International Journal of Education and Practice
2025 Vol. 13, No. 2, pp. 723-738
ISSN(e): 2310-3868
ISSN(p): 2311-6897
DOI: 10.18488/61.v13i2.4129

The integration of mathematical modeling tasks into primary education: A comparative study of Croatia and Slovenia

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Article History
Received: 24 December 2024
Revised: 10 February 2025
Accepted: 28 February 2025
Published: 7 March 2025

Keywords
Mathematics
Modelling
Primary education
Students
Task.

ABSTRACT

This study examines the practical aspects of introducing mathematical modeling into primary mathematics education in Slovenia and Croatia. It aims to explore how students approach mathematical modeling tasks, whether they rely solely on mathematical operations or integrate non-mathematical knowledge, and what strategies they employ during problem-solving. Teachers in Slovenia and Croatia were prepared by training to present mathematical modelling tasks to 231 students, ages 10 to 11. Students solved three group tasks analyzed using a three-level framework to assess solution sophistication. Most students demonstrated basic problem-solving strategies with a smaller proportion achieving advanced reasoning and multi-criteria analysis. Croatian students more frequently employed diverse strategies reflecting differences in curricular emphasis between the two countries. The ability to engage in mathematical modeling is essential in today's world, yet effective teaching approaches remain undefined. This study underscores its potential in primary education and calls for further research to refine instructional methods and enhance student outcomes. Teachers can be trained in less than three months to effectively implement mathematical modeling tasks. With this training, students can develop foundational modeling skills in fewer than six lessons, enabling them to apply mathematical reasoning to real-world problems.

Contribution/Originality: The originality of this study lies in its simultaneous focus on training teachers and assessing students in mathematical modeling, bridging a gap seen in previous research. Its contribution also includes implementing mathematical modeling education within a realistic, time-constrained framework and conducting empirical classroom research at the underexplored primary education level.

1. INTRODUCTION

Mathematical modeling is the foundation of modern scientific research in fields such as biotechnology, aerospace engineering and computer science. Mathematical models assist in solving problems making mathematical modeling a highly desirable skill in the modern world (Merritt, Lee, Rillero, & Kinach, 2017). As a result, it is increasingly recognized as a powerful tool not only for enhancing students' understanding of various scientific aspects and concepts but also for appreciating the potential of mathematics as a critical tool for analyzing important issues in their lives, society and communities (English, 2010).

When it comes to learning and teaching mathematical modeling, it often overlaps with methods such as problem-based learning, inquiry-driven instruction and project-based activities. This highlights the effectiveness of modeling in achieving deep and lasting understanding across a wide range of mathematical content (Erdem, Gökkurt, Şahin, Başıbüyük, & Soylu, 2015). When students engage in mathematical modeling activities, these scenarios are both relatable and complex for them. Thus, through mathematical modeling, we support not only learning mathematics itself but also the development of social and interpersonal skills as well as problem-solving abilities that extend beyond the classroom. In this way, mathematics helps students improve their critical thinking skills and future financial literacy and prepares them for future life challenges.

It is also very important for teachers to encourage the design of diverse mathematical problems that go beyond routine tasks. These should include real-life problems, non-routine challenges and open-ended questions that promote thinking and problem-solving skills among students (Masriyah, Rahaju, Fardah, & Hanifah, 2024; Unver, Hidiroglu, Dede, & Guzel, 2018). This approach moves beyond repetitive tasks, pushing students to engage with more complex and meaningful problems that better reflect real-world situations.

For the purposes of our study, we adopted the definition of mathematical modelling by Stohlmann and Albarracín. Mathematical modelling is an iterative process involving open-ended and real-world problems that students understand using mathematics through assumptions, approximations and multiple representations. Other sources of knowledge besides mathematics can also be used (Stohlmann & Albarracin, 2016). The literature offers various definitions of mathematical modelling because it effectively captures the main principles of mathematical modelling within educational settings. Hansen (2021) argues that introducing mathematical modeling in primary education presents challenges, mainly due to the limited experience teachers have with this approach at this level. Recently, primary school curricula have not included mathematical modeling (Brown & Ikeda, 2019) making its integration into early education even more difficult.

However, foundational skills for modelling can be developed at the primary level as young children possess the basic competencies necessary for engaging in modelling (English & Watters, 2004).

The importance of this study lies in two key aspects. First, its classroom-based findings demonstrate that mathematical modeling can be effectively implemented with younger students—a practice that has been largely overlooked in previous research. Second, the study provides a cross-country comparison of two neighboring countries. This study addresses the following critical research question: How do students approach mathematical modeling tasks and the strategies, criteria, mathematical knowledge and non-mathematical knowledge they employ when solving such tasks?

Mathematical modelling in the classroom is an interactive process where students face real-world problems and apply their assumptions, personal experiences and various representations to solve them. The primary aim of modelling is not always to find a single correct solution but to enhance understanding and critical thinking when addressing real-world issues. Students must analyze the entire problem text rather than isolated segments, identifying essential and non-essential variables. Skills like constructing, reasoning, predicting, estimating, organizing data and quantifying are becoming increasingly essential in navigating modern life. However, mathematical modeling remains underutilized in primary education despite its significant educational potential (English & Watters, 2004). Mathematical modeling is deeply interconnected with essential mathematical skills, including design, interpretation, communication, and problem-solving strategies, all of which involve advanced cognitive abilities. However, many primary school teachers perceive modeling as overly complex for young learners resulting in its limited integration into classroom practice (Asempapa, 2015). Hodnik and Manfreda Kolar (2022) also discuss how problem-solving and posing align with real-world application, fostering critical thinking and mathematical skills. They support the idea that modeling fosters deeper understanding by engaging students in authentic problem contexts and critical analysis.

2. LITERATURE REVIEW

In Slovenia and Croatia, students face mathematical word problems as early as the first grade. Students often struggle with these problems throughout their schooling as they require reading comprehension alongside mathematical reasoning despite this early introduction. According to Kolovou (2011) the primary goal of mathematics education should be empowering students to independently solve problems with comprehension being a key indicator of success. While some students excel in algorithmic calculations, they may struggle with word problems (Fuentes, 1998; Özcan & Doğan, 2018). Verschaffel and De Corte (1999) note that many weaker students hold incorrect beliefs about mathematics such as the idea that there is only one correct answer to every problem or that solving problems should take no more than five minutes. These misconceptions are corroborated by other research highlighting that solving word problems is often challenging and frustrating for many students (Bluman, 2005; Osman et al., 2018; Phonapichat, Wongwanich, & Sujiva, 2013).

Another research focus is the lack of general knowledge and comprehension in solving modelling problems. In the late 1970s, French and German researchers tested primary school students' tendencies toward unrealistic modelling using nonsensical problems. There are 26 sheep and 10 goats on a boat. How old is the captain? They found that a significant number of students (up to 60%) attempted to solve these unsolvable problems by combining the given numbers without recognizing the absurdity of the question (Baruk, 1989; Verschaffel & De Corte, 1999). This tendency persisted across grade levels illustrating the importance of reading comprehension in mathematical problem-solving (Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). According to Verschaffel and De Corte (1999), students often skip the representation phase in solving word problems and move directly to constructing mathematical expressions based on surface cues, reinforcing stereotypical beliefs about mathematics. This issue emphasizes the need for careful analysis and critical examination of problem texts (Sabo Junger & Lipovec, 2022).

Individual studies have shown that younger students as early as six years old can participate in mathematical modeling activities (English, 2010). Similarly, seven-year-olds (Albarracín, 2021) eight-year-olds (English & Watters, 2004) and nine-year-olds (English, 2013) have been found capable of handling such tasks. In their research, Zubi, Peled, and Yarden (2019) found that even fifth-grade students can actively engage in mathematical modeling tasks if their mathematical knowledge is not at the highest level. This suggests that mathematical modeling can be effectively implemented with students who may not yet have mastered all basic mathematical skills. However, this is due to the fact that the reasons appear somewhat ambiguous—it may seem that they are capable, but this is not yet clear. The reasons may lie in small patterns, specific cultural contexts, ambiguity, and the fact that students work with researchers rather than their teachers.

Research on primary school students is less prevalent, primarily because this area has received less attention until recently compared to studies focusing on older students (Turner et al., 2022). It appears that it was easier to implement highly cognitively demanding mathematical modeling tasks with older students as they already possess more advanced mathematical knowledge and skills. However, research conducted up to 2015 (Stohlmann & Albarracin, 2016) and 2017 (Jung & Brand, 2021) has demonstrated that younger students (those under 10 years old) are also capable of engaging in mathematical modeling activities. Research by Jung and Brand (2021) analyzed studies from various developed countries, including the United States, Australia, the United Kingdom, Canada and the Netherlands. The samples included students from grades 1 to 3 and in some studies, their teachers also participated. The research examined how mathematical modeling based on solving real-world problems influences the understanding of mathematical concepts. Students engaged in activities requiring them to develop models to address real-world problems such as measurement, planning or comparison. The results showed that such learning improves students' understanding of fundamental mathematical concepts, enhances critical thinking and fosters creativity. Similarly, the study by Stohlmann and Albarracin (2016) analyzed mathematical modeling in elementary classrooms in the United States. The samples included elementary school students primarily from grades 3 to 5 and their teachers. Students participated in activities where they solved real-life problems such as event planning, data analysis, and developing models for practical situations. The study found that mathematical modeling helps students better understand mathematical concepts and develops their problem-solving skills, critical thinking, and ability to connect mathematics to everyday life.

2.1. Research Question

We were interested in understanding how students solve mathematical modeling tasks. Specifically, we examined which strategies they use, whether they rely on multiple strategies or just one, how they select and apply criteria and how many criteria they consider. Additionally, we explored whether they incorporate non-mathematical knowledge into their problem-solving process or focus solely on familiar mathematical operations.

2.2. Research Problem

This research investigates students' abilities in solving mathematical modeling problems.

2.2.1. Problem-Solving Skills

How students approach a specific mathematical modeling problem with strategies and methods they use during the problem-solving process whether they use multiple strategies or focus on just one and what knowledge they apply while solving the problem.

2.2.2. Connection between Theory and Practice

Analyzing the module enables students to better connect mathematical theories and concepts with practical applications. This involves determining whether students gain a deeper understanding of how mathematics is used in the real-world contexts and whether they can transfer their mathematical knowledge across different fields and situations.

3. METHODOLOGY

3.1. Research Design

This paper presents the findings of a study conducted with students as part of a larger research. Primary school teachers from Slovenia and Croatia also participated in addition to students. The research took place between October and December 2022 in both countries.

Approval for conducting the study in schools was obtained from the Croatian Ministry of Science and Education and the Ethics Committee for Research in the Field of Organizational Sciences in Slovenia. Additionally, consent was secured from teachers, parents and principals of the selected schools.

Institutional Review Board Statement: The Ethical Committee for Research in the Field of Organizational Sciences/ University of Maribor, SLOVENIA approved this study on 19.09.2022. (Ref. No. 514-10/2022/1/902-DJ).

Institutional Review Board Statement: The Ministry of Science and Education, CROATIA approved this study on 22.10.2022. (Ref. No. 533-06-22-0004).

The research framework included three workshops on mathematical modelling held in Croatia and Slovenia culminating in a final joint session where all 15 teachers collaborated. Each workshop lasted three hours and followed an iterative and spiral learning approach. Workshops were conducted in the participants' respective languages. Croatian workshops were delivered in Croatian while Slovenian workshops were conducted in Slovenian. Teachers actively worked on and discussed specific mathematical modelling tasks, fostering a dynamic and collaborative atmosphere throughout the workshops. Once each task was presented to the teachers, they incorporated it into their mathematics lessons with their students. This classroom implementation includes the strategies and solutions employed by the students' forms the central focus of this study. In this paper, we will present three mathematical modelling tasks that were completed by teachers and their students. The first task, "waste sorting" (see Appendix A) was inspired by "the pea problem" (English & Watters, 2004). The second, " buying an electric scooter" (see Appendix B) was based on " what car to buy?" (English, 2003) adapted for younger students. The third task, "amusement park" (see Appendix C) is from Bleiler-Baxter, Stephens, Baxter, and Barlow (2017).

3.2. Research Population

Three primary schools in Slovenia and four in Croatia were selected. The schools were chosen to enable the physical implementation of the study. In Slovenia, schools were in the Maribor area while in Croatia, the schools were from Čakovec and its wider surroundings. The research included seven fifth-grade classes from Slovenia and eight fourth-grade classes from Croatia. The difference in grade levels reflects the varying lengths of primary education in Croatia. Primary school lasts eight years whereas in Slovenia, it extends to nine years. Despite this structural difference, the participating students were of similar age typically 10 or 11 years old. The study analyzed data from 231 students, i.e., 98 students (42.4%) from Croatia and 133 students (57.6%) from Slovenia.

3.3. Instrument

Students worked on each task in groups of three or four. Each group received a printed version of the tasks with a special sheet for recording their solutions. They were required to reach a consensus on their approach to solving the tasks within the group and then document their solutions on the provided paper. The total time allocated for solving each task ranged from one to two class periods depending on the teacher's decision. Once each task was completed, each group presented their solution to the rest of the class. After the presentation, they submitted their written solutions to the teacher who brought them to the next workshop. Slovenian teachers brought the solutions from their student groups to workshops held at the Faculty of Education, University of Maribor in Maribor. Croatian teachers submitted their solutions during workshops held at the Faculty of Teacher Education, University of Zagreb at the Čakovec Department.

We first divided the collected solutions into Croatian and Slovenian groups to compare potential similarities and differences. Next, we categorized the tasks based on the level of solution sophistication.

We applied the same three-level categorization framework, more precisely the students' group solutions were analyzed using the same levels of completeness to present the solutions for the first three tasks "waste sorting", "buying an electric scooter" and "amusement park". These levels were defined by the extent of mathematical reasoning applied and the sophistication of mathematical modeling strategies employed.

The first level consisted of responses that lacked substantial mathematical reasoning or detailed explanations of how students arrived at their answers. At the second level, responses demonstrated some attempts at mathematical reasoning often using basic strategies or methods. The third level comprised solutions that demonstrated advanced mathematical reasoning with students applying multiple strategies and providing thorough justifications. At this third level, we can truly speak of mathematical modeling as students demonstrated the ability to integrate various approaches, analyze multiple factors and provide thorough explanations for their reasoning across all tasks.

We present each level for specific tasks:

- Task: Waste Sorting (see Appendix A).
- First level: Students only answered which school collected more waste using basic mathematical operations such as addition and comparison of values without providing additional explanations. Students did not make a prediction for the fourth month or provide a prediction without further justification.

- Second level: Predictions for the fourth month included general and non-mathematical explanations.
- Third level: Students made a prediction with mathematical reasoning and presented simple strategies or rules (e.g., identifying growth trends based on data from previous months).
- Task: Buying an Electric Scooter (see Appendix B).
- First level: Simple solutions without mathematical reasoning. Students simply listed options or selected a scooter without providing any justification or analysis.
- Second level: Attempts to use mathematical reasoning where students considered one criterion (e.g., recommending the cheapest scooter based on price).
- Third level: More complex solutions where students considered two or more criteria (e.g., combining price and range of the scooter while also considering additional parameters such as battery life or extra features).
- Task: Amusement Park (see Appendix C).
- First level: Solutions without mathematical reasoning or explanation of how the solution was reached.
- Second level: Attempts at mathematical reasoning using basic strategies such as sequential addition or subtraction where students followed a single method without combining strategies.
- Third level: Advanced solutions using multiple strategies (addition, subtraction, multiplication, and division) and combining these strategies for different scenarios. Each step was supported by explanations and calculations.

The data were processed and analyzed using IBM SPSS Statistics 29. Descriptive statistical methods were employed in the analysis.

3.4. Validity and Reliability Tests

Particular attention was given to validity and reliability when determining task levels for student solutions. Validity was ensured by developing clear and accurate criteria, refined through discussions and piloted on a subset of solutions. Reliability was achieved by basing the criteria on previous research on younger students' modeling abilities and providing concrete examples of student solutions for each level.

4. RESULTS

The group solutions will be presented in Table 1 separated by Croatian and Slovenian students categorized by task and by the levels of solution sophistication previously described. The column f (frequency) indicates the specific number of tasks solved at a given level while f % represents the percentage of those tasks which we rounded to one decimal place.

Table 1. The results of the tasks "waste sorting" "buying an electric scooter" and "amusement park" by levels.

Tasks		Waste sorting		Buying an electric scooter		Amusement park	
		f	f %	F	f %	f	f%
Slovenia	Level 1	41	74.5	9	15.3	32	57.1
	Level 2	10	18.2	4	6.8	21	37.5
	Level 3	4	7.3	46	77.9	3	5.4
∑	∑	55	100.0	59	100.0	56	100.0
Croatia	Level 1	28	62.2	7	14.9	9	20.9
	Level 2	13	28.9	4	8.5	25	58.2
	Level 3	4	8.9	36	76.6	9	20.9
∑	∑	45	100	47	100	43	100
Combined	Level 1	69	69.0	16	15.1	41	41.4
	Level 2	23	23.0	8	7.5	46	46.5
	Level 3	8	8.0	82	77.4	12	12.1
∑	∑	100	100.0	106	100.0	99	100.0

The results were analyzed by group solutions. Differences in the number of groups arose due to the varying time required to complete individual tasks and, consequently, due to differences in the number of groups or students in certain classes caused by absences.

According to Table 1, for the first task, "waste sorting", most solutions in both Slovenia and Croatia were classified as level 1 (Slovenia 74.5% and Croatia: 62.2%). This indicates that most students simply determined which school collected more waste relying on basic arithmetic. Predictions for the fourth month were either missing or offered without any explanation showing limited use of mathematical reasoning or modelling strategies.

At level 2, the distribution of solutions was similar between the two countries with moderate improvement in reasoning. However, in level 3, where advanced mathematical reasoning and modeling were required, the proportion of students reaching this level was quite low in Slovenia (7.3%) and Croatia (8.9%). These findings suggest that few students' groups were able to apply multiple strategies or provide comprehensive mathematical explanations emphasizing the challenge of developing more sophisticated mathematical modelling skills in both countries.

For the second task: "buying an electric scooter", we can observe that a significant percentage of students, specifically 77.4% from both countries completed the task at level 3 indicating that most groups considered at least two or more criteria in determining their solutions. Only 7.5% of the groups focused on a single criterion, while 15.1% provided subjective solutions without any mathematical justification. Some specific responses at level 1, lacking any mathematical reasoning included statements such as *I think the best electric scooter is this one because I like the color*, or *this is the solution because this electric scooter is the best*. Other responses included statements such as if *I could choose, I would pick the Segway. But I'd rather buy a regular scooter. Electric ones pollute the environment and are for lazy people, so I wouldn't buy one* and *Jan wouldn't buy any because they are too dangerous*.

These responses clearly demonstrate that students at this level focused solely on non-mathematical content, often of a very personal nature without incorporating even a single mathematical argument.

An example of a level 2 solution where students selected a single key criterion to make their decision involved choosing based on speed. One group justified their choice by stating that we *should buy the Xiaomi because it's not too fast and not too slow*. They did not take other factors into account. Similarly, other groups offered answers based solely on price demonstrating a reliance on just one parameter to reach their conclusion.

Regarding level 3 solutions, most groups (76. 6% in Croatia and 77. 9% in Slovenia) demonstrated advanced reasoning. These groups provided mathematically sound justifications, ranked potential solutions in order and factored in multiple parameters when making their decisions. For instance, a Slovenian group categorized the different criteria into separate lists. They created one list for Karl and another for his mother then evaluated which scooter would be best suited for both. They even raised an open-ended question about reliability and acknowledging uncertainty about whether this could be determined from the given data.

This suggests that students recognized the possibility of alternative solutions and iterative processes, understanding that their approach was not the only valid one. Such responses indicate an appreciation for the complexity of the problem and a deeper engagement with mathematical modeling and critical thinking.

For the third task, "amusement park", we can observe that most Slovenian groups (57.1%) solved the task at level 1, meaning they either simply filled in the table with numbers without any mathematical explanation of how they arrived at those values or provided non-mathematical reasoning. Most Slovenian students approached the task without using mathematical calculations or strategies reflecting an initial stage of understanding where intuitive reasoning was predominant. In contrast, Croatian groups at level 1 represented a significantly smaller percentage (20. 9%) suggesting a notable difference in approach between the two countries.

Some examples of the non-mathematical reasoning provided by Slovenian students include statements such as *we guessed and either increased or decreased the numbers. This ride is more exciting, so there will be longer queues there*; *there will be more people in the morning, so the crowd will be bigger and fewer people in the afternoon, so the crowd will be smaller.*

We approximately doubled everything. This movie is popular, so the crowd will be bigger. We guessed based on similar numbers. The haunted house will have the shortest queue because kids are scared to enter while the main ride will have the longest queue.

These responses reveal that students primarily relied on intuition and everyday logic rather than precise mathematical methods. This approach might be attributed to the fact that students were just beginning to engage with mathematical modeling tasks leading them to use simple and familiar reasoning patterns.

At level 2, where students employed basic mathematical strategies without providing detailed explanations, the trend is reversed. 58. 2% of Croatian groups fell into this category compared to 37.5% of Slovenian groups. This level involved using one or two fundamental strategies (e.g., basic addition or subtraction) indicating a relatively straightforward yet logical approach to mathematical modelling. The higher percentage among Croatian groups suggests that Croatian students more frequently applied basic mathematical operations at the initial stage to compare results.

At level 3, where students utilized multiple mathematical strategies and provided more detailed explanations of their problem-solving processes, Croatian students outperformed their Slovenian counterparts (20. 9% versus 5. 4%). This highest level of mathematical modelling is characterized by the effective use of a broad range of mathematical operations such as addition, subtraction, multiplication, and division, all thoroughly explained and connected to the task.

When we look at the combined percentages for both countries, we see that most student groups (46.5%) reached the second level indicating the use of basic strategies and initial attempts at mathematical reasoning. The first level was achieved by 41.4% of students while only 12.1% of students reached the third level with advanced strategies and detailed explanations. This task proved to be more challenging in terms of achieving the highest levels of performance.

5. DISCUSSION

An analysis of the results for the first task "waste sorting" shows that most students solved the task at the first level where they used basic arithmetic operations to process data. This is understandable as the students faced mathematical modeling for the first time with this task. Consequently, their problem-solving relied on simple strategies such as addition and data comparison which are characteristic of beginners in mathematical modeling. Verschaffel and De Corte (1999) also states that many students approach solving mathematical problems in a superficial and inadequate manner within the school environment. Their problem-solving activity often consists of only performing one or more arithmetic operations with the numbers provided in the task paying very little or no attention to other aspects of a competent problem-solving model (Verschaffel & De Corte, 1999). The less frequent use of more complex strategies at the third and fourth levels indicates that most students were not yet prepared for deeper analyses or more abstract mathematical thinking.

The explanation for these results can be found in the nature of the task. Since the task was designed based on the model of "The Butter Bean Problem" developed by English and Watters (2004). It is understandable that students employed similar strategies to those observed in the original study. The authors found that students often search for patterns in the data and use them for predictions when solving these tasks, a finding that is also confirmed by the results of this study.

In the study by Wei et al. (2024) on the topic of mathematical modeling in Chinese textbooks for grades 3 to 6, the authors reveal that the textbooks follow a gradual progression in the development of mathematical modeling content. At lower levels, such content is focused on basic understanding while at higher levels; more complex processes such as model validation and application are gradually introduced. The authors note that the study showed the level of highly abstract tasks that develop higher-level thinking processes remains insufficient, potentially limiting students' ability to use or apply more complex problem-solving strategies.

Our research confirms these findings. Specifically, the results showed that students at higher levels of given tasks used deeper understanding or the ability to employ more complex strategies and solutions much less frequently. The emphasis in mathematical modeling must be on developing specific skills and approaches that are crucial for effective use in classrooms (D'Ambrosio, 2009). According to Shahbari and Tabach (2019) the active involvement of teachers in mathematical modeling tasks enables a deeper understanding of the very nature of mathematical modeling and its impact on students.

Task "waste sorting" represents a significant transition from simple calculations to understanding the structure and predictability of data enabling students to gain a deeper comprehension of the connections between the data they analyze. Supporting this perspective, Asempapa (2015) emphasizes the importance of integrating mathematical modeling into elementary education to enhance students' problem-solving skills and critical thinking. Furthermore, the National Council of Teachers of Mathematics (NCTM) highlights that engaging student in modeling tasks fosters a deeper understanding of mathematical concepts and their applications (National Council of Teachers of Mathematics, 2014).

In the second task, "buying an electric scooter" which required selecting the most suitable electric scooter based on different criteria; we observed progress in reasoning and mathematical justification. Most students considered at least two or more criteria such as price, speed and capacity. About 77% of students from both countries solved the task at level 3 demonstrating that they used basic criteria to make a reasoned choice although they did not always connect these criteria with more rigorous mathematical analysis. Students who solved the task at lower levels often provided personal and subjective opinions such as *I would choose this scooter because I like the color*. Nevertheless, students at level 3 demonstrated critical thinking and the ability to compare multiple parameters simultaneously. Raising questions about the reliability of data as mentioned in one of the responses by Slovenian students indicates a higher level of metacognition and awareness that the task can be solved in various ways. Supporting this finding, Armutcu and Bal (2023) demonstrated that mathematical modeling activities within STEM education positively influence the enhancement of modeling skills in high school students. The aim of this study was to examine how mathematical modeling activities within STEM education impact students' ability to develop and apply mathematical models in problem-solving. The authors conducted an experimental study with two groups: a control group following traditional teaching methods and an experimental group participating in specifically designed mathematical modeling activities. The activities in the experimental group were structured to enable students to apply mathematical concepts to solve real-world problems, foster critical thinking and enhance collaboration among students. The research relied on two primary methods of data collection. The first method involved pre- and post-intervention testing where students completed identical tests at the beginning and end of the experiment to allow researchers to assess changes in their knowledge and skills in mathematical modeling. The second method consisted of observing and analyzing student activities during the modeling tasks. Researchers monitored how students approached problems, developed models, and used mathematical concepts to support their solutions. The results of the study revealed that the experimental group achieved statistically significant improvement compared to the control group. Students in the experimental group demonstrated a better understanding of mathematical concepts as they were required to apply their theoretical knowledge to practical examples. They also developed greater creativity and innovation by exploring different approaches to problem-solving. Furthermore, the activities encouraged their independence and active engagement in learning, contributing to better management of problem-solving situations. Based on the study's findings, the authors concluded that integrating mathematical modeling into STEM education significantly contributes to the development of students' analytical and problem-solving skills (Armutcu & Bal, 2023).

The results of the third task, "amusement park" show that Slovenian students mostly solve tasks at the first and second levels while Croatian students demonstrate greater diversity and a higher percentage of task completion at the most demanding third level. The reason for such differences can be linked to the varying approaches to the curricular mathematical content of the two educational systems where the Croatian curriculum places greater emphasis on the use of multiple strategies and mathematical operations (Sabo Junger & Lipovec, 2022). The authors of the original task also report similar results and problem-solving approaches. They note that some groups focused on entire rows of data and identified additive relationships limiting their problem-solving to that one specific strategy (Bleiler-Baxter et al., 2017). This approach is characteristic of the second level where students did not use diverse strategies but adhered to a single method such as simple addition or summing values in rows indicating a more limited approach to problem-solving.

Additionally, the study conducted by English and Sriraman (2010) support these findings. The authors emphasize that initial experiences with mathematical modeling often involve simple approaches with students gradually developing more advanced skills as they are exposed to more diverse problem-solving contexts. Furthermore, Blum and Ferri (2009) highlight that students' progress in modeling tasks often depends on the complexity of the task itself and the level of support provided during the learning process.

The analysis of the task "amusement park" shows that students used various strategies to solve mathematical modeling problems. Most students relied on linear relationships between quantities and successfully applied basic mathematical operations such as addition, subtraction, multiplication and division to establish appropriate functional relationships. This approach indicates an initial understanding of relationships between quantities which is crucial for developing the concept of functions. Similar results can be observed in the study by Çelik and Güzel (2018) who confirm that modeling tasks based on real-life situations such as analyzing linear functions, encourage students' quantitative reasoning. Their study demonstrated that tasks involving contextual data and requiring the connection of two variables increase students' cognitive engagement. This approach enabled students to develop an understanding of variable quantities and to construct functional models through mathematical abstraction and interpretation.

Connecting these results highlights the importance of tasks that expose students to realistic scenarios with multidimensional data. The "amusement park" task serves as an example of effective use of mathematical modeling. The task allows students to develop an understanding of functions through quantitative reasoning and problem-solving in real-world contexts.

The novelty of this study lies in two key aspects. The first aspect combines teacher training with the evaluation of student outcomes providing a comprehensive perspective on the effectiveness of the approach. The second aspect demonstrates how students can develop the ability to engage in mathematical modeling even with a relatively short professional development program for teachers and a limited number of classroom hours.

5.1. Limitations

Our program included only four mathematical modeling tasks adapted for students aged 10 to 11 and lasted a relatively short time, specifically a total of 6 to 8 school hours. The brevity of such a program despite the fact that this time was very intensively dedicated to the program itself and the given tasks and being supported by additional preparation efforts from teachers may have influenced the depth of engagement and potential outcomes.

In our research, we did not directly assess the mathematical knowledge of the students involved in the program or their knowledge of mathematical modeling. Therefore, we could only infer their knowledge and abilities based on the evaluation and feedback obtained from their classroom teachers.

6. CONCLUSION

The results reveal how students solve given tasks. We can also observe how students perceive the given problem and how they approach solving it using different methods and strategies. The analysis of solutions allows us to compare students' understanding of mathematical modeling in both countries and suggests potential directions for further teaching methods and approaches.

From the results, we can see that Slovenian and Croatian students generally demonstrated similar abilities in solving mathematical modeling tasks with differences primarily emerging at specific levels as well as in their approaches to the tasks. Croatian students more frequently reached higher levels emphasizing multi-criteria analysis and the application of more complex approaches while Slovenian students tended to remain at the initial levels. They relied on simple strategies such as comparing values and making intuitive decisions to arrive at solutions.

Tasks such as "buying an electric scooter" highlighted the significant role of multi-criteria analysis in developing mathematical skills. Students who solved the task at higher levels demonstrated the ability to integrate data and use criteria prioritization to make decisions. Slovenian students lagged slightly in applying these strategies while Croatian students showed greater readiness for data analysis and the use of multiple strategies in problem-solving (Akapame, 2022; Armutcu & Bal, 2023). The reader has noticed the significant number of iterations required to improve the model in the "buying an electric scooter" task. Wei, Zhang, and Guo (2022) identified a similar pattern highlighting that student often recognize flaws in their chosen models through repeated reflections and peer discussions. They noted that using a single variable may be insufficient to accurately represent the data, prompting students to improve the model's effectiveness. This iterative process of evaluation and collaboration helps students refine their mathematical reasoning and modeling skills. Consequently, they need to modify the model. Teachers can expect less experienced students to develop simpler, initial models while more experienced students will refine their models through multiple iterations. Therefore, we recommend that teachers focus initially on one specific type of task (e.g., the relationship between quantities is characteristic of the "waste sorting" and "amusement park" tasks while selecting and prioritizing criteria is central to the "buying an electric scooter" task). With a consistent context, students will become progressively more proficient at modeling the chosen type of task.

The "amusement park" task revealed similar trends with Croatian students more frequently achieving the third level of task-solving while Slovenian students predominantly remained at the second level. These differences can be attributed to variations in curricular approaches where the Croatian curriculum places greater emphasis on employing diverse strategies and mathematical operations, fostering broader problem comprehension and more comprehensive analysis (English & Watters, 2004; Sabo Junger & Lipovec, 2022). The connection between task-solving levels and methods used in modeling was clear. Slovenian students more often relied on basic strategies such as value comparisons and intuitive decisions while Croatian students demonstrated a better understanding of task complexity and a greater ability to use more advanced strategies, reflecting a higher level of mathematical literacy (Blum & Ferri, 2009; Çelik & Güzel, 2018).

The overall findings indicate that mathematical modeling tasks are crucial for developing mathematical thinking and problem-solving skills in students. The results confirm the need to introduce tasks into education that encourage the use of multiple strategies and approaches as these enable a deeper understanding of mathematical concepts and their application in real-life scenarios. As research has shown, tasks based on real-world situations not only enhance problem-solving abilities but also strengthen creativity, collaboration, and critical thinking among students (Akapame, 2022; Armutcu & Bal, 2023; English & Watters, 2004).

The conclusion of the study provides valuable insights for applying mathematical modeling in practice, particularly in the classroom. Teachers can leverage these findings by structuring tasks that progressively develop students' abilities, starting with simpler, more intuitive problems and advancing to more complex scenarios that require evaluation based on multiple criteria. By emphasizing the iterative nature of problem-solving, teachers can encourage students to reflect on and refine their models, thereby fostering deeper mathematical reasoning. This iterative process enhances critical thinking and adaptability allowing students to apply mathematical concepts in real-world situations. Such an approach not only fosters problem-solving skills but also enhances students' ability to think analytically.

By regularly engaging students in such tasks, teachers can help them better connect mathematical concepts with their real-world applications, simultaneously developing skills essential for problem-solving in today's contexts. This practice not only increases students' motivation but also ensures long-term educational benefits in the field of mathematical modeling.

7. IMPLICATIONS OF THE STUDY

The results of our study with its limitations open avenues for further research and raise additional research questions. One potential direction involves extending the program over a longer period, enabling students to work on a larger and more diverse set of mathematical modeling tasks. Such a long-term intervention could provide deeper insights into how sustained practice impacts students' problem-solving strategies, adaptability, and overall mathematical modeling proficiency. Furthermore, future research could incorporate systematic evaluation of task accuracy and the use of quantitative measures for a more comprehensive assessment of mathematical modeling problem-solving skills. This would include analyzing not only the accuracy of solutions but also the depth of reasoning, creativity in approaches, and the ability to adapt models to different contexts.

Funding: This study received no specific financial support.
Institutional Review Board Statement: Not applicable.
Transparency: The authors state that the manuscript is honest, truthful, and transparent, that no key aspects of the investigation have been omitted, and that any differences from the study as planned have been clarified. This study followed all writing ethics.
Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: Both authors contributed equally to the conception and design of the study. Both authors have read and agreed to the published version of the manuscript.

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APPENDIX

Appendix A. Task "waste sorting".

Sea Primary School and Shore Primary School had a waste collection and sorting campaign in the first three months of this year. The tables show the amount of collected and sorted waste by month and type of waste.

Sea primary school				Shore primary school			
Waste	**1st month**	**2nd month**	**3rd month**	**Waste**	**1st month**	**2nd month**	**3rd month**
Plastic	9 kg	12 kg	13 kg	Plastic	5 kg	9 kg	15 kg
Paper	8 kg	11 kg	14 kg	Paper	5 kg	8 kg	14 kg
Glass	9 kg	14 kg	18 kg	Glass	6 kg	9 kg	12 kg
Metal	10 kg	11 kg	17 kg	Metal	6 kg	10 kg	13 kg

Determine which school collected more separated waste in the first three months. Predict how much separated waste will be collected in the 4th month. Explain how you made your prediction and decide which school is more efficient at waste collection. Write a letter to the winning school explaining your prediction and decision and suggest ways the school could become even more efficient in this area.

Appendix B. Task "buying an electric scooter".

Karlo and his mom were shopping for an electric scooter. Karlo wants a scooter that is fun to ride, has a large range, and is not too expensive. Karlo's mom, who will help pay for the scooter, wants it to be reliable and not too fast. Your task is to make a list for Karlo and a list for his mom, showing which electric scooters are best for them. Based on these lists, Karlo and his mom will have to decide which scooter to buy!

Information about the electric scooters:

Scooter brand	Price	Color	Range	Max speed	Weight capacity
Segway	€439.99	Red	28 km	25 km/h	100 kg
Denver	€423.99	White	15 km	25 km/h	120 kg
Xiaomi	€317.99	Black	20 km	20 km/h	80 kg
Ducati	€609.99	Black-red	25 km	25 km/h	100 kg
Tecro	€159.99	Grey	6 km	15 km/h	50 kg
Aprilia	€927.99	Black-red	50 km	25 km/h	100 kg
Energy neutron	€329.99	Silver	22 km	20 km/h	120 kg

Images of scooters by brand:

Segway	Denver	Xiamoi	Ducati	Tecro	Aprilia	Energy neutron
						

Appendix C. Task "Amusement park" by Bleiler-Baxter et al. (2017).

The amusement park manager needs your help. His intern has only filled in part of the required data in the table. Your group will complete the table with the missing waiting times for the Amusement park Smiley. Your group's decision must be mathematically justified, and you will need to clearly explain your strategy. When completing the missing data in the table, you can use drawings, words, and symbols.

Your group will present your strategy for filling in the missing information to the entire class. During the presentation, each group member must be ready to explain the group's mathematical strategy. The goal of our class is to find the best possible solution for filling in the missing data regarding waiting times.

Important note: There is not just one correct way to complete this task. Your task is to try to find the best possible strategy.

(p. 23)

Ride	Waiting time for the ride in minutes based on the crowd at amusement par smiley			
	Low	Moderate	Busy	Very busy
Production central				
Despicable me: Minion mayhem 3D	30	50	110	155
Transformers: The ride 3D		30		
Shrek 4D	10			
Hollywood rip ride rocket	20	30	75	110
New York				
Twister: Ride it out	5	10	15	20
Revenge of the mummy				90
The blues brothers show	10			
San Francisco				
Beetlejuice's graveyard revue		15		
Disaster!			35	
World expo				
Men in black™: Alien attack			55	
The Simpsons ride	15	35	60	90
Fear factor live		10		
Woody woodpecker's Kidzone				
Animal actors on location	10	15	15	20
A day in the park with barney			20	
Curious George goes to town		20		
Woody woodpecker's nuthouse caster				45
Fievel's playland			15	
E. T. adventure			45	
Hollywood				
Universal Orlando's horror make-up show 10 20 30 45	10	20	30	45
Terminator 2: 3D			35	
Lucy—A tribute	5			