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TEACHING HYPOTHESIS TEST IN AN EASY WAY

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ABSTRACT

This paper introduced an easy way for selecting null (H_0) and alternative hypotheses (H_1), finding p-values and taking decisions. Should a research hypothesis or a claim be H_0 or H_1? This paper proposed a new method to select H_0 and H_1. Considering two sets of hypotheses (a) H_0: $\theta \leq \theta_0$ against H_1: $\theta > \theta_0$ and (b) H_0: $\theta \geq \theta_0$ against H_1: $\theta < \theta_0$ for testing a location parameter θ if the absolute value of the test statistic obtained from a sample is less than the absolute critical value then the two decisions corresponding to two sets of hypotheses (a) and (b) are contradictory. This paper proposed new methods to overcome these issues and making consistent decision. This paper also introduced a general formula for finding P-value. These new methods and formula have been applied in an undergraduate class. It is observed that the performances of proposed methods are significantly better than that of the traditional methods.

Keywords: Hypothesis test, Research hypothesis, Significance level, Critical values, P-values, Rejection region.

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Contribution/ Originality:

The paper's primary contribution is finding an easy way for selecting null and alternative hypotheses. This study uses new three region hypothesis testing method and introduces new formula for finding P-values. It is one of very few studies which have investigated practically the effectiveness of the proposed method.

1. INTRODUCTION

Social science, Commerce and Business students usually have limited knowledge in Mathematics, especially in Algebra and Calculus. Mathematical Statistics basically deals with probability distributions of random variables, their properties, estimation and test of hypothesis regarding the parameters of probability distributions. In hypothesis testing the test statistic is selected in such a way that it follows some standard probability or sampling distribution when the null hypothesis is true. Question may arise, how can we select null and alternative hypotheses?

Why a selected test statistic follows a certain probability or sampling distribution? How can we find the P-value? How can we make the decision? The derivation of the distribution of the test statistic requires extensive knowledge on Algebra and Calculus. It is difficult to explain these to students with limited knowledge in Mathematics or probability distribution theory. In order to give understanding of selecting null and alternative hypotheses, finding P-value and making decision in testing hypothesis to such type of students needs some simple way. This paper proposed a simple method to address these issues which is discussed in section 3. Section 4 deals with the applications of the proposed methods. Section 5 is the concluding remarks.

2. LITERATURE REVIEW

Selection of null and alternative hypotheses were discussed by many authors such as Anderson *et al.* (2011); Lehmann and Romano (2010); Shi and Tao (2008); Rao (1973); Bickel and Doksum (1977); Bain and Engelhardt (1992); McClave *et al.* (2005) etc. Anderson *et al.* (2011) proposed a general guideline for selecting null and alternative hypotheses. They considered a particular automobile model that currently attains an average fuel efficiency of 24 miles per gallon. They have mentioned that a product research group has developed a new fuel injection system specifically designed to increase the miles per gallon rating. In this case, the research hypothesis is that the new system will provide a mean miles-per-gallon rating exceeding 24 that is, $\mu > 24$. They have proposed as a general guideline, a research hypothesis should be stated as the alternative hypothesis. Hence, the appropriate null and alternative hypotheses for this study are H_0: $\mu \leq 24$ and H_a: $\mu > 24$.

As an illustration of testing the validity of a claim, Anderson *et al.* (2011) considered a manufacturer of soft drinks who states that two-liter containers of his product have an average of at least 67.6 fluid ounces. A sample of two-liter containers was selected, and the contents were measured to test the manufacturer's claim. In this type of situation, it is assumed that the manufacturer's claim is true unless the sample evidence proves otherwise. Using this approach for the soft-drink example, they stated the null and alternative hypotheses as H_0: $\mu \geq 67.6$ and H_a: $\mu < 67.6$. They said a manufacturer's claim is usually given the benefit of the doubt and stated as the null hypothesis. In any situation that involves testing the validity of a claim, the null hypothesis is generally based on the assumption that the claim is true.

Therefore, according to Anderson *et al.* (2011) a research hypothesis should be stated as the alternative hypothesis and in testing the validity of a claim, the null hypothesis is based on the claim. These two statements are contradictory. The hypotheses were selected correctly in the above two examples but generalization are confusing and unacceptable. This paper proposed some easy methods to select null and alternative hypotheses.

It is believed that people who live in northern Illinois are less likely to return sweepstakes entries than people who live in southern Illinois. Three hundred sweepstakes entries were sent to people in northern Illinois, and 300 entries were sent to people in southern Illinois. In response, 54% of the people in northern Illinois returned the entries, and 50% of the people in southern Illinois returned the entries. Let P_1 and P_2 be the proportions of peoples returning the sweepstakes entries for northern and southern Illinois respectively. Here we state the null and alternative hypotheses as H_0: $P_1 - P_2 \geq 0$ and H_a: $P_1 - P_2 < 0$. Then the test statistic is,

$$Z = \frac{(\overline{P}_1 - \overline{P}_2) - (P_1 - P_2)}{\hat{\sigma}_{\overline{P}_1 - \overline{P}_2}}, \text{ where } \hat{\sigma}_{\overline{P}_1 - \overline{P}_2} = \sqrt{\overline{P}(1-\overline{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \text{ and } \overline{P} = \frac{n_1\overline{P}_1 + n_2\overline{P}_2}{n_1 + n_2},$$

and the value of the test statistic is

$Z = 0.98$ and P-value = $P(Z < 0.98) = 0.8365$. For $\alpha = 0.05$, reject H_0 if P-value < 0.05 or, $Z < -1.645$. Therefore, do not reject H_0. That is people who live in Northern Illinois are not less likely to return sweepstakes entries than people who live in Southern Illinois. For the same problem if we are interested in testing the hypothesis that people who live in northern Illinois are more likely to return sweepstakes entries than people who live in southern Illinois. Then we would state the null and alternative hypotheses as H_0: $P_1 - P_2 \leq 0$ and H_a: $P_1 - P_2 > 0$. The value of the test statistic is $Z = 0.98$ and P-value = $P(Z > 0.98) = 0.1635$. For $\alpha = 0.05$, reject H_0 if P-value < 0.05 or, $Z > 1.645$. Therefore, do not reject H_0. That is people who live in Northern Illinois are not more likely to return sweepstakes entries than people who live in Southern Illinois. Therefore, using the same information we come up with two contradictory decisions. This paper proposed three regions, hypothesis testing procedure to avoid these contradictory decisions.

3. PROPOSED METHODS

3.1. Selecting Null and Alternative Hypotheses

In any research study, we usually interested in testing the validity of a claim or statement. In order to setup the null and alternative hypotheses, first we should identify the claim or statement that we are going to test in language or in Mathematical form. Then write down the alternative statement or claim in such a way that both statements/claims accommodate all the possibilities. Then the statement/claim that indicates or includes the equality sign will be the null hypothesis and other statement/ claim is the alternative hypothesis.

3.2. Finding P-Values

An easy and straight forward way to find P-value for testing location parameter is stated below:

(i) For one-tailed test P-value = $P\{T >$ (or < depending on H_a) $T_{obs}\}$.

(ii) For two-tailed test P-value = $2P\{T > |T_{obs}|\}$.

3.3. Three Regions Hypothesis Testing Procedures

Let ($x_1, x_2, x_3, \ldots, x_n$) be a random sample of size n drawn from a population the distribution of which has a known mathematical form say $f(x/\theta)$ but involves some unknown location parameter θ. Let T be a function of $x_1, x_2, x_3, \ldots, x_n$ and is used as a test statistic to test (i) H_0: $\theta \le \theta_0$ and H_a: $\theta > \theta_0$ or, (ii) H_0: $\theta \ge \theta_0$ and H_a: $\theta < \theta_0$, or, (iii) H_0: $\theta = \theta_0$ and H_a: $\theta \ne \theta_0$. Here the parameter space can be divided into three regions $\Omega_L = \{\theta < \theta_0\}$, $\Omega_0 = \{\theta = \theta_0\}$, and $\Omega_U = \{\theta > \theta_0\}$. That is in (i) H_0 is the union of Ω_L and Ω_0, in (ii) H_0 is the union of Ω_0 and Ω_U, and in (iii) H_a is the union of Ω_L and Ω_U. These hypotheses can be tested in two ways: comparing the observed value of the test statistic T with the critical value or comparing P-value with the significance level α.

For testing (i), if the observed value of T (T_{obs}) obtained from the sample is less than $T_{1-\alpha}$ then conclude that $\theta < \theta_0$ and if $T_{1-\alpha} < T_{obs} < T_\alpha$ then conclude that $\theta = \theta_0$ and if $T_{obs} > T_\alpha$ then reject H_0 and conclude in favour of H_a that is $\theta > \theta_0$. Alternative way for testing (i): if P-value > 1-α then conclude that $\theta < \theta_0$ and if α < P-value < 1-α then conclude that $\theta = \theta_0$ and if P-value ≤ α then reject H_0 and conclude in favour of H_a that is $\theta > \theta_0$.

For testing (ii), if $T_{obs} < T_{1-\alpha}$ then reject H_0 and conclude in favour of H_a that is $\theta < \theta_0$, if $T_{1-\alpha} < T_{obs} < T_\alpha$ then conclude that $\theta = \theta_0$ and if $T_{obs} > T_\alpha$ then conclude that $\theta > \theta_0$. Alternative way for testing (ii): if P-value ≤ α then reject H_0 and conclude in favour of H_a that is $\theta < \theta_0$ and if α < P-value < 1-α then conclude that $\theta = \theta_0$ and if P-value > 1- α then conclude that $\theta > \theta_0$.

For testing (iii), if $T_{obs} < T_{1-\alpha/2}$ then conclude that $\theta < \theta_0$ and if $T_{1-\alpha/2} < T_{obs} < T_{\alpha/2}$ then conclude in favour of H_0 that is $\theta = \theta_0$ and if $T_{obs} > T_{\alpha/2}$ then conclude that $\theta > \theta_0$. Alternative way for testing (iii): if P-value ≤ α and the sign of T_{obs} is negative then conclude that $\theta < \theta_0$ and if P-value > α then conclude in favour of H_0 that is $\theta = \theta_0$ and if P-value ≤α and the sign of T_{obs} is positive then conclude $\theta > \theta_0$.

4. APPLICATION OF THE PROPOSED METHODS

4.1. Selecting Null and Alternative Hypotheses

In Automobile model problem, the research group has developed a new fuel injection system specifically designed to increase the miles per gallon rating which is currently 24 miles per gallon. Here we are interested in testing the hypothesis that the new system will provide a mean miles-per-gallon rating exceeding 24 that is, $\mu > 24$. Other possibility is $\mu \le 24$. Out of these two possibilities, the one with equality sign is the null that is H_0: $\mu \le 24$ and H_a: $\mu > 24$.

In soft drink containers problem, the manufacturer claimed that two-liter containers of his product has an average of at least 67.6 fluid ounces that is $\mu \ge 67.6$. Other possibility is $\mu < 67.6$. Out of these two possibilities, the one with equality sign is the null that is H_0: $\mu \ge 67.6$ and H_a: $\mu < 67.6$.

Microsoft Outlook is believed to be the most widely used e-mail manager. A Microsoft executive claims that Microsoft Outlook is used by more than 85% of the internet users. However, a study on 300 sample observations reported 270 use Microsoft Outlook. What can you say about the Microsoft executive claim at 2% level of significance? Let P be the proportion of internet users those used Microsoft Outlook. The Microsoft executive claim is $P > 0.85$. Other possibility is $P \leq 0.85$. Out of these two possibilities the one with equality sign is the null that is H_0: $P \leq 0.85$ and H_a: $P > 0.85$.

4.2. Finding P-Values

In the above example of returning sweepstakes entries for testing H_0: $P_1 - P_2 \geq 0$ against H_a: $P_1 - P_2 < 0$, the value of the test statistic $Z_{obs} = 0.98$.

P-value = $P(Z < 0.98) = 0.8365$.

In the same example of returning sweepstakes entries for testing H_0: $P_1 - P_2 \leq 0$ against H_a: $P_1 - P_2 > 0$, the value of the test statistic $Z_{obs} = 0.98$.

P-value = $P(Z > 0.98) = 0.1635$.

In the Microsoft Outlook example for testing H_0: $P \leq 0.85$ against H_a: $P > 0.85$, the test statistic is

$$Z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}}, \text{ where } \sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.85(1-0.85)}{300}} = 0.021, \ \bar{p} = 0.90,$$

$$Z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{0.90 - 0.85}{0.021} = 2.4.$$ P-value = $P(Z > 2.4) = 0.0082$.

4.3. Three Regions Hypothesis Testing Procedures

In the above example of returning sweepstakes entries for testing H_0: $P_1 - P_2 \geq 0$ against H_a: $P_1 - P_2 < 0$, the value of the test statistic $Z_{obs} = 0.98$. At $\alpha = 0.05$ the critical value is -1.645 and the rejection rule is (i) reject H_0 if $Z_{obs} < -1.645$, (ii) conclude $P_1 = P_2$ if $-1.645 < Z_{obs} < 1.645$ and (iii) conclude $P_1 > P_2$ if $Z_{obs} > 1.645$. As $Z_{obs} = 0.98$, conclude $P_1 = P_2$. Alternative way is, as $0.05 <$ P-value $= 0.8365 < 0.95$, conclude $P_1 = P_2$.

On the other hand for testing H_0: $P_1 - P_2 \leq 0$ against H_a: $P_1 - P_2 > 0$, the value of the test statistic is $Z_{obs} = 0.98$. At $\alpha = 0.05$, the critical value is 1.645 and the rejection rule is (i) reject H_0 if $Z_{obs} > 1.645$, (ii) conclude $P_1 = P_2$ if $-1.645 < Z_{obs} < 1.645$ and (iii) conclude $P_1 < P_2$ if $Z_{obs} < -1.645$. As $Z_{obs} = 0.98$, conclude $P_1 = P_2$. Alternative way is, as $0.05 <$ P-value $= 0.1635 < 0.95$, conclude $P_1 = P_2$. Therefore, the decisions in both cases are same.

In the Microsoft Outlook example for testing H_0: $P \leq 0.85$ against H_a: $P > 0.85$, the test statistic is $Z = 2.4$. At $\alpha = 0.02$, the critical value is 2.055 and the rejection rule is (i) reject H_0 if $Z_{obs} > 2.055$, (ii) conclude $P = 0.85$ if $-2.055 < Z_{obs} < 2.055$ and (iii) conclude $P < 0.85$ if $Z_{obs} < -2.055$. As $Z = 2.4$, reject H_0. Alternative way is, as P-value $= 0.0082 < 0.02$, conclude $P > 0.85$. Therefore, the claim is true.

4.4. Practical Application

We offered Business Statistics course STAT 1811 for undergraduate students in each semester at our University. In spring 2012, I taught two Sections of this course named section 10 (S-10) and section 20 (S-20). There were 40 students in each section. Statistics for Business and Economics written by Anderson *et al.* (2011) was the text book. In section 10, I taught the methods mentioned in the text book and in section 20, I taught the methods proposed in this paper. In one test we set two questions similar to Automobile model problem and soft drink containers problem and checked how many of them answered correctly. The results are given in Table 1.

Table-1. Test results of two sections for comparing the two teaching methods

Teaching Method	# of students attended	Hypotheses selected correctly	P-value calculated correctly	Conclusion made correctly
Traditional (S-10)	36	20	15	14
Proposed (S-20)	34	27	23	23

Source: STAT1811 Course file, Department of OMBS, CEPS, SQU, Spring 2012

The results in Table 1 indicate some improvement for applying proposed method. To test whether this improvement is significant or not we performed two sample proportion test and consider the following hypotheses.

H_0: $P_1 - P_2 \geq 0$ against H_a: $P_1 - P_2 < 0$, where P_1 is the proportion of correct answer using traditional method in Section 10 (S-10) and P_2 is the proportion of correct answer using proposed method in Section 20 (S-20).

Test statistic, $Z = \dfrac{\overline{P}_1 - \overline{P}_2}{\hat{\sigma}_{\overline{P}_1 - \overline{P}_2}}$, $\hat{\sigma}_{\overline{P}_1 - \overline{P}_2} = \sqrt{\overline{P}(1-\overline{P})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}$ and $\overline{P} = \dfrac{n_1\overline{P}_1 + n_2\overline{P}_2}{n_1 + n_2}$

The test results are given in Table 2,

Table-2. Hypotheses testing results for comparing the two teaching methods

Method	Hypotheses selected correctly	P-value calculated correctly	Conclusion made correctly
$\overline{p}_1$	0.556	0.417	0.389
$\overline{p}_2$	0.794	0.676	0.676
$\overline{p}$	0.671	0.543	0.529
Z	-2.125	-2.176	-2.412
P-value	0.017	0.015	0.008
Conclusion	Significant	Significant	Significant

Source: Summary of calculations using test statistic and P-value formula for the data in Table 1

The test results indicate that the proposed methods significantly improve the hypothesis testing concept of the students. The proposed methods are better than traditional methods.

5. CONCLUSION

The proposed methods are simple, easy and straightforward. These methods will be very useful for students/researchers with limited knowledge in Mathematical Statistics to select null and alternative hypotheses, finding P-values and making decision for any practical problem without confusion. Three regions hypothesis testing procedure would give consistent decisions for all practical situations. The performances of the proposed methods are better than that of traditional methods.

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