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Constructivism-based visual instructions for students learning the concept of a continuous function at a point

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ABSTRACT

The concept of a continuous function at a point involves a key calculus idea, to help students learn about the content in calculus. Therefore, developing a learning environment where students can independently build concepts and related knowledge when learning this topic is crucial. Previous studies show challenges for teachers in designing lessons, however, there exist fewer number of studies discussing effective solutions to support teachers. This study fills this gap through the exploration of constructivism based visual instructions. This study employed an action research methodology involving 12 pre-service teachers at Sai Gon University, organized into three stages namely identifying issues when teaching the lesson of "a continuous function at a point", exploring solutions, and finally discussing the best application. The findings demonstrated that using visual models in teaching helped students understand and verify hypotheses, reject wrong ones, and construct new knowledge about the continuous function at a point in an effective way. Four perspectives—numerical, graphical, linguistic, and algebraic—were used during the teaching process to solve the challenge of learning the concept of a continuous function at a point.

Contribution/Originality: This study contributed the use of visual models in teaching students understand to verify hypotheses, reject wrong ones, and construct new knowledge about the continuous function. The students could now learn how to develop a learning environment where they could independently build concepts and related knowledge. A major contribution in this study was to provide a teaching process and specific examples, to suggest a small model in constructivist teaching so that it can be applied to other subjects as well.

1. INTRODUCTION

The concept of "a continuous function at a point" is important in the high school Mathematics program for Calculus. When presenting this concept to students in mathematics, Vietnamese textbooks typically introduce the definition before presenting illustrative examples. The definition of this concept is fairly simple, but the question is how teachers should organize the teaching and learning activities so that students should be able to form and be aware of the conceptual understanding for them. Teaching practice shows that using visual models can help students construct their knowledge of a continuous function at a point more easily. However, there exists a lack of studies in the Vietnamese context to clearly describe the process of teaching and learning with the lesson of "a

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function continuous at a point". Consequently, teachers have reported challenges when teaching this lesson, like feeling uncomfortable when reading the existing textbook of instruction and requesting innovative teaching pedagogies for this crucial lesson to ensure student learning outcomes. Furthermore, when students express dissatisfaction and complain about simple teaching process for complicated knowledge resulting in frequent mistakes when practicing exercises or doing exams, this lesson needs to be concerned with innovative products in pedagogies for enhancing the student learning outcomes. To solve this problem, there is a need to conduct research to understand deeply the student learning process when they study the lesson of "a continuous function at a point" and based on this result, a model of teaching and learning for convenient application will be developed and an explanation on how to use it in the classroom.

To reach the objectives, this paper focused on the following two research questions:

1) How can students construct knowledge of the concept of a continuous function at a point effectively?

2) What model teachers can practice in guiding students to discover the continuous function?

2. LITERATURE REVIEW

2.1. Constructivism Theories in Teaching and Learning

Constructivist theory is widely accepted as a major theory in education science, especially in teaching and learning. Constructivism has become the most valuable guiding principle for science teachers as well as for researchers in mathematics education. Proulx (2006) stated some potential implications of constructivism for teaching and pedagogy, as follows:

2.1.1. Prior Knowledge

The learner is not a blank canvas for a constructivist since they interpret and use new experiences concerning their prior knowledge. Everything is interpreted in terms of how it functions and how it relates to this prior knowledge and experiences, which are fundamental to the learning event. This implies the significance of incorporating this knowledge into instruction.

2.1.2. The Learner Plays a Role in the Communication

For constructivists, learners are neither blank slates nor passive sponges in accumulating knowledge because knowledge is perceived as actively construed by the learner, only explaining notions and concepts will not automatically make the learner understand a concept. A distinct view of communication is provided by constructivism because the presence of a receiver is explicitly acknowledged, rather than just the sender and the receiver's central role in understanding the message transmitted, since it is a separate concept of communication. This acknowledgment has a significant impact on the teacher's position in terms of speaking, acting, listening, and other behaviors.

There is a shift away from miraculous recipes that guarantee comprehension toward "attempts" and tentativeness that may hold for potentially meaningful creations. It entails taking into account the presence of an active receiver, which alerting a message to the sender signifying the role and presence of the learner's prior knowledge and significance of words and sentences uttered when teaching.

2.1.3. The Meaning of Mistakes

Constructivists value mistakes because they are a crucial component of the adaptive process that characterizes learning. Constructivists believe that mistakes present abundant chances for advancing learning and understanding. Mistakes made by learners are not regarded as failures or negative actions; rather, they are seen as a normal component of learning and a chance to learn.

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Learning from mistakes is not the same as simply correcting errors. It means that we are studying the error and almost relishing it as a learning opportunity (Jalongo, 1991). Mistakes greatly inform the learning process and allow for a better understanding of the domain or concepts under consideration.

2.1.4. Interest in Creativity and Invention

Teachers become interested in students' knowledge creation when mistakes are seen as acceptable (in the sense that they are not seen as failures) and when knowledge is seen as helpful and functioning. In essence, they are interested in repeating or simply reproducing what has been done and what is already standardized and creating a space where students can express their creativity and "inventions."

The term creation is used here in the sense defined by Jalongo (1991), that is, the ability to generate new ideas through the combination or juxtaposition of previously existing elements. The idea is not that students must reinvent the wheel or redo, reconstrue, and reinvent everything because there is no curriculum or knowledge already established; rather, teachers must demonstrate an openness to the development and production of knowledge from their students, "an opening of new spaces of possibilities by exploring current spaces" (Davis, 2004).

Inquiring about things that are taken for granted, simplifying certain features of concepts, and so forth are all common behaviors of learners. These concepts help teachers develop an interest in students' productions and construal since students frequently use unexpected methods to incorporate and apply pre-existing, standardized knowledge (Desautels & Larochelle, 2004). This suggests that, rather than being seen as "things of replication," students are seen as "subjects of production."

2.1.5. The Importance of Verbalization

For a constructivist, the act of verbalization — putting explications into words — is central for the individual to help him or her in the knowledge construal and the meaning-making creation. When someone explains what they understand, they highlight the significance and the strength of what they claim, become aware of any inconsistencies and potential flaws in what they are currently explaining, and have the opportunity to forge new connections with other ideas: "The act of verbalization requires a review of what is to be verbalized" (Von Glasersfeld, 2001). Since verbalizing necessitates ongoing evaluation, analysis, and development of what is presented, it can be seen as a learning opportunity. In terms of education, a pedagogy that is affected by this perspective may have a great desire to have students talk so they may clarify what they understood and how they arrived at that understanding. The teacher's objective now becomes dual: not only to check for understanding and engage with students' learning but also to give students the tools they need to continue learning, i.e., to give them continual learning opportunities.

2.2 Constructivism-Based Learning

All human cognition, in Kant's words, "begins with observations, moves on to conceptions, and culminates in ideas" (Polya, 1965). The words observation, conception, and notion were used in this statement. "Learning begins with action and perception, goes from there to words and concepts, and should culminate in beneficial mental habits," is how Polya (1965) rephrased the original line (p.103). As a result, understanding a notion involves three stages including three stages such as exploration, formalization, and assimilation.

An initial exploratory phase proceeds more intuitively and heuristically and is closer to action and perception. A second formalizing step introduces vocabulary, concepts, and proofs while moving up to a more conceptual level. The final stage is assimilation, which involves making an effort to understand the "fundamental ground" of things. The knowledge acquired during this phase prepares the learner for applications on one hand and greater generalizations on the other by mentally digesting what was learned. "For effective learning, an exploratory phase

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should precede the phase of verbalization and idea construction, and, subsequently, the material learned should be integrated into and contribute to the learner's integral mental attitude" (p. 104).

Teachers employ a variety of mathematical representations to put the aforementioned principle into practice and help students understand basic mathematical ideas. Multiple representational modalities, according to Vui (2009), "improve transitions from concrete manipulation to complex thought, and offer a framework for knowledge building" (p.2). In the teaching of mathematics, visual representation is essential. "Thinking through what is represented (as a thinking method), recording what was thought through the representations (as a recording method), and an essential factor for communication" are these important functions, respectively (p.3).

Arcavi (2003) put forth the following definition of visualization: "Visualization is the ability, the process, and the result of creation, interpretation, use, and reflection upon pictures, images, and diagrams, in our minds, on paper, or with technological tools, to depict and communicate information, thinking about and developing previously unrealized ideas, and advancing understandings" (p. 217).

Mathematical concepts can be explored using a computer, which is a rich source of computational and visual pictures. Tall (1991) provides more information on these concepts, highlighting the significance of visualization in calculus, outlining the advantages and disadvantages of visual pictures, and suggesting a computer-based graphical approach to calculus. According to him, to reject visualization is to reject the foundations of many of our most important mathematical concepts. Visualization was a key source of inspiration in the early phases of the creation of the theory of functions, limits, continuity, and related fields. Students would be cut off from the subject's historical roots if these concepts were denied to them (p.105).

It is crucial to establish the right conditions to support students' "self-discovery" of the continuous function and related information when organizing instruction within a broad constructivist framework. Therefore, any "finding" should be located in a learning environment with pedagogical aims properly organized by the teacher to make the construction of knowledge successful and to achieve high results without consuming too much time.

3. RESEARCH METHODOLOGY

This study employed an action research methodology. The action research design approach fulfills the requirements to reach the research objectives regarding teacher professional development. According to Feldman, Rearick, and Weiss (2001), action research aims to improve practice and understanding of educational situations in which practice is embedded, as well as professional development for educators. In this study, the educational situation was improved by teaching pedagogies for the lesson of “a continuous function at a point” (Mathematics curriculum - grade 11 in Vietnam). This action research methodology was implemented through collaborative conversations (Feldman, 1999; Hollingsworth, 1994) and enhanced normal practice mechanisms in collaborative action research groups (Feldman, 1999). This study includes three stages as follows: planning; implementation and monitoring; reflection. The planning stage was organized at Saigon University, with 12 pre-service teachers in Mathematics. In the third year, Saigon University had a meeting and discussion of experiences in teaching and learning the lesson of “a continuous function at a point”. A self-developed tool was used in conversation to identify what challenges for teachers in teaching and the barriers a student has when learning. What learning outcome do students need to achieve after finishing the period of learning in the classroom? What pedagogies should be used for students to facilitate conceptual understanding clearly because this natural knowledge is abstract and complicated for students in comprehensive? After discussion, the learning activity forms were designed to apply in teaching practice in the real class in the school. The pre-service teacher who does teaching practice needs to predict in response to the students’ incorrect answers and focus on the important key conceptions of the continuous function, not focusing too much on general teaching strategies or overall descriptions of the continuous function. Besides that, she or he should give concrete examples that are familiar and easy for students to understand to help them understand the continuous function.

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The implementation and monitoring stage was undertaken on the students in Grade 11. This class of 32 students was divided into eight groups of four students. Each group discussed and completed the tasks, and wrote the results on a hand out. In this section, we focused on four outcomes based on research by Nam and Stephens (2013): interactions among students, teachers' support for students to face difficulties, teachers' treatment of correct results, and teachers' response to incorrect results. When the pre-service teachers came together to do their practice teaching, other pre-service teachers listened to and observed the class and responded in several ways, took notes, and prepared opinions for the next meeting.

The reflection stage started with a meeting organized at Saigon University. All pre-service teacher who participated in the classroom shared their opinions how they improved the lesson design through these activities, how they gained emotion in teaching, confidence and how they would apply this knowledge in changing their class with the direction of active teaching for student learning and development effectively (Skovsmose & Borba, 2004).

4. RESULTS

4.1. Constructivism-Based Lesson Design

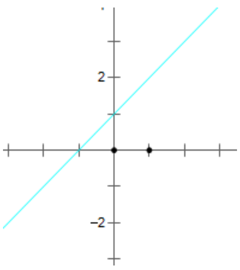
According to Nam and Stephens (2013), it is important to choose appropriate mathematical tasks and activities for students where: the first task (1) was designed to actively engage students in mathematical thinking; and the second (2) task took into account students' previous knowledge and experiences.

Task 1:

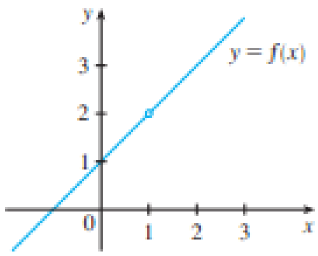
This work aimed at two objectives. The first was that students should understand concepts visually. The second was that from the graphs of functions, they could see the results immediately. This approach helped students save time studying. Such visual understanding of concepts was the important premise that students would proceed to construct a formal definition of the concept. During the implementation process, students obtained certain ideas about the concept of continuous function at a point and related knowledge.

Task 1 Given four functions and their corresponding graphs as below:

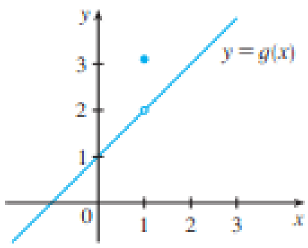
a) $h(x) = x+1$



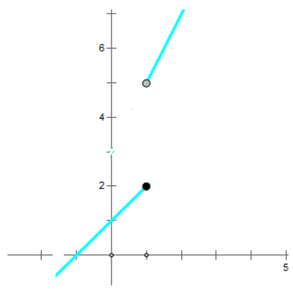
b) $f(x) = \dfrac{x^2-1}{x-1}$



c) $g(x) = \begin{cases} \dfrac{x^2-1}{x-1} & if \quad x \neq 1 \\ 3 \quad if \quad x = 1 \end{cases}$



d) $g(x) = \begin{cases} x+1 \ \ if \quad x \leq 1 \\ 2x+3 \ \ if \quad x > 1 \end{cases}$



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Look at the graphs and answer the following questions:

Question 1: Define $\lim_{x \to 1} f(x)$ each case.

Question 2: In Case *c*, what number should replace 3 to make the graph uninterrupted?

Question 3: What causes the graph to be broken in Case *c*? What conditions should the functions in Cases *b*, *c*, and *d* satisfy for their graphs to be uninterrupted?

After the students constructed the concept of continuous function, they were asked to implement Task 2.

Task 2 provided them an opportunity to present the said ideas which were designed for students to express their understanding and also to help them to connect previously learned knowledge with new knowledge and create a premise for discoveries about the learned knowledge.

Task 2: Explanation of the concept of continuous functions at a point effectively.

Question 1. What does "$f(x)$ continuous at point x_0" mean?

Question 2. When is $f(x)$ discontinuous at the point x_0?

Question 3. Use the sign "$\Rightarrow, \Leftrightarrow$" to express the relationship between the concepts: continuous function at x_0, the limit of the function $f(x)$ as *x* gradually tends to x_0 limit of sequence $f(x_n), x_n$ as gradually tends to x_0. Then give your ideas about the previous knowledge.

4.2. Visual Instructions

4.2.1. Interactions among Students

Following up on the group work, most of the students were found to actively work to get their group's joint results. The visual images were fairly clear and therefore caused no obstacle for students to understand the models.

4.2.2. Teachers Support Students to Face Difficulties

For Task 1, in the experimental process to address Question 1, there was one group that was at a loss to remember the definition of *limit*. The reason was that the group members were familiar with the calculation of a concrete limit of a function. In this case, the researcher asked: "From the graph, what number does $f(x)$ tend to as *x* tends to 1?" Some of the members faced difficulties in answering Question 3. In that case, the teacher might support students by asking them to compare two of the graphs: *a* and *b*, *a* and *c*, *a* and *d*.

In the implementation of Task 2, for Question 1, when some students faced difficulty in answering, the researcher put the following supporting question: "Can you express the definition of this concept in another way?" For Question 3 in Task 2, some groups were unable to connect the concepts, so the researcher suggested asking more questions or offering additional requirements to support students, such as: "What is a sufficient condition for $f(x)$ to be a continuous function at the point x_0?", "What is a sufficient condition for the limit $f(x)$ to exist when *x* tends to x_0?", "What is the necessary condition $f(x)$ that has the limit when *x* tends to x_0? Is it a necessary condition for $f(x)$ to be continuous at x_0?"

4.2.3. Teacher's Treatment of Correct Results

For Task 1, many correct results were given by the groups. When asked to explain the results, some groups' responses were based on the graph, whereas other groups used theorems on the calculation of limits to provide the reasons. For Question 2, all of the groups decided to replace 3 with 2 so the graph would not be interrupted. When asked to explain, one group could not answer; the remaining groups stated, "Such a replacement will fill the hole in the graph, then the graph would not be interrupted". For Question 3, all groups stated that the reason the graph was interrupted was that "the function was not defined at $x=1$". 6 groups identified the cause as "$\lim_{x \to 1} f(x) \neq f(1)$", and there were 4 groups that said, "because the limit of $f(x)$ when x tends to 1 does not exist".

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For Task 2, in the experimental process to answer Question 1, some groups said, "$f(x)$ continuous at point x_0 means $f(x)$ is close to $f(x_0)$ if x is close to x_0". This result connected the concept of a continuous function with the concept of the limit of a function. One group said, "The left and right limits exist, and $\lim_{x\to x_0^-} f(x) = \lim_{x\to x_0^+} f(x) = f(a)$". This result connected the concept of a continuous function with the one-sided limit of a function. Another group said, "Since $\lim_{x\to x_0} x = x_0$, the condition $\lim_{x\to x_0} f(x) = f(x_0)$ means that $\lim_{x\to x_0} f(x) = f(\lim_{x\to x_0} x)$". When asked to give the meaning of this result, one group said, "If we consider $f(x)$ as an operation, f acts on x, $\lim_{x\to x_0} f(x) = f\left(\lim_{x\to x_0} x\right)$ means that the operation of taking limit commutes; that is, the order of taking f and taking the limit can be interchanged".

This result gave students an important idea to solve problems. For Question 2, some results that were given were as follows: " $f(x)$ is undefined at x_0", $\lim_{x\to x_0} f(x)$ does not exist", and " $\lim_{x\to x_0} f(x)$ does not exist but does not equal $f(x_0)$". For Question 3, some groups linked only the concept of continuous function at a point to the limit of the function at a point. Some groups could link the conditions of a function to be definite at a point: "$f(x)$ continuous at $x_0 \Longrightarrow f(x)$ has a limit at the point $x_0 \Longrightarrow$ every sequence $(x_n), lim x_n = x_0$ then $lim f(x_n) = f(x_0) \Longrightarrow$ $f(x)$ defined at all points belonging to a neighborhood of x_0 (including the point x_0)". There was one group that gave the following results: "$f(x)$ has a limit when x gradually tends to $x_0 \Longrightarrow$ every sequence $(x_n), lim x_n = x_0$ then $lim f(x_n) = f(x_0)$". In this case, the students were asked to try checking whether the reverse was true. This result did help the students recognize the important ideas in calculus, which was the transition to the limit for continuous functions. These ideas helped the students solve many difficult problems in the Olympiad exams. The problems may not be only in calculus, but also in algebra, geometry, and arithmetic.

4.2.4. Teachers Responding to Incorrect Results

When implementing Task 1, in answering question 1, one group stated that cases b, c, and d had no limit to exist when x gradually tended to be 1. Being asked to explain, the group said that "because the function undefined at x=1. To help students recognize these mistakes, we asked them to answer the question: "when moving gradually *from* x to 1, would $f(x)$ tend to whatever number?". The purpose of this question was to help students to realize their mistakes from the visual images. Then, we requested students to repeat the concept of the finite limit of a function when x gradually tends to a. Such a request was to help students to remember that, when considering the limit of a function as x gradually tends to a, then whether the function is defined as not important and the function would still reach a finite limit when x gradually tends to a value which does not belong to the definition domain of the function.

In answering questions of Task 2, some groups gave wrong results as follows: " (x) continuous at the point x_0 $\Leftrightarrow$ $f(x)$ has a limit at the point x_0". To help students recognize this mistake, we requested them to check the results in cases b, and c. " (x) continuous at $x_0 \Longrightarrow f(x)$ has a limit at the point x_0 and is defined at that point". To help students to recognize the mistake, we requested them to check the results in the case c graph. " (x) has a limit when x gradually tends to $x_0 \Longrightarrow f(x)$ defined at the point x_0". To help students to recognize this mistake, teachers requested them to check the results in the case c graph.

5. DISCUSSION

This study aimed to examine the effectiveness of the application of the constructivism-based visual in teaching the concept of a continuous function at point for Vietnamese students. The most notable feature in research findings

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showed that when teachers used the constructivism-based visual pedagogy, students were able to self-construct the new knowledge regarding the function at a point and graphing the function (in Task 1). Furthermore, Task 2 created conditions for students to make new discoveries, such as recognizing the important thought in calculus, which was turning the limit. This assisted students in developing knowledge from textbook basics. Teachers also understood how to organize learning tasks that encouraged students to actively participate in the creation of new knowledge. This finding supported previous studies like Jalongo (1991); Proulx (2006), when emphasizing the process of constructing new knowledge in teaching and learning.

During the research process, it was discovered that the most difficult aspect of applying constructivist theory was designing activities that were appropriate for students' abilities. Each student had a different capacity; in order to work effectively with students, teachers must first understand them, what they already know and what they can do next. Furthermore, suggestions when students were having difficulties and responses when students made mistakes were required and necessitated pedagogical support as well as flexibility on the part of teachers.

Furthermore, time was limited, and not all students could quickly reach out to the teacher for assistance. They also could not receive timely guidance, which left them bored, confused, and uninterested in learning. However, this was a small number that necessitated a combination of factors. This result was different from previous studies, which may be explained because the number of students in Vietnamese class was quite big and one teacher could not manage everything.

The lesson of "continuous function" was selected in this study as a representative function because it is linked to many important concepts in the Mathematic curriculum, such as the limit of the series, the limit of the function, and the graph of the function. In particular, with the help of function graphs and images, teachers facilitated students' construction, whereas continuous function knowledge helped them understand the expression in analytical language "solid graphs", where "the graph was interrupted at a point". The understanding of continuous functions established a logical link between the limit of a sequence, the limit of a function, and a continuous function, resulting in an important idea in analysis known as the transition to the limit. This concept also aided in problem solving. Teaching according to constructivist theory is not new in comparison to previous studies, but the new point in this study was to provide a teaching process and specific examples, so this can be considered a small model in constructivist teaching so that it can be applied to other subjects and topics.

6. CONCLUSION

Generally, understanding the nature of the concept of a continuous function at a point is a necessary prerequisite for mastering later topics in analysis, such as the concept of a continuous function on an open or closed interval. To address this challenge, four perspectives—numerical, graphical, linguistic, and algebraic—were used during the teaching process. Furthermore, constructivism-based learning and constructivism-based lesson design, which relied on and supported visual models, enabled students to investigate important mathematical ideas associated with the concept of the continuous function. More practice and the chance to demonstrate their skills allowed the students to accurately forecast the qualities of the idea of a limit as well as develop their sound comprehension of it. Through these experimental activities, concepts for solving problems such as applying the definition of a continuous function to determine a limit and limit transition commutes for continuous functions were also developed. Along with the right responses, some were inaccurate or lacking. These gave teachers the chance to act appropriately to assist pupils in gaining the proper understanding and avoiding misconceptions. A visual model is a crucial tool for teaching and understanding abstract ideas. Follow-up research could be done to assess how well students learned from this session while teachers can make improvements in their teaching methods.

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