For this reason, many researchers are interested in this area (Alajmi, 2012; Fan & Zhu, 2007; Haggarty & Pepin, 2002; Park & Leung, 2006; Yeap, Ferrucci, & Carter, 2006; Zhu & Fan, 2006). The 13th ICMI (International Commission of Mathematics Instruction) study includes a number of comparisons of textbooks from East Asia and the West (see Leung, Graf, and Lopez-Real (2006)) whilst the TIMSS (Third International Mathematics and Science Study) collects, analyses and compares curricula and textbooks of many countries throughout the world every four years.

Probability and statistics are important modules in the mathematics curricula and textbooks because they are indispensable parts of real life and efficient tools to solve many problems. Previous studies have shown that probability and statistics have broad applications in various areas of science and technology as well as in the real life such as insurance, education, health, security, social services, politics, medicine, etc., which leads to the prediction that in the 21st century the cultured man should understand something about probability and statistics (Freudenthal, 1970; Isaac, 1995; Rényi, 1987; Tijms, 2007; Woolfson, 2008).

One cause of education is to satisfy the needs of society. That is a principal demand. Hence, under conditions of a society where probability and statistics curricula are widely applied, some basic knowledge and skills in these areas have to become elements of a general culture of all society members. That is the reason why many countries pay attention to probability and statistics in their curriculum reforms (Wu & Zhang, 2006).

This study shows the similarities and differences in the probability and statistics curricula in the high school textbooks of Vietnam and Germany. The two countries were chosen because Vietnam belongs to the group of East Asia countries and Germany belongs to the group of Western countries. Textbooks from these two groups of countries are often compared because the differences in the cultural traditions of the East Asia and the Western countries lead to the differences in textbooks' presentation and students' achievement (Leung et al., 2006). This comparison is hoped to contribute to the development of textbooks and the quality of teaching and learning mathematics.

2. LITERATURE REVIEW AND RESEARCH QUESTIONS

In the history of educational research, studies on textbooks analysis focused on *the physical characteristics of the books* (the size, number of lessons, percentages of pages for specific topics, the location of topics), *the selection of content* (the main topics covered, the order of topics), and *the characteristics of content* (the structures of lessons, the depth of mathematical inquiry, the features of mathematics problems, the contexts related to content, the pedagogy related to content) (Alajmi, 2012; Hong & Choi, 2014; Jones & Tarr, 2007; Park & Leung, 2006; Piyatissa, Johar, & Tarofder, 2018; Vicente et al., 2020; Wang et al., 2017; Wang & Lu, 2018).

Alajmi (2012) analyzed the presentation of fractions in elementary textbooks of Kuwait, Japan, and the USA. The study focused on the physical characteristics of the textbooks, the structure of the lessons, and the nature of the mathematical problems presented. The analysis showed that USA and Kuwaiti mathematics textbooks are larger than Japanese textbooks because USA and Kuwaiti textbooks include a great deal of repetition. Different from the USA and Kuwaiti textbooks where fractions are introduced from the first grade, the Japanese textbooks introduce fractions from the third grade. Textbooks of all the countries concentrate on standard algorithms as the main computational methods. Jones and Tarr (2007) researched on the level of cognitive demand in probability tasks in middle school textbooks. Their finding suggests that a majority of mathematics tasks require a high level of cognition. Hong and Choi (2014) identified the similarities and differences in the content of the quadratic equations lessons of Korean and American secondary textbooks. The studies focused on the methods of introducing and developing the topics, the use of worked examples, representations and contexts used in mathematics problems, algorithms and formulas found, types of responses and levels of cognitive demand required by problems in the

textbooks. Vicente et al. (2020) compared the word problem solving approaches proposed by primary mathematics textbooks from Spain and Singapore based on the presence of the problem solving steps. The study revealed that the Singapore textbooks provided better scaffolding for high quality learning of word problem solving than Spanish textbooks. Wang et al. (2017) described a comparison of how England and Shanghai textbooks presented possible learning trajectories toward understanding linear function. The study found that in the English textbooks the structural aspect of understanding linear function due to a point-to-point view of function was constrained, while in the Shanghai textbook a variable view of function overemphasizing the algebraic approach was focused. Wang and Lu (2018) showed the similarities and differences in the selection of content and the presentation of content in high school mathematics textbooks of China, France, Germany, Japan, Russia, United Kingdom, United States and discussed the implications of development of Chinese mathematics textbooks.

According to Wang et al. (2017) Eastern textbooks have concentrated on pure mathematics knowledge, while Western textbooks considered real life situations. For example, (Park & Leung, 2006) compared the textbooks in the grade 8 of Eastern countries (China, Japan, Korea) and Western countries (England, the USA) and found that East Asia textbooks usually contained minimal real world contexts and concentrated on the mathematical content itself, while various real world situations presented the mathematical content in the Western textbooks. Moreover, East Asian textbooks always used the deductive approach in which general concepts were introduced before specific examples were given. On the contrary, Western textbooks took the inductive approach which first presented various activities and explorations in a realistic context, then step by step introduced the mathematical concept. In addition, East Asian textbooks included a limited number of components or features, and concentrated mostly in examples, explanation and exercises while Western textbooks had various features such as projects, practice and application exercises, technology, career connections, etc. (Park & Leung, 2006). Furthermore, after analyzing the characteristics of problems presented in textbooks. Zhu and Fan (2006) revealed that Chinese textbooks should present more authentic application problems whose conditions and data were from their daily lives, whereas USA textbooks should consider more challenging problems for students involving more steps in the solution. Besides, USA textbooks included more visual information than Chinese textbooks. In addition, how to present the content was emphasized in studies. Comparisons of the content presentation of the addition and subtraction of integers between American and Chinese mathematics textbooks showed that the Chinese textbooks included more high level mathematics content problems (Li, 2000). The English textbooks contained more questions connecting to real world situations, while Chinese textbooks had more emphases on knowledge depth (Wang et al., 2017).

Motivated by the previous studies of textbook analysis and the textbook comparisons between Eastern and Western countries, this study focused on *the selection of content* and the *characteristics of content*, including realistic situations used to explore the content, contexts related to content, approaches presenting content, and tools to consolidate or deepen content.

Therefore, the research questions are:

1. What are the similarities and differences in *the selection of content* of probability and statistics in the Vietnamese and German mathematics textbooks?
2. What are the similarities and differences in *the characteristics of content* of probability and statistics, including realistic situations used to explore the content, context related to content, approaches presenting content, and tools to consolidate or deepen content in the Vietnamese and German mathematics textbooks?

3. METHODOLOGY

3.1. Textbook Sampling

Vietnam has centralized national curriculum and textbooks. The curriculum and textbooks are approved by the Minister of Education and Training in order to be used formally in teaching and learning in school. Vietnamese school textbooks are mainly published by the Education Publishing House, the official publisher appointed by the

Government for the entire country. There are two high school textbooks series in Vietnam. The probability and statistics curricula in these two series are not too far and not too deep in compare to each other. Therefore, it is enough to consider only one of them. This study chose the textbook series that contained more probability and statistics contents. The chosen textbooks at Vietnamese high school level included:

- Doan, Nguyen, Nguyen, Dang, and Tran (2009a).
- Doan, Nguyen, Nguyen, Nguyen, and Dang (2009b).

On the contrary, Germany has a decentralized curriculum. The Federal Republic of Germany includes 16 states and each state has its own curriculum and textbooks. For each curriculum, there are many series of textbooks which are published by different publishing houses. Therefore, collecting and studying all series of textbooks in Germany is virtually impossible.

To compare with the Vietnamese system, high school textbooks of the two richest and most populous states: Nordrhein – Westfalen and Bayern were chosen. Thus, the textbooks sampled for this study are as follows:

- Griesel and Postel (2003).
- Griesel and Postel (2004).
- Feuerpfeil and Heigl (2003).

Table 1. Considerable aspects and tasks.

Aspects	The selection of content	The characteristics of content		
		Approach presenting the content	**Context relating the content**	**Tools to consolidate or deepen the content**
Tasks	Analyzed the main content of probability and statistics in the textbooks, showed similarities and differences in the content in the textbooks of the two countries.	Examined which approach was used in the textbooks of the two countries to present the probability and statistics contents: a *deductive approach* (where the concept was introduced first, then illustrated examples are given) or an *inductive approach* (where various examples and explorations in a realistic context are presented at the beginning to help students adapt to the new concept step by step) (see Park and Leung (2006)).	Examined how the contents are related in the textbooks of the two countries, whether the contents were mentioned in a contextualized or decontextualized way.	Examined how the probability and statistics content in the two countries were consolidated and deepened, whether the textbooks used various and suitable tools or not.
Analysis method	The inductive method was applied to analyze similar and different topics presented in the textbooks of the two countries.	Analyzed the percentages of inductive and deductive methods used to approach the concepts, rules and formulas in the textbooks of Vietnam and Germany.	Focused on the concepts and rules, identified how they are presented in the textbooks of the two countries, analyzed the average number of realistic situations used in German and Vietnamese textbooks to introduce or demonstrate concepts and rules of probability and statistics in each lesson.	The inductive method was applied to analyze the similar and different tools used to consolidate and deepen content in the textbooks of the two countries.

3.2. Research Procedure and Framework

The first step of the research procedure was searching the literature. Various comparative studies of mathematics textbooks, especially studies of comparisons between mathematics textbooks in East Asian and in Western countries were collected. The inductive method of analysis was used to figure out the main aspects considered by the literature. These aspects were divided to two categories: the selection of content and the characteristic of content. Based on these categories, the research questions and the frame work of the study were established. The considerable aspects of comparisons were explored. The tasks and analysis methods for each considerable aspect were identified. Table 1 shows the aspects of the textbooks that the study concentrated on and the tasks, the analysis methods for each aspect.

4. RESULTS

4.1. The Contents

4.1.1. Probability and Statistics Main Contents in Textbooks of the Two Countries

a) Main Contents of Probability and Statistics in Vietnamese Textbooks

The high school level in Vietnam includes three grades: Grade 10, 11, 12. In the textbooks the probability and statistics contents are distributed into two chapters (other chapters in the textbooks include the contents of Algebra or Analysis):

- Chapter 5 in Algebra 10 of which the content is *descriptive statistics*, although its title is only *statistics* (Doan et al., 2009a).
- Chapter 2 in Algebra - Analysis 11 of which the content is *combinatories and probabilities* (Doan et al., 2009b).

Table 2 presents the contents of statistics in grade 10 (Chapter 5 of Algebra 10):

Table 2. Main contents of descriptive statistics in grade 10 in Vietnam.

Topics	Sub-topics
§.1 Some initial concepts	1. What is statistics? 2. Data sample
§.2 Representation of a data sample	1. Absolute / relative frequency table 2. Absolute / relative grouped frequency table 3. Chart
§.3 Typical values of a data sample	1. Average value 2. Median 3. Mode 4. Variance, standard deviation
Alternative:	Using calculator in statistics

Source: Doan et al. (2009a).

Table 3 summarizes the contents of probability statistics in Grade 11 (Chapter 2 of Algebra and Analysis 11).

b) Main contents of Probability and Statistics in German Textbooks

In Germany, the high school level is called the secondary level II which includes Grade 11, 12 and 13. In some states of the country, this level includes only grade 11 and 12 (Basic Structure, 2017). Among the chosen textbooks of the two states Nordrhein – Westfalen and Bayern, some books have probability and statistics included with other topics (Griesel & Postel, 2003, 2004) and some have two topics in a separate book (Feuerpfeil & Heigl, 2003).

Table 4 presents the main contents of probability and statistics of the chosen German textbooks (the order of the topics is presented differently by the textbooks' authors).

4.1.2. Similarities in Probability and Statistics Contents of German and Vietnamese High School Textbooks

The analysis of mathematics textbooks of the Vietnamese and German high schools showed that the probability and statistics contents in the textbooks of Vietnam and Germany have a similar part comprising the following areas:

(i) *Descriptive statistics* including basic knowledge of this area:

- The frequency and relative frequency / grouped frequency and relative grouped frequency.
- Representations of statistical data by distribution tables and by graphs.
- Compression of statistical data by using typical values: mean, median, mode, variance and standard deviation.

Table 3. Main contents of combinatorics and statistics in Grade 11 in Vietnam.

A. Combinatories	**Sub-topics**
§1 Two basic counting rules	• Addition rule • Multiplication rule
§2 Permutations, arrangements and Combinations	• Permutations • Arrangements • Combinations
§3 Newton's binomial	• Newton's binomial formula • Pascal's triangle
B. Probability	**Sub-topics**
§4 Event and probability of an event	• Event • Probability of an event, Laplace's and statistical approach of probability
§5 Probability calculation rules.	• Union of events, mutually exclusive events, addition theorem and complementary events • Intersection of events, independent events and multiplication theorem
Alternative:	Using calculator for calculation of combinatories and probabilities
§6 Discrete random variable	• Concept of discrete random variable • Probability distribution of a discrete random variable • Mathematical expectation, variance and standard deviation of a discrete random variable
Alternative:	Relationship between discrete random variable and statistics

Source: Doan et al. (2009b).

Table 4. Main contents of probability and statistics *in the considered German textbooks.*

Topics	**Sub-topics**
Descriptive statistics	• Processing and representation of statistical data • Average value • Measures of dispersion • Linear regression • Manipulation
Probability concept and probabilities calculations	• Laplace experiment and Laplace's definition • Statistical approach • Axiomatic approach • Addition and multiplication rules • Combinatories • Conditional probability • Bayes's formula • Independence
Probability distributions	• Random variable • Probability distribution • Expectation and standard deviation of a random variable • Discrete random variable • Bernoulli experiment and binomial distribution
Applications of the probability calculation	• Testing hypothesis • Estimation

Source: (Feuerpfeil & Heigl, 2003; Griesel & Postel, 2003, 2004).

(ii) *Probability calculus*, namely:

- Probability concept: trial and event, probability of event, basic properties of probability, independent events.
- Formula for addition of probabilities, formula for multiplication of probabilities.
- Combinatories as a mean for probabilities calculation.

(iii) Discrete random variable consisting of the following knowledge concepts:

- Concept of discrete random variable.
- Probability distribution of a discrete random variable.
- Mathematical expectation, variance and standard deviation of a discrete random variable.

All the aforementioned contents were available in the mathematics textbooks of the high schools in Vietnam. They existed in principle also in the textbooks of the considered German states. If the mentioned textbooks of any state lacked any of aforementioned contents, then it is because this content has already been treated in a lower grade.

4.1.3. Differences in Probability and Statistics Contents Between German and Vietnamese High School Textbooks

There are some differences in the probability and statistics content between German Vietnamese textbooks:

(i) In *descriptive statistics*, there was no content of *linear regression* and *correlation* in Vietnamese textbooks, whereas German textbooks included this content.

(ii) German textbooks introduced more variety of *representing data*: table, pictogram, column chart, pie chart, block diagram, histogram, stem-leaf diagram, box-plot diagram, relative frequency polygon, Vietnamese textbooks mentioned only table, column chart, pie chart and relative frequency polygon. In addition, while Vietnamese textbooks had no explanation of the qualities that make graphs effective, German textbooks had many discussions of the characteristics of each type of graph and when each type is effective, such as:

- "Pie charts are more appropriate if one must read directly from a graph which presents majorities (example: after elections)" (Griesel & Postel, 2003).
- "Block diagrams are more suitable with the comparison of results from different surveys" (Griesel & Postel, 2003).
- "Column charts can be read more easily than block diagrams and pie charts, which represent frequencies of different values" (Griesel & Postel, 2003).
- "Stem-leaf diagrams contain on the one hand many details of a survey and on the other have almost the character of a graph (a replacement for a column chart)" (Griesel & Postel, 2003).
- "It is customary to represent frequency distributions of grouped data in the form of histograms; these are column charts in which there are no gaps between the individual columns" (Griesel & Postel, 2003).

(iii) In probability, Vietnamese textbooks did not cover the topic of conditional probability and Bayes formula, as in German textbooks.

(iv) The *statistical* (or *frequential)* approach to the probability concept was introduced explicitly or implicitly into the German and Vietnamese textbooks. This can be recognized through the following statements:

- Suppose that a trial T is executed N times and A is an event related to each trial T. "It is proved that the larger N is, the nearer the relative frequencies of the event A and a fixed number which is called probability of A in the *statistical* meaning (this number is also $P(A)$ in the classical meaning)" (Doan et al., 2009b).
- The content of the empirical law of large numbers is explicitly defined: In the execution of a long series of experiments, so in the frequent repetition of a random trial the relative frequencies of an outcome lie in the proximity of the probability of the outcome (Griesel & Postel, 2004).

However, whereas the Vietnamese curriculum did not pay due attention to the *statistical* approach to the probability concept and the relevant Vietnamese textbook halted at the aforementioned introduction, this approach

together with the classical one was taken into due account by the considered German textbooks. Indeed, a number of contents were presented and many exercises were given, in which relative frequencies are estimated probabilities (Feuerpfeil & Heigl, 2003; Griesel & Postel, 2004).

(vi) Whereas the Vietnamese curricula and textbooks paid no attention to the *axiomatic* approach to the probability concept, the *axiomatic definition of probability* was realized in the Bayern textbooks (Feuerpfeil & Heigl, 2003) although this approach was not found in the curricula or textbooks in Nordrhein-Westfalen.

(vii) Whereas the Vietnamese curricula and textbooks paid no attention to the *inferential statistics*, this area was taken into due account by relevant German textbooks, which serve to increase the application of the probability and statistics in real life situations. The topics *estimating* and *testing* were not found in the Vietnamese curricula and textbooks, but they were basic methods for the inferential statistics in German textbooks (see (Feuerpfeil & Heigl, 2003; Griesel & Postel, 2004)). The German students learn at least one of these two topics.

4.2. Characteristics of the Contents

4.2.1. Approach to Present the Contents

Mathematical content can be presented by inductive or deductive approaches. In the deductive approach, the concepts, rules, or formula are introduced first, then the demonstrative examples are given. This approach is in contrast to the inductive approach, where various examples and explorations in realistic context are presented at the beginning to help students explore the new concept gradually.

Table 5 shows the percent of probability and statistics concepts, rules, and formula presented by inductive and deductive approaches in the Vietnamese and German textbooks:

Table 5. Percent of probabilities and statistics concepts, rules, and formulas presented by inductive and deductive approaches in the Vietnamese and German textbooks.

Text books series / Approaches	Vietnamese textbooks	German textbooks	
		Nordhein Westfalen textbooks	Bayern textbooks
Inductive approach	33.33%	88.46%	68.42%
Deductive approach	66.67%	11.54%	31.58%

From the Table 5, it is realized that Vietnamese textbooks mainly used the deductive approach to present the probabilities and statistics content (66.67% concepts, rules, and formulas were presented by this way), while the inductive approach was dominated in introducing probability and statistics content in the German textbooks (88.46% concepts, rules, and formulas in the Nordhein Westfalen series and 68.42% of them in the Bayern textbooks were presented by the inductive approach). For example, German and Vietnamese textbooks had different ways of presenting the concept of *average value* in descriptive statistics: In the Vietnamese textbook, the definition of *average value* was introduced first, then an example to calculate average length of 74 leaves was given (see Doan et al. (2009a)). This deductive approach is different from the inductive approach used by German textbooks, where the definition of *average value* was introduced gradually by asking students to determine the average length of school time of 80 high school graduate students, average number of children in a household, the average household's monthly income on the basis of given data tables. Through these activities, students can explore the formula of *average value*, and this leads to the definition of *average value* (see Griesel and Postel (2003)). To compare the inductive and deductive approach, Singh (2017) showed that inductive approach is a method of constructing knowledge with the help of reasonable number of concrete examples, which leads students from examples to general rule, or from concrete to abstract. On the contrary, deductive approach brings learners from general to particular, from formula to example. In this method, students do not need to explore knowledge, they have to perform only calculations or substitutions of known values in the given formula (Singh, 2017). According to Park and Leung (2006) inductive approach takes into consideration student's psychological state and different levels of

understanding and allows students to familiarize themselves with new concepts step by step, in a rather easy and natural manner whereas deductive approach forces students to accept the knowledge without any prior evidence.

4.2.2. Context Relating the Content

a) Context Used to Introduce the Content

The language used for the presentation of knowledge is not introduced arbitrarily. On the contrary, the topic is mentioned in contexts of content. The context used in Vietnamese textbooks to introduce probability and statistics contents was often more simple and shorter than in German textbooks. For example, German and Vietnamese textbooks had different contexts of mentioning the concept of *average value* in descriptive statistics:

In Germany textbook, three paragraphs of newspapers in real life were given, after that the concept of *average value* was mentioned:

Paragraph 1: **Household income per year**

In Western Germany, the household income per year is 55100 DM on average, which range from social assistance receivers (average value is 26200 DM) to self-employed (average value is 128300 DM).

Paragraph 2: **Life expectancy higher in girls**

The average life expectancy in Germany is 72.8 years in newborn Boys while it is 79.3 years in newborn girls.

Paragraph 3: **Cost of living increases**

From 1991 to 1995 the cost of living rose in Western Germany on Average by 12.5%; the most strongly increase is the rents (21.3%), the lowest is energy costs (4.9%).

In all three newspaper cuttings, *average values* indicated:

- Average values summarize frequency distributions in a single number.
- They give important information about a considered quantitative property.
- They are easily comprehensible and representative.
- They allow rough comparisons ("Women live longer than men"), it is although clear to everybody that such comparisons do not have to be correct in the individual case.

In this section we deal with different types of average values, their calculation and their properties (Griesel & Postel, 2003) translated from German by the author). In Vietnamese textbooks, the concept of *average value* was introduced after only one simple sentence: "To quickly grasp important information contained in data samples, we work with some numbers called typical values of the data sample" (Doan et al., 2009a). It is possible to realize that the above context used in German textbook is in more details and more natural than in Vietnamese textbook. The context in German textbook gives students a general picture of the concept which they are going to learn and its specific applications in their real life. It helps students to understand how this concept is important and why they should learn it. In contrast, the context used in Vietnamese textbook is in an abstract way. With this context, students have no imagination about the new concept and cannot understand why it is important. In Vietnamese textbooks even some concepts and rules were introduced directly in pure mathematical statements, symbols and formulas such as concept of *expectation*, *variance* and *standard deviation* of a *discrete random variable*, basic rules for calculations of probabilities.

b) Realistic Contexts Related to the Content

Table 6 shows the average number of realistic situations used in German and Vietnamese textbooks to introduce or demonstrate concepts and rules of probability and statistics in each lesson. From this table, the differences between Vietnamese and German textbooks can be recognized.

Table 6. Average number of realistic situations used in German and Vietnamese textbooks per lesson.

Text books series / Approaches	Vietnamese textbooks	German textbooks	
		Nordhein Westfalen textbooks	Bayern textbooks
Average number of realistic situations per lesson	3.89	6.06	7.00

While in German textbooks many basic concepts and rules of probabilities and statistics were approached through various realistic situations, Vietnamese textbooks used minimal realistic situations and concentrated on the mathematical content itself. For example, to present the concept of "random test", the Vietnamese textbooks used only situations of rolling a die and throwing a coin (Doan et al., 2009b) whereas the German textbooks used various realistic situations, such as rolling a die, throwing a coin, playing card, Lotto game, Roulette game, gender of a new born child, HIV test, efficacy of a new drug test (Feuerpfeil & Heigl, 2003).

4.2.3. Tools to Consolidate and Deepen Content

To consolidate or deepen the probability and statistics content, Vietnamese textbooks used examples, explanations, and exercises whereas German textbooks utilized more various tools. Besides examples, explanation and exercises, German textbooks used other appropriate tools including project work, simulation and technology.

a) Project Works

While no project work was found in Vietnamese textbooks, several such works existed in the part of *descriptive statistics* in German textbooks. Each project is aimed to strengthen and consolidate the knowledge of some concept or skill in *descriptive statistics*. For example:

- Project about surveys of optional course chosen by students in one class, choice of foreign language, music trend, attractive TV channels of the youth (Griesel & Postel, 2004) is to help students practice and consolidate the ability of collecting data, determining *absolute frequency* and drawing graphs.
- Project about surveys of pocket money per month of the youth and how much they pay for going to the cinema (Griesel & Postel, 2004) is to help students practice and reinforce the ability of dividing data in groups, determining the distribution of grouped data.
- Project about surveys of traffic payment, pocket money per month, time students spend watching TV ... (Griesel & Postel, 2004) helps students deepen and consolidate their knowledge of *average value*.
- Project about surveys of the heights of students, pocket money per month, popularity ranking of subjects... (Griesel & Postel, 2004) helps students strengthen and reinforce their knowledge of *median*.
- Project about surveys of relationship between heights and elbow lengths, heights and span of arms, shoe sizes and hand spans, heights and waist measurements... Griesel and Postel (2004) helps students deepen and consolidate their knowledge of *linear regression* and *correlation*.

In each project there are oriented-questions and guidelines to help students understand what they have to do. These questions and guidelines also express the aim of the project. For instance, when the aim of the project is to deepen and reinforce the ability of dividing data in groups and determining the distribution of grouped data, the following questions and guidelines are given:

(1) How much pocket money do teenagers receive per month? Implement a survey in your class and determine the distribution for the following groups: Less than 15€, 15 to less than 20€, 20 to less than 25€, 25 to less than 30€, 30 to less than 40€, 40 to less than 75€, 75€ and more.

(2) How much money do teenagers pay for the cinema? Implement a survey in your class and then divide the data into groups. (Griesel & Postel, 2003) translated from German by the author).

In teaching statistics, there is a focus on statistical literacy and reasoning, which is the ability to interpret, critically evaluate, and communicate about statistical information and message. In order to promote statistical reasoning, project –based learning environment is encouraged in the teaching process (Ben-Zvi, 2014). Moreover, using project works is an appropriate method for teaching the probability and statistics contents because it encourages co-operative learning, activates students and helps them understand applications of probability and statistics knowledge in real life.

b) Simulation and Technology

Modeling is encouraged in teaching probability (Chaput, Girard, & Henry, 2011; Eichler & Vogel, 2014; Prodromou, 2014). Simulation is one of the fundamental probabilistic knowledge that play a key role in the history of probability and is the basis for the modern theory of probability (Batanero, Chernoff, Engel, Lee, & Sánchez, 2016; Heitele, 1975; Watkins, 1981). Simulation provides an opportunity for practical experience and experiment for students (Everton, 1994). It can reveal empirical solutions to many probability problems (Inhelder, 1981). Some probability and statistics experiments (e.g., experiment of tossing a die to find the approximately estimated values for the probabilities of the appearance of each side 1,2,3,4,5,6) require so many repetitions that students find difficult in performing them. Such kind of experiment is executed better by simulation. According to Bryan (1989) simulation is an ideal mechanism for providing the teacher with the opportunity to develop a systematic progression from estimating probabilities to drawing conclusions and making inferences (p.99). Shaughnessy (1981) recommended that the study of probability and statistics in the schools should emphasis on simulation, both as a tool to model experiments and as a problem-solving technique (p.96). Inhelder (1981) concluded that computer simulation can help students discover and develop probabilistic truths in a realistic problem. He emphasized that without computer simulation, gathering sufficient experimental data to investigate this problem would be so time-consuming that it would not be feasible for the classroom (p.224). German textbooks agree with the aforementioned ideas when they required and showed simulations of some probability and statistics situations by using computer programs, random equipment or software. More specifically, the following are simulations in German textbooks:

- Simulation of throwing a dice by using Pascal program (Feuerpfeil & Heigl, 2003).
- Simulation of the Galton board with 14 rows by using Pascal program (Feuerpfeil & Heigl, 2003).
- Monte Carlo simulation with support of Pascal program or pocket calculator (Feuerpfeil & Heigl, 2003).
- Simulation of tossing a coin by using Excel software (Griesel & Postel, 2004).
- Simulation of rolling a die by using CAS- calculator (Griesel & Postel, 2004).
- Simulation of binomial distribution by using Excel software (Griesel & Postel, 2004).

In contrast to German textbooks, Vietnamese textbooks did not consider the idea of simulation. No simulation of probability and statistics situation was found in Vietnamese textbooks.

5. SUMMARY AND CONCLUSION

Comparisons of probability and statistics curricula between Vietnamese and German textbooks showed similarities and differences in the content itself and in the way it is mentioned, presented, consolidated and deepened. The probability and statistics content in Vietnamese textbooks was much less than in German textbooks. All topics in Vietnamese textbooks were included in German textbooks but many topics in 'German textbooks were not covered in Vietnamese textbooks, such as topics of conditional probability, Bayes formula, Bernouli experiment, Binomial distribution and inference statistics. There was also a difference between Vietnamese and German textbooks in the number of approaches to probability concept. Textbooks of the state Bayern introduced and paid due attention to three different conceptions of probability: the classical, statistical and axiomatic one. Textbooks of the state Nordrhein-Westfalen introduced and focused on the classical and statistical conceptions of probability whereas Vietnamese textbooks introduced the classical and statistical conceptions they only concentrated on the

classical conception of probability. According to Koirala (1998) "the teaching of probability only through one approach is harmful for students' conceptual understanding" (p.141). Chaput et al. (2011) have the same opinion as of Koirala (1998) when they emphasized that focusing only on the classical approach led to reducing the understanding of probability to the counting of elementary events and to the overuse of combinatory; but focusing only on the statistical approach generates didactic issues like the confusion between the observation of reality and theoretical knowledge. Therefore, using a multiplicity of approaches in teaching probability is encouraged because each approach has its own advantages and using a variety of approaches helps students to develop a conceptual understanding of mathematics (Chaput et al., 2011; Hawkins & Kapadia, 1984; Koirala, 1998; Shaughnessy, 1992). The three approaches need to be blended together to provide students with an appropriate frame work for formal probability, rather than merely focus on one or two approaches which may well conflict with the student's expectation and intuitions (Hawkins & Kapadia, 1984). Varieties of problems that can be solved by using different approaches should be provided, the strengths and weakness of each conception should be discussed so that students have preparation in solving varieties of problems in future (Koirala, 1998).

German textbooks introduced various types of representing data whereas Vietnamese textbooks mentioned only the most common types. Moreover, while Vietnamese textbooks had no explanation of how to use each type of graph, German textbooks had many discussions of features of each type of graph, when each type is effective. Latour (1990) concluded that graphs are powerful due to the fact that they are easily understood, easily transported and reproduced, and most importantly, extremely persuasive. The effective use of graphs is an essential tool for communicating research results (Cleveland, 1984; Hulsizer & Woolf, 2009; Pittenger, 1995). Therefore, introducing limited types of graphs without any further discussion of how to use them effectively like Vietnamese textbooks can limit students in understanding applications of graphs in real life. The contexts relating the contents in German textbooks are often more details and more natural than in Vietnamese textbooks. In Vietnamese textbooks even some concepts and rules were introduced directly in pure mathematical statements, symbols and formulas. Such a decontextualized way is economic but fails to motivate students to learn. On the other hand, mathematics is born out of human activities, and so it is fitting that mathematics be learned "in context" (Park & Leung, 2006).

German textbooks used inductive approach in which various activities and explorations in a realistic context are given first to encourage students to adapt gradually to new knowledge. Vietnamese textbooks often used a deductive approach to force students to accept new knowledge without any prior evidence. An inductive approach wastes more time than a deductive approach but it constructs knowledge in a natural manner.

Vietnamese textbooks used minimal realistic situations to approach concepts and rules of probability and statistics. On the contrary, in German textbooks many basic concepts, rules of probabilities and statistics were introduced or explored through various realistic situations. One of the most important aims of teaching probability and statistics is to help students know how to solve various problems in real life. This can be achieved most effectively by putting students in realistic contexts. Dinges (1982) emphasized that in the teaching of probability and statistics, applications should not be mere illustrations suitable for motivation. Rather, they should provide an opportunity to use mathematics to understand more precisely aspects of realistic situations (p. 50).

German textbooks encouraged co-operative learning by introducing some project works in the part *descriptive statistics*. Each project in German textbooks is aimed to deepen and consolidate the knowledge of some concept or skill in *descriptive statistics*. Unlike German textbooks, there was no project work in Vietnamese textbooks although this kind of work has many advantages: like giving students the opportunity to explore real world problems and challenges, improving their ability in co-operative learning, and developing their statistical literacy and reasoning.

Many educators confirmed the essential role of simulation in the study of probability and statistics curricula in schools. As Wolpers and Götz (2002) mentioned: "Experience of probability and statistics situations is absolutely required to be able to develop their models or to understand the models, and such experience can be best achieved by simulations" (p.204). The German textbooks agree with these opinions when they introduced simulations of

probability and statistics situations with the support of computer programs, random equipment or software. In contrast to German textbooks, no simulation of probability and statistics situation was found in the Vietnamese textbooks. The absence of simulation in Vietnamese textbooks has also removed the opportunity to collect "direct experimental experiences" of students. In general, the differences in the probability and statistics contents in the high school mathematics textbooks of Vietnam and Germany come from the different views on the nature of mathematics: "naked mathematics" (Vietnamese textbooks) and "dress-up mathematics" (German textbooks) (see Park and Leung (2006)). The Vietnamese textbooks focused on the mathematical content itself whereas the German textbooks focused on applications of the content in real life. In Vietnamese textbooks, probability and statistics knowledge was considered as the absolute truth and was presented by combinations of concepts, symbols and rules in simple and short contexts. Vietnamese students tend to be forced to accept the knowledge through deductive approaches. On the contrary, German textbooks had the view that abstract mathematical knowledge need to be "dressed up" in realistic situation to return them to their true nature, because mathematics is origin from real life. For this reason, probability and statistics knowledge in German textbooks was presented by inductive approach through various realistic situations. German students are encouraged to explore the knowledge in a natural way.

In the Vietnamese side, the comparison showed advantages in probability and statistics contents of the German textbooks that the Vietnamese textbooks can learn for the development of future textbooks. More specifically, in future the Vietnamese textbooks should consider about supplementing necessary probability and statistics contents which have many applications in real life such as conditional probability, Bayes formula, Bernoulli experiment, Binomial distribution and inference statistics. The Vietnamese textbooks should focus on not only classical probability but also statistical and axiomatic conceptions of probability. The number of realistic situations should be increased and the inductive approach should be used more in exploring and introducing probability and statistics content. Project work, simulation and technology should be utilized in Vietnamese textbooks as appropriate tools to consolidate and deepen probability and statistics knowledge.

Funding: This study received no specific financial support.
Competing Interests: The author declares that there are no conflicts of interests regarding the publication of this paper.

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