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Manipulative interactive didactics with puzzles: An effective solution for overcoming junior high school students' learning obstacles to quadrilateral materials

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ABSTRACT

The study of learning obstacles is essential to develop students' competencies and improve the quality of learning that is responsive, inclusive, and relevant in the 21st century. The purpose of this study is to identify learning obstacles related to misconceptions regarding quadrilateral material and students' mathematical creativity, and to find effective empirical didactic designs to overcome these learning obstacles. The research methodology used is didactic design research (DDR), which consists of four frameworks: Initial data collection, prospective analysis, metapedidactic analysis, and retrospective analysis. These stages are cyclical. The study involved 172 junior high school students and six model teachers. Data was collected using the quadrilateral writing creativity test, interviews, documents, and field notes. Data analysis was conducted simultaneously using quantitative and qualitative approaches. The study produced empirical didactic designs, namely manipulative interactive didactics with puzzles, which are equipped with a guide for teachers to manage learning. This empirical didactic design includes seven frameworks that have been shown to be effective in overcoming didactic and epistemological barriers—scaffolding, questioning skills, brainstorming, learning opportunities, conceptual representation of mathematical creativity, misconceptions, and concept representation. Teachers can implement or adapt this empirical instructional design for use in geometry subtopics or other mathematics materials. For instance, they can use it for subtopics such as flat shapes with the same area, spaces with the same volume, or spaces with the same surface area.

Contribution/Originality: The empirical analysis found that previous studies have not proceeded to the stage of developing instructional designs and identified learning obstacles related to students' misconceptions and mathematical creativity. This study contributes to the development of learning theories and is an alternative for teachers to implement innovative and interactive mathematics learning.

1. INTRODUCTION

This research was inspired by observations made at a secondary school in Makassar, South Sulawesi, Indonesia. This observation was part of a preliminary research study funded by the Ministry of Education, Culture, Research and Technology, Indonesia. In this observation, students were asked to draw as many quadrilaterals as possible in 15 minutes. The results showed that only three types of quadrilaterals—rectangles, squares and parallelograms—were drawn correctly. Some students drew pentagons and hexagons, indicating learning obstacles related to the concept of quadrilaterals, and barriers in generating different geometric ideas or mathematical creativity in the domain of geometry (Assmus & Fritzlar, 2022; Singer, Voica, & Pelczer, 2017; Siswono, 2010).

Geometry is one of the branches of mathematics that is complex and challenging for students because of its abstract, figurative, and conceptual aspects. As a result, many students experience misconceptions (Gridos, Avgerinos, Mamona-Downs, & Vlachou, 2022; Hohol & Miłkowski, 2019; Siemon, Barkatsas, & Seah, 2019), cognitive conflict between concept description and concept definition (Duval, 2019; Fischbein & Nachlieli, 1998), difficulties distinguishing between important and unimportant geometric attributes (Vinner, 2018) and challenges combining convergent and divergent thinking to generate different ideas or mathematical creativity (Aqda, Hamidi, & Rahimi, 2011; Liu, Pang, Guo, & Zhang, 2022; Tursunai et al., 2019). To overcome the above-mentioned barriers, relevant and appropriate didactics are needed that integrate the concept and figural definitions of geometry (Cesaria & Herman, 2019; Kuzle, 2023; Nurwijayanti, Budiyono, & Fitriana, 2019). This highlights the important role of teachers in overcoming these barriers. However, field findings show that teachers tend to focus only on figural definitions and neglect to emphasize geometric concepts based on their properties. This is in line with the findings of studies conducted by experts including Ulusoy (2021) and Tsamir, Tirosh, Levenson, Barkai, and Tabach (2015) who found that one of the factors contributing to students' learning obstacles in geometry is the inadequate learning design implemented by teachers. Teachers often emphasize the memorization of figural aspects and provide limited opportunities for students to explore their own knowledge. In addition, Hershkowitz, Tabach, and Dreyfus (2017); Beghetto and Sriraman (2017) and Tabach and Friedlander (2013) stated that many mathematics teachers still adhere to traditional procedures that limit students' conceptual understanding, mathematical creativity, and ability to overcome cognitive conflicts. Based on the description above, there is a need for research that can reveal the obstacles experienced by students in learning quadrilaterals. In addition, it is necessary to develop an empirical didactic design (EDD) that can be applied by teachers to overcome the learning obstacles experienced by students. Therefore, this research is important for both aspects. Based on the points that have been presented, the two questions raised in this study are:

1. What are the learning obstacles experienced by students regarding quadrilateral material?
2. How can an effective empirical didactical design overcome students' learning obstacles on quadrilateral material?

2. THEORETICAL FRAMEWORK

2.1. Learning Obstacles

2.1.1. The Concept of Learning Obstacles

Learning barriers refer to various factors or limitations that impede the development of a competency. These barriers come in many forms and affect the ability to understand, remember and apply new knowledge or skills (Prabowo, Suryadi, Dasari, Juandi, & Junaedi, 2022; Rudi, Suryadi, & Rosjanuardi, 2019).

2.1.2. Categories of Learning Obstacles

Experts categorize student learning barriers into three types: (1) Ontogenetic barriers, which arise from individual developmental factors such as learning anxiety, self-confidence, different initial abilities, learning styles, and lack of understanding of the value of education (Fulei, 2010; Suryadi, 2013); (2) Epistemological barriers, which arise from students' conceptual frameworks or epistemological misconceptions around mathematical representation, connections, communication, problem solving in different contexts, and creative and critical thinking in mathematics; and (3) Didactic barriers, which refer to the challenges students face due to the teaching methods used by educators. These barriers include sub-optimal teaching skills, methods, strategies and approaches to learning that may not be suited to students' needs. This category also includes learning resources such as student worksheets, learning media, textbooks, and curriculum (Pratiwi, Herman, & Suryadi, 2019; Subroto & Suryadi, 2018).

2.2. Didactical Design Research (DDR)

2.2.1. Concept of Didactical Design Research (DDR)

DDR is a type of research that aims to identify and overcome learning obstacles through the development of effective didactical strategies (Rudi et al., 2019).

2.2.2. Didactical Design Research (DDR) Paradigm

DDR operates within two research paradigms: (1) The interpretive paradigm, in which the ontological aspect examines the impact of teachers' didactical designs on students' individual development (Margolinas, 2022) and the epistemological perspective focuses on how students assimilate knowledge in a learning context (Daher, Baya'a, & Jaber, 2022); and (2) The critical paradigm, in which the ontological and epistemological elements assess the need for small, large, or fundamental changes, leading to the formulation of a didactic hypothesis framework based on relevant literature (Novotná & Hošpesová, 2022).

2.2.3. Didactic Design Research (DDR) Frameworks

Rudi et al. (2019) proposed three DDR frameworks: (1) Prospective analysis. This stage comprises a thorough analysis of the observational data to identify student learning barriers, a systematic literature review, and the formulation of the empirical didactic design hypothesis (EDDH-1); (2) Metapedidactic analysis. In this phase, the EDDH-1 was piloted, and data was collected; and (3) Retrospective analysis. In this stage, the effectiveness of the EDDH-1 pilot test is analyzed and evaluated. If the results of the pilot show that the EDDH-1 is not effective, adjustments are made as needed, and EDDH-2 is formulated. These stages are carried out cyclically to produce an effective empirical didactic design (EDD).

3. METHOD

3.1. Research Design

This study adapted the didactic design research (DDR) with four frameworks.

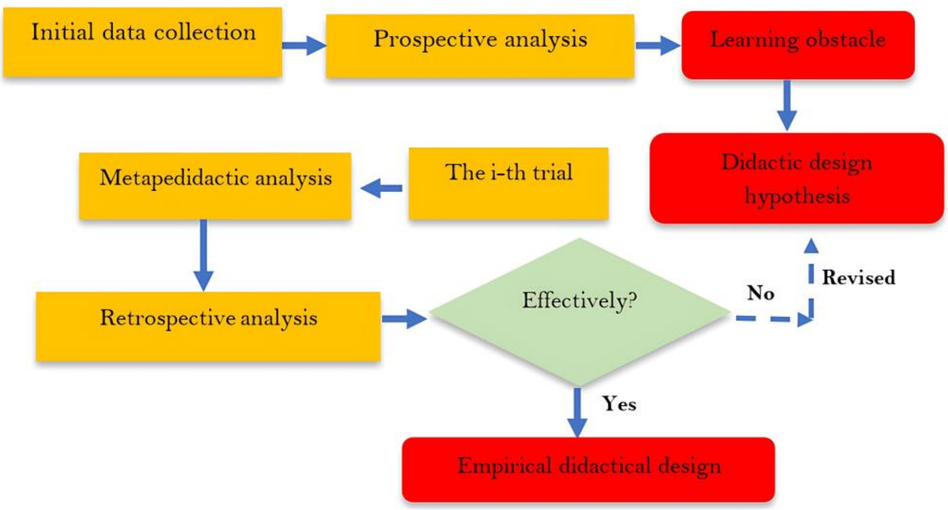


Figure 1. Flowchart of research design.

First, initial data collection; second, prospective analysis, i.e., identifying students' learning obstacles and developing didactic design hypothesis-1 (DDH-1); third, DDH-1 pilot and metapedidactic analysis, i.e., implementing DDH-1 and identifying epistemological barriers, and didactic barriers; and fourth, retrospective analysis, i.e., developing DDH-2 by revising DDH-1. All four stages are conducted cyclically until an "empirical didactical design" is obtained. The flowchart of the DDR is shown in Figure 1.

3.2. Research Subject

The sampling technique used was stratified cluster random sampling. Junior high schools in Makassar City, South Sulawesi, Indonesia, were stratified based on school status (public and private) and school accreditation (A, B, and C), resulting in six strata, and one school was randomly selected from each stratum. One VII class was randomly selected from each school, resulting in six VII classes, comprising a total of 172 students as research subjects. In addition, teachers from the selected classes participated as model teachers or experimental teachers. Table 1 shows the profiles of the research subjects.

Table 1. Profiles of research subjects.

School	No. of classes	No. of students based on gender		Total number of students
		Male	Female	
A-PJHS	1	13	17	30
B-PJHS	1	12	16	28
C-PJHS	1	11	16	27
A-RJHS	1	12	18	30
B-RJHS	1	14	16	30
C-RJHS	1	12	15	27
Total	6	74	98	172

Note: A-PJHS: A accredited public junior high school.
B-PJHS: B accredited public junior high school.
C-PJHS: C accredited public junior high school.
A-RJHS: A accredited private junior high school.
B-RJHS: B accredited private junior high school.
C-RJHS: C accredited private junior high school.

3.3. Research Instruments

3.3.1. The Quadrilateral Constructing Creativity Test (QC2T)

The QC2T (see Figure 2) went through a rigorous validation process involving three expert raters, consisting of two professors and one Ph.D. The three experts unanimously agreed on the relevance of the instrument, achieving a content validity ratio (CVR) of 100% and a content validity index (CVI) of 100%. These metrics signify full agreement among experts regarding the substantial coverage of the instrument for the concept or domain being measured (Singh et al., 2022). In addition, the QC2T results administered to a group of 60 students showed a Cronbach's alpha value of 0.814, surpassing the generally accepted threshold of 0.70. This reinforces the reliability of the QC2T (Hair et al., 2021).

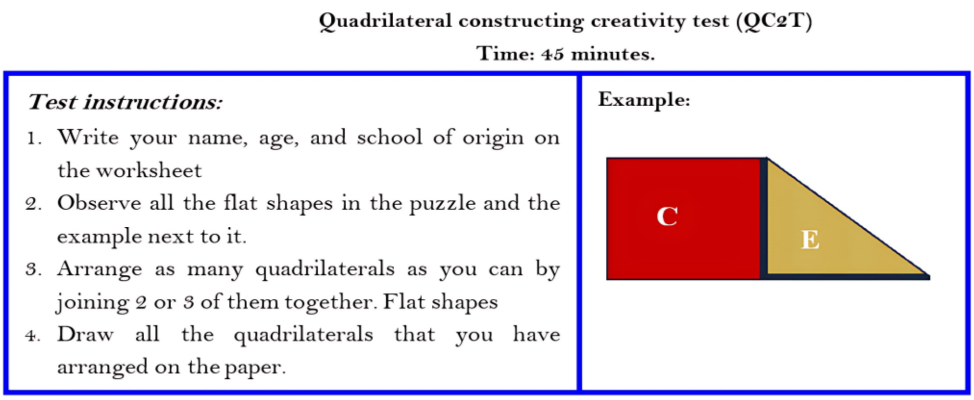


Figure 2. Quadrilateral from the combination of two flat shapes (c and e).

The puzzle given to students is illustrated in Figure 3.

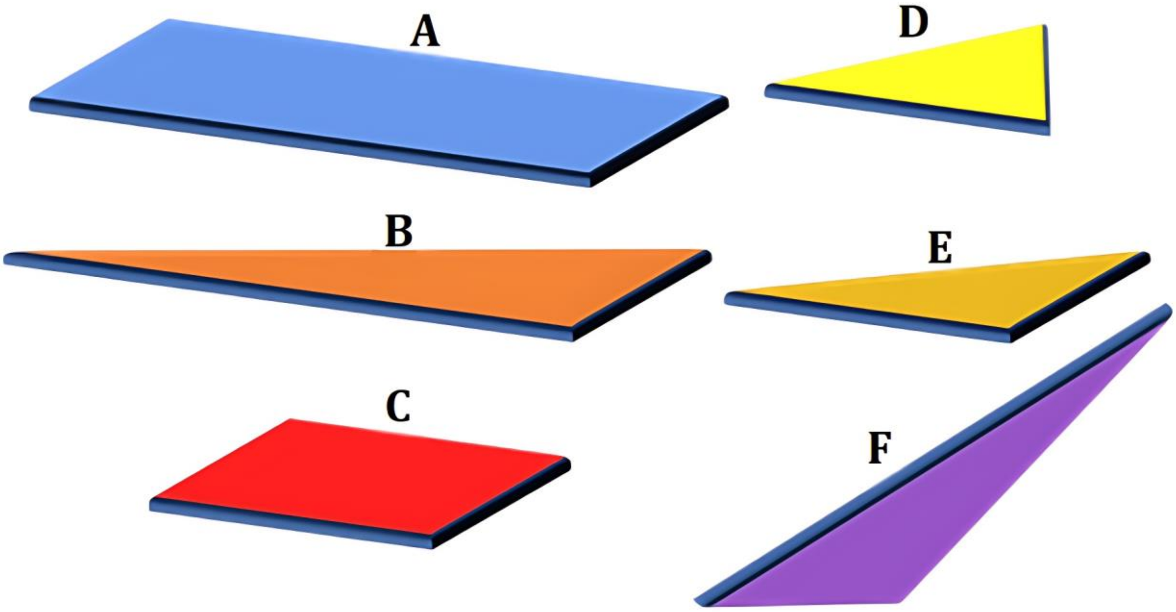


Figure 3. Puzzle illustration.

Note: Figure caption
A: Rectangle (10 cm x 5 cm).
B: Right triangle (side lengths: 10 cm, 5cm).
C: Square (5 cm).
D: Equilateral triangle (side length: 5 cm).
E: Right isosceles triangle (side length: 5 cm).
F: Isosceles triangle (side lengths: 10 cm, 10 cm, 5 cm.

The criterion for mathematical creativity is measured based on the number of quadrilaterals that can be created by combining two or three flat shapes. The minimum score is zero and the maximum score is 22.

3.3.2. Qualitative Instruments

The qualitative data was collected using semi-structured interviews, observation sheets, student test documents, and field notes.

3.4. Data Analysis

This study used both quantitative and qualitative approaches for data analysis. Quantitative analysis uses measures of data centering to explain the central tendency of the data distribution, facilitating identification and understanding of the degree of variation of the data from the central value. Statistical calculations were performed using the Statistical Package for the Social Sciences (SPSS). In addition, to assess the significance of the increase in QC2T scores before and after the pilot test, the Normalized N-Gain method was applied. This method offers a comprehensive picture of the improvement in participants' understanding or performance after an intervention or experiment, which takes into account the initial variability in knowledge or scores before the intervention (Coletta & Steinert, 2020). The N-Gain formula is presented as follows:

Formula	Formula Caption		Effectiveness Criteria	
$\langle g \rangle = \frac{\langle post \rangle - \langle pre \rangle}{22 - \langle pre \rangle}$	$\langle g \rangle$:	N-Gain	$\langle g \rangle \geq 0.7$:	High
	$\langle post \rangle$	Average score of QC2T after the pilot	$0.3 \leq \langle g \rangle < 0.7$:	Medium
	$\langle pre \rangle$:	Average score of QC2T before the pilot	$\langle g \rangle < 0.3$:	Low
	22:	Maximum QC2T score		

The qualitative data analysis was directed toward comprehending, interpreting, and ascribing meaning to the descriptive, non-numerical data within the context of the learning obstacles that the students encountered. The qualitative analysis employed followed an adaptation of Creswell (2014) comprising distinct stages: (1) preparing data for analysis, involving the organization of field notes and the transcription of teacher-student interactions; (2) carefully reading all data, focusing on pertinent information by simplifying and abstracting data; (3) coding data, assigning specific codes to frequently occurring and relevant research-related information; (4) grouping coded data with a similar nature or meaning into thematic categories; (5) presenting data; (6) interpreting data; and (7) drawing conclusions.

4. RESULTS

4.1. Initial Data Collection and Prospective Analysis

The initial data collection was designed to explore the obstacles to student learning on the quadrilateral material more thoroughly and to verify the symptoms of student learning obstacles identified during the observations described in the introduction. The QC2T test was administered to 172 students, with each student's work evaluated and scored based on the number of flat shapes successfully created. The students' individual scores were collected, and the data was tabulated to obtain the frequency distribution of the different scores obtained. Next, a data centering measure was calculated using SPSS statistical software. The purpose of this calculation process is to identify the center of the distribution of student scores to provide a clearer picture of the overall performance of the group. The statistical results of the calculation are presented in detail in Table 2. The Normalized N-Gain method was used to determine the increase in students' QC2T scores. In addition to the quantitative analysis, a qualitative analysis of the students' work was conducted to identify the variety of flat shapes they created. The qualitative approach provides in-depth insight into the students' understanding of the quadrilateral material and can provide valuable information for the development of hypothetical didactic designs.

Table 2. Distribution of QC2T test scores.

Interval score of QC2T	Freq.	%	Mean	Modus	Median	SD	Max.	Min.	Range
0–3	5	2.9							
4–8	107	62.2							
9–13	58	33.7	7	5	7	3	14	3	11
14–18	2	1.2							
19–22	0	0							
Total	172	100							

Table 2 shows that the average QC2T score achieved by the students was 7. Out of 172 students, 107 (62.2%) were only able to create four to eight quadrilaterals. In addition, only two out of 172 (1.2%) were able to create 14 to 18 quadrilaterals, and no students were able to create 19 to 22 quadrilaterals. Information regarding the minimum score, maximum score, range, median, mode, and standard deviation showed that the students' scores were centered around the mean, with a dense distribution score of 5. Based on the results of this analysis, it was concluded that the students had difficulty arranging two or three flat shapes into a quadrilateral. A qualitative analysis of the students' work documents was also conducted, focusing on the characteristics of the flat shapes they created. The analysis results showed that out of 172 students, 101 (58.7%) made inappropriate flat shapes. The next step was to abstract, categorize, and code based on the similarity of the characteristics of the flat shapes created by the students, followed by the creation of themes based on the flat shapes. The results of this process are shown in Figure 4.

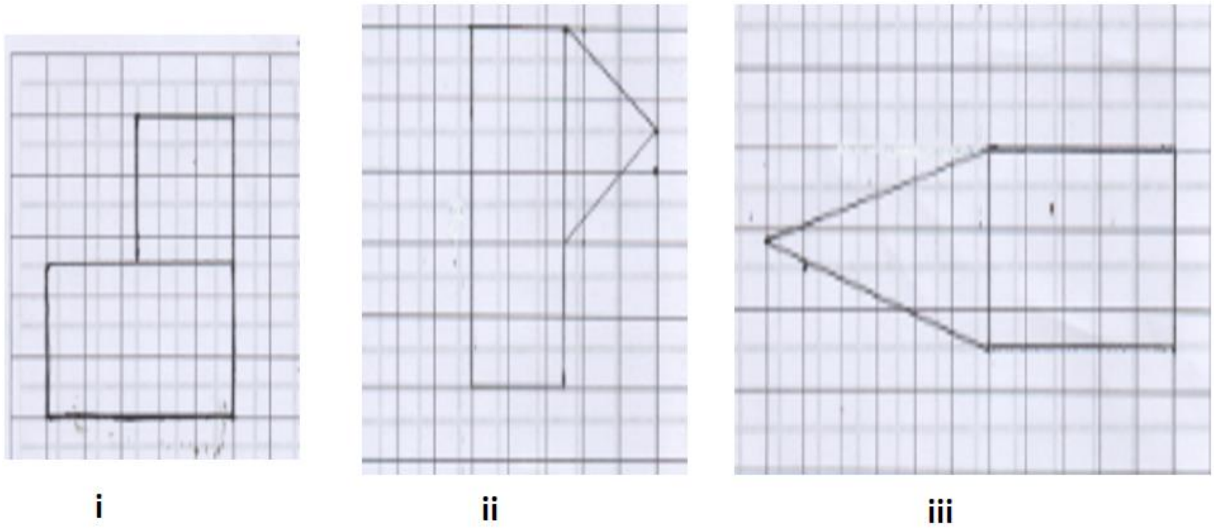


Figure 4. Student error themes.

To explore the learning obstacles to the concept of quadrilaterals, a group discussion was conducted which was attended by nine students representing figures (i), (ii), and (iii). The main questions in the discussion were: (1) Are you sure that this picture is a quadrilateral? (while showing the picture); (2) Explain why you say this is a quadrilateral; (3) Explain your habit in learning flat shapes. Is it with pictures, or by paying attention to its properties?

The method used in the group discussion is brainstorming, which involves the exchange of ideas and opinions among students. For example, do you agree with your friend's opinion, do you have another opinion? etc. All data was recorded, organized into transcripts, abstracted, coded and themed based on the nature of the responses. The data organized into themes, exposure, and interpretation are presented in Table 3.

Table 3. Data presentation and data interpretation.

Data collection		Data interpretation
Theme	**Data**	
Confidence	• The majority of students felt that their answers were correct. • A small number of students had doubts about the correctness of their answers.	This data reflects that students' schemas have formed knowledge of quadrilateral concepts that contradict theory, in other words, students are experiencing quadrilateral misconceptions.
Rectangle	"Explain why you believe this figure is a quadrilateral." • Because there is a rectangle. • There is a side of the rectangle whose length is equal to the base of the triangle.	This data shows that students' understanding of quadrilaterals depends on having or not having rectangles in the picture.
Angle	"Explain why you believe this figure is a quadrilateral." • Because this image has four corners of the same size. • Because there are equal angles. • Because adjacent angles are equal.	This data shows that students have an understanding of the concept of a quadrilateral in terms of the existence of equal angles.
How to learn	"How do you learn flat shapes, especially quadrilaterals, whether through pictures or properties of flat shapes or both?" • Often focus on visual understanding by looking at pictures. • Never explore the properties of sides and angles of flat shapes. • In general, teachers start by showing and explaining pictures.	This data reflects that students gain an understanding of quadrilaterals through pictures they see or are taught by teachers, but they do not always represent them in the concept of definitions based on their properties.

Based on the data analysis, it can be concluded that the obstacles faced by students in understanding the concept of quadrilaterals include misconceptions, partial approach in obtaining knowledge information, difficulty in representing concept images to concept definitions, and a weak understanding of quadrilateral concepts.

Furthermore, to explore the learning process of teachers on quadrilateral material, a group discussion forum was conducted with six model teachers. The discussion focused on issues such as: (1) teaching strategies used for quadrilateral material; (2) use of learning media; (3) strategies to encourage the active participation of students in the learning process; (4) how to relate the material to the real world. All information was recorded, transcribed, abstracted, and categorized based on the themes of the responses. This data is presented in Table 4 and provides an overview of the themes, data interpretation, and conclusion.

Table 4. Data presentation and interpretation of teacher learning data.

Data collection		**Data interpretation**
Theme	**Data**	
Learning strategy	Learning strategies used by teachers on quadrilateral material, include: ● Using pictures to explain the concept of quadrilaterals. ● Giving concrete examples of objects in the classroom that relate to quadrilaterals. ● Work together to complete the student activity sheet, a learning tool that includes assignments and activities designed to enhance student competencies.	The teacher does not link the concept of drawing and the concept of definition based on its properties in the learning steps.
Learning media	Media used by teachers related to quadrilateral learning: ● Activity sheets. ● The teacher uses images on white paper to help students visualize concepts. ● Objects in the classroom.	This information illustrates that the media used by teachers is not interactive enough to motivate student participation.
Encouraging students to be active	Strategies to activate students in quadrilateral learning: ● Cooperation in groups to complete the task on the worksheets. ● Conducting presentation sessions.	This information shows that the teacher has provided opportunities for students to collaborate to complete the task, but the way of facilitating is not optimal.
Participation linking the material to the real world	Relate the concept of quadrilaterals to concrete objects or objects in the classroom.	This data illustrates that teachers have linked concepts with context but are still limited to linking the concept of images with the concept of concrete objects.

From the interpretation of the data, it can be concluded that the way teachers manage learning is not optimal because it does not involve students, and the media used is not interactive and does not represent the concept of images to the concept of definitions.

4.2. Learning Obstacles

Through prospective analysis, students' obstacles to learning quadrilateral material were found to be epistemological obstacles, which includes creativity in constructing quadrilaterals using two or the figures and how students acquire knowledge that is still partial (only through images); misconceptions of quadrilateral concepts; representation of concept images to concept definitions; and weak understanding of quadrilateral concepts. Didactic obstacles include linking the concept of the image to the concept of the definition based on its properties. The media used is not interactive and lacks student involvement in linking concepts to context, which does not provide opportunities for students to explore their knowledge.

4.3. Didactic Design Hypothesis-1 (DDH-1)

At this stage, DDH-1 was developed with the involvement of the research team, model teachers and expert professors. The development process included group discussion activities focused on data analysis, reflection by model teachers, and study of relevant literature, especially the Van Hiele theory introduced by Pierre Marie van Hiele and Dina van Hiele-Geldof (MdYunus, Ayub, & Hock, 2019). This theory emphasizes the importance of instructional experiences in moving from one level to the next based on the age or biological maturity of the students (Atebe & Schäfer, 2011). This theory has proven to be a useful framework for identifying and addressing student difficulties in school geometry (Tan, Ahmad, Yunus, & Mohd, 2015). The recommendation from the group discussion is to implement interactive manipulative learning using geometry puzzles, hereafter referred to as DDH-1. The DDH-1 framework is described as follows:

1. Construct a geometry puzzle consisting of seven flat shapes in the form of cardboard pieces. An illustration of the seven flat shapes is shown in Figure 5.

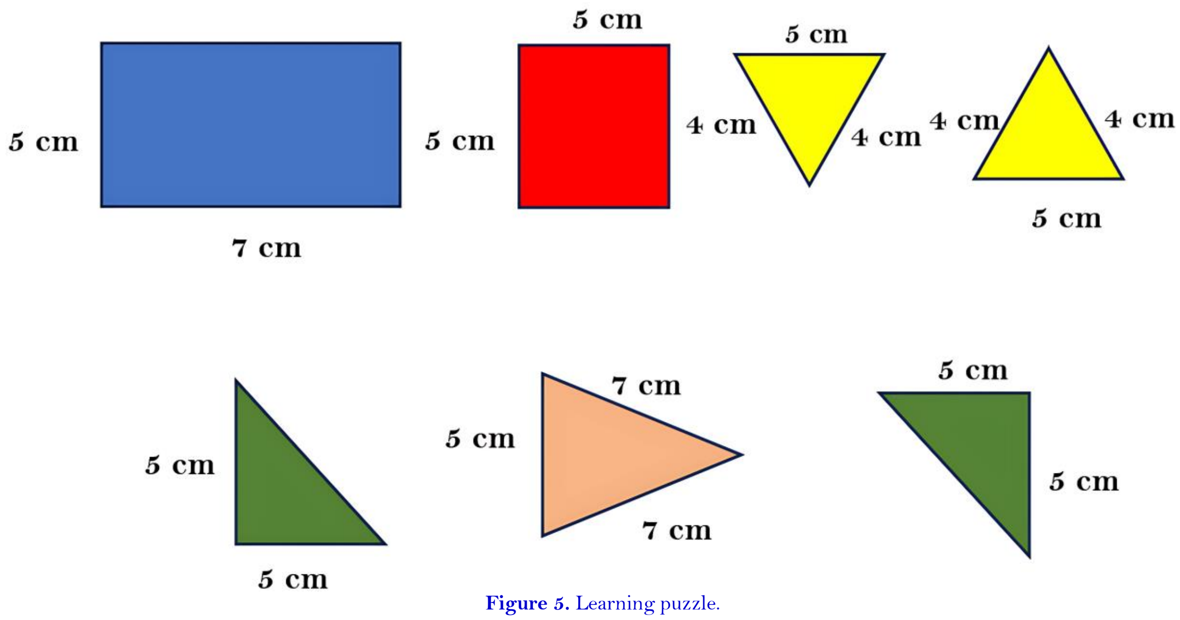


Figure 5. Learning puzzle.

2. Developing the DDH-1 Framework. Table 5 explains the extent to which we developed the DDH framework in this research.

Table 5. DDH-1 framework.

No.	Teacher activity	Student activity
1.	The teacher organizes the students into small groups (4–5 people) that are heterogeneous according to the initial abilities of the students.	Students sit in groups in the designated places.
2.	• The teacher motivates the students. • The teacher communicates the outline of the learning process that the students will take part in. • The teacher communicates the learning objectives to be achieved. • The teacher conducts a brainstorming session about supporting materials that the students have learned that are related to the material to be learned.	Students listen to the teacher's explanation, ask questions, and respond to the teacher's questions.
3.	• The teacher distributes puzzles to each group and gives explanations/Instructions related to learning using puzzles.	Students listen to the teacher's explanation, ask questions and

No.	Teacher activity	Student activity
	• You are asked to arrange two or three flat shapes on the puzzle to make as many quadrilaterals as possible. • Draw the quadrilaterals that you have arranged on the prepared paper.	respond to the teacher's questions.
4.	The teacher goes around the class, monitoring discussions and providing assistance to groups that are having difficulty.	• Students engage in group discussions. • Students engage in three-way interactions: students with students, students with teachers, and students with learning resources. • Students engage in discussions to agree on the results of the project assignment in the form of a written report (One group, one project assignment).
5.	• The teacher instructs each group to present their project results. • The teacher facilitates discussion between groups. • The teacher identifies students' misconceptions.	Each group presents the results of their project, and the other groups give feedback.
6.	• The teacher assists students to address identified misconceptions by establishing connections between concepts such as parallel sides, equal angles, and other relevant properties through the application of guiding and probing questions. • The teacher and students summarize the learning material.	• Students listen and respond to the teacher's questions. • Students realize their mistakes and correct them. • The material is summarized.

4.4. Pilot 1 and Metapedidactic Analysis

DDH-1 was piloted in three classes (87 students); viz B-PJHS (30 students), C-PJHS (27 students), and A-RJHS (30 students). The piloting was conducted by the teachers in their respective classrooms and observed by the research team. Figure 6 illustrates pilot 1.

Figure 6. Learning activities for Pilot 1.

4.4.1. Preliminary Data from Pilot 1

The students' QC2T test scores before Pilot 1 are presented in Table 6.

Table 6. Preliminary data of the QC2T scores in Pilot 1.

Interval QC2T	Freq	%	Mean	Modus	Median	SD	Max.	Min.	Range
0–3	3	3.45	7.5	5	7	2.8	14	3	11
4–8	53	60.92							
9–13	30	34.48							
14–18	1	1.15							
19–22	0	0.00							
Total	87	100							

In addition, other data showed that 53 out of 87 students, or 60.9%, made flat shapes that were not quadrilaterals.

4.4.2. Metadidactic Analysis of Pilot 1 Findings

4.4.2.1. Didactical Obstacles

Data collection was conducted by two researchers during the experimental process using an open-ended observation sheet.

Observations focused on activities that occurred during the learning process, including teacher-student interactions, teacher instructions or questions, and mastery of material. In addition, the data collected was analyzed qualitatively through the following steps: (1) preparing data for analysis—organizing field notes and making transcripts of teacher and student interactions; (2) reading all data carefully—focusing relevant data by simplifying and abstracting the data; (3) coding data—incidences that frequently occur and are relevant to the research are given specific codes; (4) coded data with the same nature or meaning are grouped into a theme; (5) presenting data; (6) interpreting data; and (7) drawing conclusions.

Data presentation and data interpretation are shown in Table 7.

Table 7. Data presentation and data interpretation.

Data collection		Data interpretation
Theme	**Data**	
Scaffolding	When responding to students' problems, the teacher provides assistance by: ● Taking flat shapes and arranging them. ● Drawing a quadrilateral and then asking the students to arrange the flat shapes according to the drawing.	When helping students who are having problems, the teacher does it directly and then asks students to model it, thus positioning the teacher as the problem solver.
How to ask questions	The teacher's way of asking and what is being asked: ● Is this a quadrilateral? ● Do you understand it? ● What don't you understand? ● Why hasn't anyone found this shape before? ● Who already understands?	The questions asked by the teacher are closed-ended with answers, and levels of recall and understanding. The types of questions asked do not stimulate students to develop analysis, evaluation and creation. For example, what if, explain the difference, and so on.
Brainstorming	When a student asks, "Is this right?" the teacher replies: ● That's right, good. ● Still wrong, try again. ● When the student asks, "Can I make a quadrilateral by combining these?" the teacher answers, "No, you can't" or "Yes, you can."	The teacher does not give other students the opportunity to respond to their friends' questions, or the teacher monopolizes the answers, so there is no brainstorming activity in the classroom, for example, "Who can answer your friend's question?".

Opportunity to learn	The data presentation below was done by one teacher; the other two teachers experienced the same case but did not respond to it. (T = Teacher, S = Student) T: Why didn't anyone find the trapezium flat? S: What is the shape of a trapezium? T: Just so you know, take shapes B, E, and G, and put the sides (Not the hypotenuse) of shapes E and G on the side of B. S: The students do the experiment and then show the teacher the results. T: Good, what you found is a trapezium. Next, draw.	The teacher gives explicit instructions to the students in conducting the experiment. For example, the teacher said, "That is not the hypotenuse". In addition, after the students showed their work, the teacher stopped to explore students' understanding and critical thinking. For example, what if only B and E were used, can it still be a trapezoid?
Conceptual representation	T: Who can show me a picture of a quadrilateral? S: (Shows a flat shape). T: What is the name of the flat shape? S: (Says the name of the flat shape). T: Good, now who can show me a flat shape that is not a quadrilateral? S: (Shows a flat shape). T: Good.	The teacher only explains the concept of quadrilaterals and does not perform conceptual representations such as comparing two or more flat figures based on their sides or angles. .

The didactic obstacles found in Experiment 1 were scaffolding, questioning, brainstorming, opportunity to learn, and conceptual representation.

4.4.2.2. Epistemological Obstacles

After the implementation of Pilot 1, 84 students (3 absent) participated in the QC2T test. Each student's work was evaluated and scored based on the number of flat shapes successfully created. The students' individual scores were then collected, and data tabulation was done to obtain the frequency distribution of the various scores. Furthermore, measures of data cantering were calculated using SPSS statistical software. The results of the statistical calculations are presented in detail in Table 8. Furthermore, the N-Gain was calculated to analyze the increase in QC2T scores before and after Pilot 1. In addition to quantitative analysis, qualitative analysis of students' work was also conducted with the aim of identifying the variety of flat shapes deconstructed by the students. The qualitative approach provides a deeper insight into the students' understanding of the material and can provide valuable information to improve DDH-1.

Table 8. Students' QC2T test scores after Pilot 1.

Interval QC2T	Freq.	%	Mean	Modus	Median	SD	Max.	Min.	Range
0–3	0	0.00	10.3	7	9	3.5	19	6	13
4–8	34	40.48							
9–13	33	39.29							
14–18	15	17.86							
19–22	2	2.38							
Total	84	100							

Note: N-Gain = (10.3-7.5)/(22-7.5) = 0.19.

Table 8 shows an increase in the average QC2T score from 7.5 before Pilot 1 to 10.3 (37%) after. Meanwhile, the minimum score increased from 3 to 6 (100%) and the maximum score increased from 14 to 19 (35.7%). The median, mode, and standard deviation showed that the scores were centered around the mean with a dense distribution score of 7. However, the N-Gain was still low at 0.19. Furthermore, the qualitative analysis results showed that 27 out of 84 students (32.1%) made improper flat figures. This shows that students' misconceptions about quadrilaterals decreased from 60.9% to 32.1%, and the variety of errors also decreased. Based on the results of the statistical analysis, it is concluded that after Experiment 1, students still experience epistemological obstacles, such as creativity in constructing various quadrilaterals and a limited understanding of quadrilateral concepts, and there are still misconceptions.

4.4.3. Retrospective Analysis

A retrospective analysis was conducted to follow up on the results of the metapedidactic analysis. The strategy used was a reflective group discussion forum, which was attended by the entire research team and all model teachers. The activities included evaluation, review of relevant literature, and revisions deemed necessary. The resulting recommendations: (1) *Didactic Design Hypothesis-2 (DDH-2)* added one framework, namely Framework 5, so Frameworks 5 and 6 in DDH-1 became Frameworks 6 and 7 in DDH-2 (see Table 9). This framework provides opportunities for students to develop critical thinking skills and creativity, and conduct assessments; (2) DDH-2 is equipped with additional instructions for teachers to manage learning (see Table 10).

Table 9. DDH-2 framework 5.

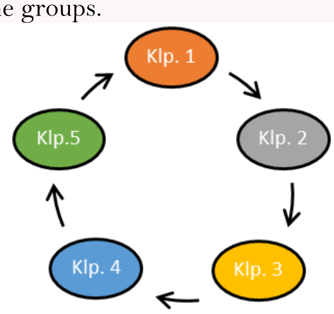
Teacher activity	Student activity
The teacher instructs each group to criticize the other groups' project results according to the mechanism determined by the teacher (see picture), then, the teacher facilitates a discussion between the groups. 	The students hold intra-group discussions to provide critical arguments for the other groups' project results and write a report on the results of the discussion to be presented.

Table 10. Learning management supplement.

Didactic obstacles	Alternative solution
Scaffolding	Very limited help: • "How do you think this quadrilateral can be made from three flat shapes?" • "Think of a way to arrange it using the angles and sides of the flat shapes." Limited support: • "What should you consider when arranging the three flat shapes to make a square?" • "You may need to rotate or move some of the pieces." Partial support: • "Try putting flat shape A in the lower left corner and see if you notice any changes. • "Why do you think you need to start from a particular corner/Side?" Almost full help: • "Now take flat B and place it on the side parallel to flat A. Do you see the pattern?" Full assistance: • The teacher provides full assistance by giving detailed instructions and telling the students how to arrange the three flat shapes into a quadrilateral. The teacher provides immediate feedback on the student's steps and offers positive encouragement.
• Questions • Brainstorming • Conceptual representation	Teacher: Good question, can anyone answer your friend's question? (Allows a few seconds for students to respond). Student A: (Answers the question). Teacher: Good, thank you, student A. Any other thoughts? (Gives other students a chance to contribute). Student B: I have a different opinion. I think... (Gives student B time to express his opinion). Teacher: Thank you, student B, for expressing a different perspective. Now, as a class,

	whose opinion do you think is correct? (Invites students to participate by raising their hands or voting). Student C: I agree with student A because... Teacher: Good, thank you, student C. Does anyone have a different opinion or something to add? (Allows other students to speak). Student D: I think that... Teacher: Thank you, student D, for adding another idea. Now, why do you think your answer is correct? (Encourages students to give reasons and explain their thinking). Student E: I am sure because... Teacher: Okay, thank you, student E. Does anyone want to respond or add anything? (Gives other students one last chance to contribute). Teacher: Let's summarize our discussion. Are there any key points you would like to share before we move on? (Invites students to summarize ideas or conclusions from the discussion). Next, the teacher can ask, "What if it's just flat shapes B and E? Does it still form a trapezium?" and "What about the flat shapes A and G? Do they form A, E, and G?" The teacher should direct the students to explore the different flat shapes in the puzzle. (The discussion continues as needed, and the teacher makes sure that each student feels heard and valued. At the end, the teacher can give positive feedback on the students' participation and emphasize the importance of listening and understanding others' points of view).
Opportunity to learn	• Shows two visualizations of different flat shapes, then guides students to abstract the similarities and differences of the 2 flat shapes based on their sides and angles. • Next, the teacher guides the students to define all flat shapes based on their side properties. • The teacher reinforces students' understanding by guiding students to analyze the relationship between flat shapes based on their properties. For example, by asking questions such as "Is a rectangle a parallelogram? Does a square contain a rectangle? Is a rhombus a kite, and so on? • Together with the students, summarize the concept of quadrilateral based on its properties (Angle, size).

4.5. Pilot 2 Test and Metapedidactic Analysis

DDH-2 was tested in three classes (85 students), namely B-PJHS (28 students), C-PJHS (27 students), and A-RJHS (30 students) by the model teacher. Figure 7 illustrates Pilot 2.

Figure 7. Learning activities for Pilot 2.

4.5.1. Preliminary Data from Pilot 2

Subject data before Pilot 2 was conducted and is described as follows in Table 11.

Table 11. Baseline data of the QC2T scores from Pilot 2.

Interval QC2T	Freq.	%	Mean	Modus	Median	SD	Max.	Min.	Range
0–3	2	2.4	7.5	5	7	2.7	14	3	11
4–8	54	63.5							
9–13	28	32.9							
14–18	1	1.2							
19–22	0	2.4							
Total	85	100							

In addition, 48 out of 85 students, or 56.5%, made flat shapes that were not quadrilaterals.

4.5.2. Metadidactic Analysis of the Pilot 2 Findings

4.5.2.1. Didactical Obstacles

Overall, DDH-2 was well implemented in Pilot 2. Specifically, the teacher successfully implemented questioning, guiding, and exploring activities, facilitating brainstorming, performing conceptual representations, and providing opportunities for students to construct their own knowledge. However, in the scaffolding aspect, teachers still used partial support to guide the students to solve problems. It is concluded that DDH-2 effectively overcomes didactical obstacles, namely scaffolding, questioning, brainstorming, opportunity to learn, and conceptual representation.

4.5.2.2. Didactical Epistemology

After conducting Pilot 2, the pilot participants took the QC2T test with the participation of 83 students (two students were absent). Each student's work was evaluated and scored based on the number of flat shapes successfully created. The individual student scores were then collected and the data tabulated to obtain a frequency distribution of the various scores obtained. In addition, to gain a deeper understanding of the data, measures of data centering were calculated using SPSS statistical software. The results of the statistical calculations are presented in detail in Table 12, which provides useful information for further analysis.

A qualitative analysis of the students' work documents was also conducted to identify the variety of flat shapes that they deconstructed. The qualitative approach provides a deep insight into the students' understanding of the material and can provide valuable information for the improvement of DDH-2.

Table 12. QC2T test scores of the students after Pilot 2.

Interval QC2T	Freq.	%	Mean	Modus	Median	SD	Max.	Min.	Range
0–3	0	0.0	14.5	14	14	2.9	21	10	11
4–8	0	0.0							
9–13	32	38.6							
14–18	41	49.4							
19–22	10	12.0							
Total	83	100							

Table 12 shows an increase in the mean pre- and post-test QC2T scores from 7.5 to 14.5 (93.3%). There was also an increase in the minimum score from 3 to 10 (233%) and the maximum score from 14 to 21 (50%). The median, mode and standard deviation showed that the scores were centered around the mean with a dense distribution. The N-Gain was moderate and tended to be high, at 0.48.

Qualitative analysis showed that only three out of 83 students (3.6%) made flat figures that were not quadrilaterals. This indicates a decrease in students' misconceptions about quadrilaterals from 56.5% to 3.6%.

Based on the results of the metadidactic analysis, it can be concluded that DDH-2 effectively overcomes epistemological obstacles, namely creativity in constructing various flat quadrilateral shapes and a limited understanding of quadrilateral concepts, as well as misconceptions. Although there were still three students (3.6%) who constructed shapes that were not quadrilaterals, this was considered insignificant to state that the didactical design was not effective.

Therefore, a third pilot was considered unnecessary. It was concluded that this study found an empirical didactical design, namely manipulative interactive didactics with puzzles included with teacher instructions, effectively overcoming didactical and epistemological obstacles.

5. DISCUSSION

This research has resulted in an empirical didactic design (EDD), namely "Manipulative Interactive Didactics with Puzzles", complemented by a Teacher's Guide. The EDD has proven to be effective in overcoming didactic and epistemological barriers.

The EDD helps students acquire knowledge through manipulative concrete objects and provides opportunities for active involvement in constructing quadrilaterals. This ensures that students do not just passively receive information but construct their own knowledge through direct experience. It gives students the opportunity to differentiate between figurative and conceptual definitions, allowing for a more effective resolution of student misconceptions. In addition, it provides students with opportunities to develop different ideas when constructing two or three plane figures to create different types of quadrilaterals. These activities train students' creative thinking skills.

The didactic barriers that were successfully addressed include scaffolding, questioning skills, opportunity to learn, expressing opinions, and conceptual representation. The EDD provides teachers with guidance on how to scaffold according to students' needs. In addition, it provides guidance on how a teacher should ask questions that stimulate students to develop higher-level thinking skills. Another feature of the EDD is guidance for teachers on how to lead small group discussions that engage students in analyzing the relationships among plane figures based on their properties. The EDD provides opportunities for students to organize their own ideas and build collective knowledge.

The development of thee EDD was informed by Van Hiele's theory, which focuses on the stages of geometric understanding, namely the levels of recognition, analysis, abstraction, deduction, and integration (Razak, Sutrisno, & Immawan, 2018). This theory also emphasizes the importance of instructional experiences in moving from one level to the next depending on the student's age or biological maturity (Atebe & Schäfer, 2011). Many experts, such as Tan et al. (2015); Alex and Mammen (2016) and Sari, Machromah, and Zakkiyah (2020) have used the Van Hiele theory to develop frameworks, instructional strategies, and geometry learning modules. The theory has also proven beneficial in analyzing students' levels of creative and critical thinking in solving geometric problems and in assessing students' conceptual understanding of geometry (Primasatya & Jatmiko, 2018; Solaiman, Magno, & Aman, 2017; Wulandari et al., 2021).

The results of this study are consistent with those of previous studies, such as Bruno, Da Silva, Da Silva, and Teixeira (2019); Fiantika, Maknun, Budayasa, and Lukito (2018) and Ubah and Bansilal (2019) which indicate that geometry learning based on concrete or real objects, such as puzzles, can help students connect theoretical concepts to everyday situations. This helps students to overcome difficulties in understanding abstract geometry. Other studies, such as those conducted by Klymchuk (2017); Hasna, Fajriyah, and Saputra (2021); Costa (2017) and Qomaria (2021) confirm that the use of puzzles in learning can develop conceptual understanding, literal thinking skills, critical and creative thinking skills, and stimulate students' curiosity. These findings are consistent with the research of Maheux and Roth (2015) who concluded that the ability to manipulate geometric objects to create new ones is an integral part of high-level mathematical creativity. Furthermore, Siew and Chong (2014) found that the use of manipulative materials in geometry instruction is effective in developing students' conceptual understanding of geometry and critical thinking skills.

Therefore, the EDD emerges as a holistic and effective learning approach to address the challenges that students may face in understanding geometric concepts, especially those related to quadrilaterals.

6. CONCLUSION

The results of this study present an empirical didactical design (EDD), namely "Manipulative Interactive Didactics with Puzzles", which consists of seven frameworks and is equipped with a teacher's guide. This EDD has been proven effective in overcoming didactical and epistemological barriers. Teachers can implement or adapt this EDD for use in geometry or other mathematics subjects.

This study has limitations. First, the findings are limited to the quadrilateral topic, so we recommend further research to expand the geometry learning content by focusing on constructing spatial figures with the same area or spatial figures with the same volume/surface area. Second, it is limited in considering other factors that influence students' problem solving, so future research can analyze the effect of teaching methods on other variables, such as student motivation, information retention, and the development of mathematical problem-solving skills.

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