S-DIMENSIONAL ASSETS PORTFOLIO EVALUATION

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ABSTRACT

An approach to calculate the upper limit to any given portfolio with s assets (corporate stocks or government bonds) in it is provided, which does not require the relative weight of each asset in the portfolio. The value obtained is contrasted with the traditional weighting approach to calculate the portfolio's value. The process followed is the scientific method, starting with observation and hypothesis and after analyzing two examples, a synthesis is performed by generalizing the concepts and the main thesis that the Pythagorean approach here proposed constitutes an upper limit for the portfolio's value.

Keywords: Economics, Management, Finance, Assets, Portfolio, Evaluation.

Contribution/Originality

This study uses a new estimation methodology to calculate the upper limit value of a portfolio of assets without the need to incorporate the weight of each asset. If there are no weights assigned to each asset, this upper limit may be the only way to calculate the portfolio's value.

1. INTRODUCTION

There are two main commonly accepted approaches to understand reality from a philosophical point of view: science and religion. Religion approaches what is true based on what it states as true as a dogma of faith, that is, it constructs its beliefs based on a set of dogmas and statements of what should be true. Science, on the other hand, engages in a never-ending process of truth finding. Nevertheless, what usually happens in science is that what is not true becomes clear so that what remains must be the truth, at least until another theory proves otherwise.

The general process of science (Handelsman *et al.*, 2004) is illustrated in Figure 1. The beginning of the process is observation of reality, which, combined with the corresponding hypothesis of what is believed is the true leads to a given thesis. For such thesis arises an antithesis, which challenges the current thesis. Based on an analysis carried out through experimentation and empirical evidence, an analysis is done to decompose the problem in its parts

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and study them in detail. After that, a process of synthesis is used to create another observation and the corresponding hypothesis and the thesis, and so on.

Science requires the use of two almost contradictory ways of thinking: skepticism and curiosity (Sagan, 1996). Most people in the world are very curious about their reality, but they do not understand it in the way science or even well-thought religion can. There is a considerable amount of pseudoscience out there, from astrology to demon-haunting.

The purpose of this paper is to provide a novel way at estimating the upper value of a set or portfolio of assets (being corporate stocks or government bonds) by simply using the price at any given time of these assets.

Figure-1. The process of thesis and antithesis, analysis and synthesis, in science.
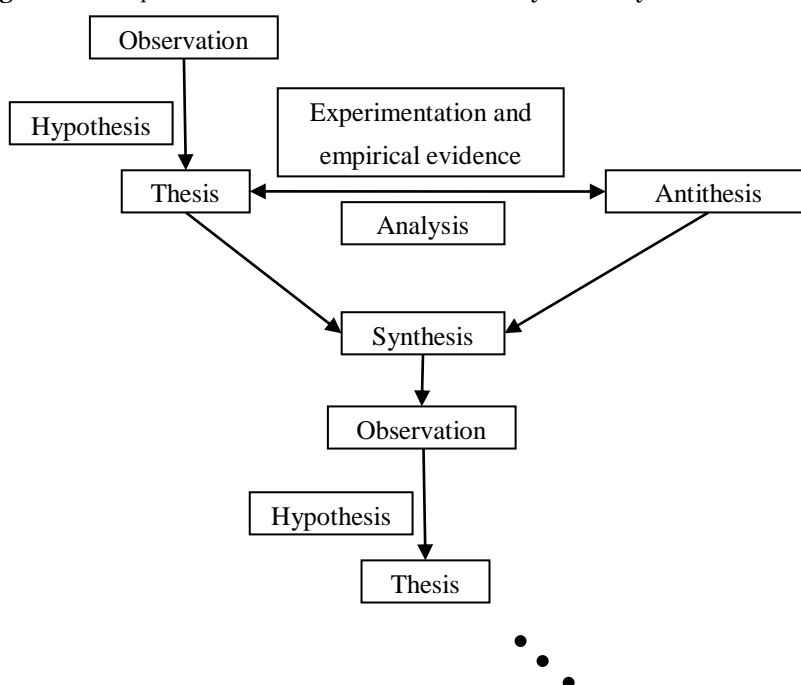
2. THEORETICAL BACKGROUND

Modern Portfolio Theory (MPT) as its name implies is not so modern (Maginn *et al.*, 2007; Project Management Institute (PMI), 2013). It was originally developed during the 1950s and further developed during the 1970s. The main idea behind MPT is that assets should not be selected based on their own merit, but rather on how the price of one changes with respect to the others. There is a tradeoff decision to make between expected return and risk. The fundamental assumption in MPT is that assets follow a normal (Gaussian) distribution. Thus, MPT is a form of diversification. Under a given set of assumptions and specific considerations, MPT explains how to select the best possible diversification strategy, that is, the best possible portfolio of assets (Lubatkin and Chatterjee, 1994).

In any case, the main concept to keep in mind when selecting a portfolio of assets (in the case of this discussion corporate shares or government bonds) is diversification or hedging. Arguments such as the ones of behavioral economics have been developed to argue against MPT (Shefrin and Statman, 2000). In order to trade corporate stocks/bonds a stock market (Kelly, 2013) is used.

In Finance, diversification (Scott, 1993) means reducing the risk of the portfolio by choosing a given set of different assets. The main idea behind diversification is "do not put all your eggs in one basket". That is, choose different shares from different areas of the economy, so that if one share falls, other(s) would rise, thus compensating the loss.

Hedging (Bychuk and Haughey, 2011) is choosing stocks that, historically, show negative correlations among them, so that if one falls, the other should rise. Hedging is not the same as diversification, since the latter is more concerned with choosing a wide variety of shares or bonds, regardless of their correlations.Behavioral economics and behavioral finance (Cartwright, 2011) study the effects of psychological (social, cognitive, and emotional) factors on the economic decisions of individuals and institutions and the derived consequences for market prices, returns, and resource allocations. Prevalent themes in behavioral finance are heuristics (Tversky and Kahneman, 1974; Gigerenzer, 2008), framing and market inefficiencies (Ritter, 2003). Heuristics (Gilovich *et al.*, 2002) are the sets of rules of thumb decision-makers use to decide upon their portfolios. Framing (Gilovich *et al.*, 2002) is the collection of anecdotes and stereotypes that constitute the mental emotional filters of individuals. Market inefficiencies (Zacks, 2011) include miss-pricings and non-rational decision-making.

The central issue in behavioral finance is to explain why decision-makers participating in the stock market make systematic errors, which affect prices and returns, creating market inefficiencies. The stock market (Fontanills and Gentile, 2001) is a specially-designed market to trade stocks (shares), government bonds and even derivatives, such as futures and options. The stock market is part of what is called financial markets, which are public markets for exchanging securities. The largest stock market in the United States, by market capitalization, is the New York Stock Exchange (NYSE). In Canada, the largest stock market is the Toronto Stock Exchange (TSE). The most important European examples of stock markets are the Amsterdam Stock Exchange, the London Stock Exchange, the Paris Bourse, and the Deutsche Börse (Frankfurt Stock Exchange). In Africa, examples include the Nigerian Stock Exchange, JSE Limited, among others. Asian examples include the Singapore Exchange, the Tokyo Stock Exchange, the Honk Kong Stock Exchange, the Shangai Stock Exchange, and the Bombay Stock Exchange. In Latin America, there are the BM&F Bovespa (Brasil), the BMV (México), among others. In Australia, there is the Australian Securities Exchange.

3. SIMPLE OBSERVATION AND HYPOTHESIS

Suppose there are two assets with prices p_1 and p_2. Imagine the first asset is worth \$300 and the second one is worth \$400. How much should the portfolio of both assets be worth? Clearly, the sum of both (\$300+\$400 = \$700) is not the answer, because the portfolio could contain different percentages of each and it is not a matter of a simple sum but rather a weighted sum. Assume there is an equal weight for both assets. Thus, the portfolio might be worth 0.5x\$300+0.5x\$400 = \$350. Is this correct? Suppose that the portfolio has a total of s assets. If each of the two assets has half that amount, there are s/2 of each. For the weighting way of calculating the portfolio's worth, equation (1) has to be applied.

$$p = \left(\frac{s}{2}p_1 + \frac{s}{2}p_2\right)\frac{1}{s} = \frac{1}{2}p_1 + \frac{1}{2}p_2$$

But, who says it is correct to find the value of the average asset? Equation (1) calculates the total value of all assets and then divides that value by the number of assets to find the value of a "typical" asset. This way of calculating the value of the portfolio seems not to be necessarily the best way to go about this requirement. The value obtained according to equation (1) of \$350 when compared to the sum of all asset prices (\$700) is \$350/\$700 = 0.5 = 50% of the maximum value possible, which not surprisingly is the weight of each asset.

The hypothesis here presented is that having a portfolio of assets means a greater value than simply the weighted sum of the respective prices. Also, that there is no need to specify the relative weights of each asset in the portfolio in order to calculate the portfolio's worth. How could such value be calculated? If the value of the first assets represents a first dimension (x) and the value of the second asset represents a second dimension (y), then the value is simply the result of the Pythagorean distance between the origin and these two dimensions, as illustrated in Figure 2.

Figure-2. The Pythagorean distance in a two-dimensional space of two assets for valuing the portfolio.
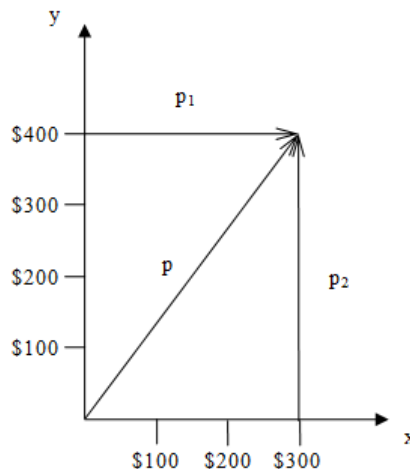
According to the Pythagoras' Theorem (Amari, 2001), p is given as indicated in equation (2).

$$p^2 = p_1^2 + p_2^2$$

By taking the square root on both sides, the value of the portfolio can be obtained as indicated in equation (3).

$$p = \sqrt{p_1^2 + p_2^2}$$

Substituting the values of p_1 ($300) and p_2 ($400) into equation (2) results in a value for the portfolio, p, of $500. Does this value make better sense than the one obtained using simple weighting ($350)? That is probably the most important question to make. Clearly, $500 is less than the sum of both values ($700), so it is not unreasonable. The hypothesis here presented is that this is the correct way of calculating the portfolio's value based on the prices of its two assets, because the values of the assets are somehow added when considered together in the portfolio in a different way than simple weighting, and also because it does not depend on the relative weight of each asset in the portfolio. The value obtained ($500) when compared to the sum of the values of the assets ($700) is $500/$700 = 0.7143 = 71.43% of the way to the sum of the assets value. Notice that the portfolio's value calculated according to equation (3) is possibly a limit to the maximum value for the portfolio as s → ∞. If the second asset is 100% of all the shares and the first asset is 0%, the portfolio's value would be $400, which is merely $100 less than the value obtained according to equation (3).

4. EXTENDING THE IDEA TO THREE DIMENSIONS

Now consider three assets. The first one is worth $200, the second one is $300, and the third one is $600. According to the weighting idea and assuming equal weights for all three assets, the value of the portfolio would be given according to equation (4).

$$p = \left(\frac{s}{3}p_1 + \frac{s}{3}p_2 + \frac{s}{3}p_3\right)\frac{1}{s} = \frac{1}{3}p_1 + \frac{1}{3}p_2 + \frac{1}{3}p_3$$

By applying equation (4) to the three prices gives a portfolio's value of $1,100/3 ≈ $366.67. The sum of all assets is $200+$300+$600 = $1,100, so the percentage represented by the value obtained to the sum of the values is 1/3 ≈ 33.33%, which is exactly the equal weight given to all three assets.

What about the Pythagorean approach to the portfolio's limit in its value? Figure 3 illustrates the new situation. Now, the price considering only the first and second prices, p_1 and p_2, is p_{xy}, as indicated in equation (5).

$$p_{xy} = \sqrt{p_1^2 + p_2^2}$$

Because there is a 90° angle between p_{xy} and p_3, the relationship between p, p_{xy} and p_3 is given according to equation (6), because the Pythagorean Theorem applies again, as indicated in equation (6).

$$p^2 = p_{xy}^2 + p_3^2$$

Figure-3. The Pythagorean distance in a three-dimensional space of three assets for valuing the portfolio.
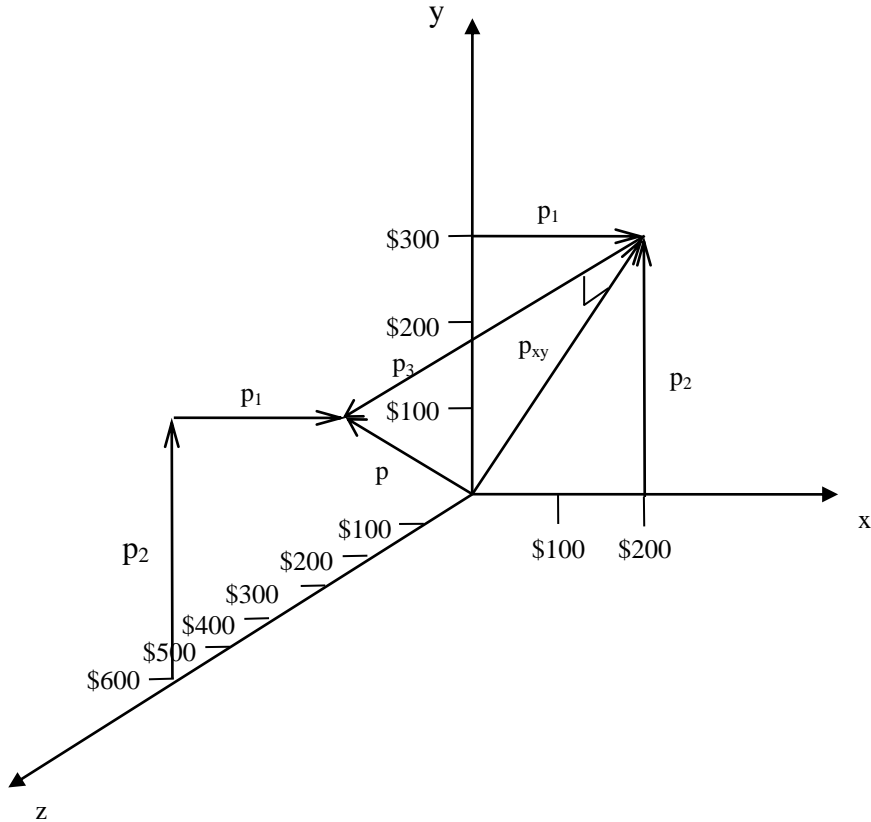
Substituting equation (5) into equation (6) yields equation (7).

$$p^2 = p_1^2 + p_2^2 + p_3^2$$

Simplifying p from equation (7) yields equation (8).

$$p = \sqrt{p_1^2 + p_2^2 + p_3^2}$$

By substituting into equation (8) p_1 = $200, p_2 = $300, and p_3 = $600, yields p = $700. This value is quite far from the value obtained by applying equation (4) of approximately $366.67. The total sum of all prices is $200+$300+$600 = $1,100. The value obtained using the weighting approach is 1/3 or approximately 33.33% of that total sum, whereas the value obtained according to equation (8) is $700/$1,100 ≈ 63.63%.

5. GENERALIZING INTO S DIMENSIONS

As can be seen, as the dimensions increase, the value obtained according to the Pythagorean approach becomes farther away from the simple sum of the values of all assets. Apparently, the value p calculated according to the Pythagorean approach as opposed to the weighting approach is an upper limit to the portfolio's value.

Generalizing the concept to s assets that are included in a portfolio, the upper limit for the portfolio's value is given according to equation (9), where p_j is the value of asset j for all j = 1,…,s. This value can be contrasted to the value obtained according to the generalization of the weighting approach for s stocks, where p_j is the price of stock j and w_j is the relative weight in the portfolio for such stock, as indicated by equation (10).

$$\lceil p \rceil = \sqrt{\sum_{j=1}^{s} p_j^2}$$

$$p = \sum_{j=1}^{s} w_j p_j$$

6. DISCUSSION AND CONCLUSION

It is somewhat suspicious that if the same weight is used for all the assets in a portfolio with size s, the rate of the portfolio's value according to the weighting approach to the sum of all the asset prices in the portfolio is the same as the equal weight used, whereas the same rate applied to the upper limit calculated according to the Pythagorean approach proposed in equation (9) tends to decrease as the portfolio's size increases.

It may be too bold to say that the value obtained using equation (9) is the true value of the portfolio, but it is reasonable to say that such value is the upper limit for the portfolio's value, p, calculated according to equation (10) regardless of the weights of each and every asset in the portfolio.

What happens when s → ∞. Supose p_j = 1 for all j = 1,…,s. Also suppose the weight is the same for all asset prices, so that if there are s assets, the weight of each is 1/s. According to the weighting approach shown in equation (10), substituting results in equation (11).

$$p = \sum_{j=1}^{s} \frac{1}{s}(1) = \sum_{j=1}^{s} \frac{1}{s} = \frac{s}{s} = 1$$

However, s = ∞, and substituting into equation (11) results in equation (12).

$$p = \frac{s}{s} = \frac{\infty}{\infty}$$

But what is the value of ∞/∞? It is also ∞, because ∞x∞ = ∞, just like 6/3 = 2 because 2x3 = 6. There is apparently a contradiction between the results from equation (11) and equation (12), but that is not so, simply because it is not possible to cancel out the infinities.

On the other hand, according to the Pythagorean approach, and taking equation (9) and substituting results in equation (13).

$$\lceil p \rceil = \sqrt{\sum_{j=1}^{s} 1^2} = \sqrt{s}$$

Substituting ∞ for s yields equation (14).

$$\lceil p \rceil = \sqrt{s} = \sqrt{\infty} = \infty$$

So as can be seen in this simple example, both approaches give a value of ∞ for p and $\lceil p \rceil$ when s → ∞. So it seems the Pythagorean approach is valid in theory as an upper limit.

Further research may consider the future and past value for the upper limit according to the Pythagorean approach as well as the portfolio's value calculated according to the weighting approach, given some internal rate of return i.

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