



SOME NON-LINEAR PROBLEMS IN ACCOUNTING AND FINANCE: CAN WE APPLY REGRESSION?

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ABSTRACT

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Recent studies have indicated that many decision problems in accounting and finance can be better modeled by non-linear models in practice. However, existing literatures have also shown that managers and decision makers are not very conversant with non-linear models as compared to linear models because of the simplicity of linear models. In this paper, attempts are made to transform some non-linear models in accounting and finance which conform to exponential and power functions to their equivalent linear forms. The resulting equivalent linear models are subjected to regression analysis. The paper documents interesting practical non-linear problems in accounting and finance where it is possible to apply regression, and provides technical interpretations of coefficients of resulting regression equations. Some non-linear problems which have been documented in this analysis include; depreciation of non-current assets, the learning curve model, life cycle costing, compounding, discounting and exponential growth bias. Although logarithmic transformation of non-linear functions is not a novel idea in literature of accounting and finance, there is no evidence in literature that scholars have proposed particular cases in finance and accounting where these linear transformations and their resulting regression equations would yield meaningful results that can enhance management decision making. This paper fills this gap by documenting practical non-linear problems in finance and accounting where linearization and subsequent application of regression analysis generates useful results for management decision making purposes.

Contribution/Originality: In the literature of accounting and finance specifically; firstly, this paper originates new formulae for some non-linear problems. Secondly, it's among very few papers which examined regression of non-linear problems. Thirdly, this is the first paper to document practical cases where regression of log transformed variables generates useful results.

1. INTRODUCTION

Economists and Business analysts have long assumed linear relationships among many business and economics variables. A simple Example is seen in Keynesian economics where consumption is expressed as a linear function of income. In finance, the popular Capital Asset Pricing Model (CAPM) expresses the expected return on any security as a linear function of Return of the market in excess of the risk free asset. Moreover, in international finance, regression technique can also be used to assess the firm's economic exposure by analyzing historical cash flows and exchange rate data (Madura, 2015). In cost analysis, total costs of a firm are always assumed to be a linear function of the number of units produced. In business, regression analysis is used for predictive purposes, for example sales forecast. Assumption of the linearity relationship may of course limit the applications of regression because many

business and economic problems are non-linear in nature. In such circumstances, some form of non-linear or curve-linear models can be more suitable.

Recent empirical studies in finance and accounting such as [Jaisinghani and Kanjilal \(2017\)](#); [Vatavo \(2016\)](#); [Thanatawee \(2016\)](#) and [McMillan \(2012\)](#) have indicated very strong evidence of non-linear relationships among many finance and accounting variables. There are also evidences that non-linear models in some circumstances are more accurate in practice than linear models for simulating business operations. Although these models are gaining a lot of popularity in practice and theory, papers such as [Wu and Li \(2017\)](#) argued that managers and other decision makers find a lot of difficulties in applying these models because of their complexities as compared to linear models. In order to ease this problem, it is advisable that the scholarly community, once again, emphasize the need to transform some non-linear models to their approximate linear forms. Logarithmic transformation of non-linear models into linear form is not a novel idea in literature and indeed, the procedures and mechanics of transformation are well illustrated by [Benoit \(2011\)](#); [Rusov, Misita, Milanovic, and Milanovic \(2017\)](#) and [Lucey \(2002\)](#). The scholarly community comprehensively emphasized the relevance and applications of these transformations for simplifying some economic problems. For instance, [Benoit \(2011\)](#) and [Ayenew \(2016\)](#) noted that logarithmic transformation of non-linear models results to linear log regression models whose slope parameters (slope coefficient of log regression model) give a direct measure of elasticity instead of marginal effect, therefore, economists use this technique to measure the elasticity of one variable with respect to another (measure of percentage change in one variable for a given percentage change in another variable). Apparently, the applications of logarithmic transformations of non-linear models in finance & accounting decisions have been under looked.

This paper tries to fill this gap by analyzing specific non-linear problems in finance and accounting whose transformations into linear forms can yield meaningful results for management's decision making purposes. Specifically, the paper addresses non-linear problems that conform to exponential functions and power functions. This technique permits the application of regression analysis as a simple management's decision making tool. With the current developments in information technology, in practice, the decision maker would simply run these regressions using statistical soft wares and obtain various results. The paper also tries to give technical interpretations of coefficients of resulting log regressions in the context of accounting and finance. Although few cases have been discussed, the techniques applied here can be extended to all non-linear problems in finance and accounting which are exponential in nature or appear in power form.

The specific objectives of this paper are as follows:

- i. To linearize some non-linear functions in accounting and finance.
- ii. To apply regression analysis after the transformation using practical examples.
- iii. To interpret the coefficients of the resulting log regression equation.

2. LITERATURE REVIEW

Recent literatures in empirical finance and accounting have shown the existence of non-linear relationship among many financial and accounting variables. In the study of optimum capital structure, [Jaisinghani and Kanjilal \(2017\)](#) and [Vatavo \(2016\)](#) showed that relationship between capital structure and the firm's profitability can be best explained by non-linear model. Similarly, [Thanatawee \(2016\)](#) studied the relationship between share repurchase and liquidity of 75 firms listed in the stock exchange of Thailand and found the relationship to be non-linear. Specifically, at low level share repurchases, repurchasing more shares increases liquidity but at high level share repurchases, acquiring more shares impair liquidity. In service sector, [Saha and Yap \(2015\)](#) further found a non-linear relationship between corruption and the demand for tourism. In the study of exchange rate uncertainty and trade growth, [Herwartz and Weber \(2005\)](#) compared the accuracy of linear and non-linear models in terms of their forecast errors and noted that non-linear models outperformed linear models for forecasting purposes. Similarly, [Wu and Li \(2017\)](#) argued that non-linear models are sometimes believed to simulate business operations better in

practice, thus enabling managers to make accurate decisions. The paper presented different theories and methods of non-linear models that can be directly applied in cost and management accounting. Similarly, in life cycle costing, many researchers have shown the relevance of non-linear model in analyzing the total costs of the product or project throughout its life. For instance, [Lapašinskaitė and Boguslauskas \(2006\)](#) proposed that the total life cycle cost of a product is an exponential function of time.

Despite the prevalence of non – linear models in finance and accounting, [Wu and Li \(2017\)](#) argued that many managers and decision makers are not very conversant with these models and instead prefer linear models because of their simplicity. Fortunately, some of these non-linear models can be transformed to linear models using logarithms. Papers such as [Benoit \(2011\)](#) and many other literatures on basic econometrics have demonstrated how to transform non-linear models to their approximate linear forms using logarithms. However, the general level of competence in this paper involves linearization of non- linear models that conform to exponential functions and power functions. Thus, non-linear specifications involving quadratic functional relationship and cubic functions (for example cubic cost functions), frequently encountered in empirical finance and accounting are outside the scope of this paper, and in fact they cannot easily be linearized through logarithmic transformation. Although logarithmic transformation of non- linear functions is not a novel idea in literature of accounting and finance, there is no evidence in literature that scholars have proposed practical cases in finance and accounting where these linear transformations would yield meaningful results that can aid management decision making. This paper fills this gap by pointing out practical non-linear problems in finance and accounting where linearization and subsequent application of regression analysis generates useful results for management decision making purposes.

3. THEORETICAL FRAMEWORK OF REGRESSION MODEL

The regression equation can be defined as a mathematical expression used for showing the linear relationship between the independent and dependent variables. The equation is given in the form:

$$Y = a + bX \quad (1)$$

Where

y = dependent variable example sales.

x = independent variable example advertising expenditures.

a = is a constant.

b = slope of the regression equation.

The task here is to find the values of the constants a, and b in order to make the equation complete. We can find the values of a, and b using least square method as indicated in [Johnston and DiNardo \(1997\)](#) and many other literature on statistics. Let's consider [Equation 1](#) for further analysis.

When we use β as an approximate value of b in [Equation 1](#), the resulting regression equation will not generate a perfect accurate estimate of Y. The resulting regression equation will be associated with an error term, e as shown below:

$$Y = a + \beta X + e \quad (2)$$

The aim of the Least square method is to choose the values of a, and β which minimize the sum of squared error terms e. [Equation 2](#) can alternatively be expressed as:

$$e = Y - a - \beta X \quad (3)$$

On squaring and summing both sides of [Equation 3](#), the sum of squared error term can be expressed as below:

$$\sum e^2 = \sum (Y - a - \beta X)^2 \quad (4)$$

From differential calculus, we shall partially differentiate [Equation 4](#) with respect to β and a, and set these derivatives to zero as below:

$$\frac{\partial \Sigma \epsilon^2}{\partial \beta} = -\Sigma 2X(Y - a - \beta X) = 0$$

It follows that:

$$-\Sigma 2xy + 2a\Sigma x + 2\beta\Sigma x^2 = 0 \quad (5)$$

Equation 5 gives the first normal equation; it can be simplified by cancelling 2 throughout and making the summation of xy the subject.

Again, we partially differentiate Equation 4 with respect to a , and set that partial derivative to zero.

$$\frac{\partial \Sigma \epsilon^2}{\partial a} = -\Sigma 2(Y - a - \beta X) = 0$$

$$-2\Sigma y + 2\Sigma a + 2\beta\Sigma x = 0$$

On re arranging and cancelling 2 throughout, we generate the second normal equation given by Equation 6.

$$\Sigma y = \Sigma a + \beta\Sigma x \quad (6)$$

The term Σa in Equation 6 is the summation of n – constant values which is equivalent to multiplying this constant value a by n , thus giving rise to Equation 7

$$\Sigma y = na + \beta\Sigma x \quad (7)$$

Equation 7 gives the second normal equation. We shall solve the two normal equations Equation 5 and 7 simultaneously.

From Equation 7, we note that the value of the constant, a is now given by Equation 8. Equation 8 gives an expression for the intercept of the regression equation.

$$a = \frac{\Sigma y}{n} - \frac{\beta\Sigma x}{n} \quad (8)$$

We now Substitute for a , in Equation 5 and solve for β with some few algebraic manipulations shown in Equation 9 through Equation 13.

$$\Sigma xy = \Sigma x \left(\frac{\Sigma y}{n} - \frac{\beta\Sigma x}{n} \right) + \beta\Sigma x^2 \quad (9)$$

The bracket term on the right of Equation 9 is a direct consequence of substituting for a in Equation 5.

$$\Sigma xy = \frac{\Sigma x \Sigma y}{n} - \frac{\beta(\Sigma x)^2}{n} + \beta\Sigma x^2 \quad (10)$$

On opening the bracket term of Equation 9, we generate Equation 10 which is now less compact as compared to Equation 9.

$$n\Sigma xy = \Sigma x \Sigma y - \beta(\Sigma x)^2 + \beta n\Sigma x^2 \quad (11)$$

To avoid quotient terms in Equation 10, we multiply it throughout by the denominators, n and this gives rise to Equation 12.

$$n\sum xy - \sum x \sum y = \beta(n\sum x^2 - (\sum x)^2) \quad (12)$$

After factoring out the common term, β from Equation 11, we shall obtain Equation 12.

$$\beta = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \quad (13)$$

From Equation 12, the slope parameter, β can therefore be expressed as a function of x , y and n as given by Equation 13.

Suppose we have the non-linear function, example an exponential function for forecasting exponential growth or decay where the variable is expected to grow or decline by the same proportion or percentage in each period. This function can be given by:

$$y = ab^x \quad (14)$$

Where

y = the variable to be predicted (shown on vertical axis).

a & b = constants

x = denotes the number of period (Shown on horizontal axis).

Introducing logs to both sides of Equation 14 and applying the laws of logarithms, we shall have the linear function of the form:

$$\text{Log } y = \log a + x \log b \quad (15)$$

From Equation 15, we note that dependent variable, y has been transformed to logged variable, and constants a , & b have also been transformed to logs. However, the independent variable x remains unchanged. This transformation is therefore called semi log transformation. However, we may also have non-linear problem on the form:

$$y = ax^\beta \quad (16)$$

Where

y is dependent variable (variable to be predicted).

a and β are constants.

x is the independent variable, in some cases it may represent the time periods.

Unlike in the previous case where only dependent variable would undergo logarithmic transformation, in this case, both dependent and independent variables will be logged, thus the name log-log transformation. By introducing logs to both sides of Equation 16, we shall have:

$$\text{Log } y = \log a + \beta \log x \quad (17)$$

From Equation 17, the dependent variable $\log y$, is now a linear function of independent variable, $\log x$, with $\log a$ as the intercept and β as the slope coefficient.

4. APPLICATIONS

This section provides different illustrations of how to apply linear regression model to non-linear problems in accounting and finance. Some specific non-linear problems discussed here includes: Depreciation of non-current assets using reducing balance method, Compounding, and the learning curve model.

4.1. Depreciation of Non-Current Assets – Reducing Balance Method

According to Francis (2004) if the book value, B , is subjected to reducing balance depreciation at the rate $100i\%$ over n period, the depreciated value at the end of the n -th time period is given by:

$$D = B(1 - i)^n \quad (18)$$

Where

D = depreciated value at the end of the n-th time period.

B = original book value (original purchase price of the asset).

i = depreciation rate as a percentage.

n = number of time period normally years.

We recognize that Equation 18 is an exponential function of the, $y = ab^x$ where in this case $y = D$, $a = B$, $1-i = b$ and $n = x$. On applying semi logarithmic transformation on Equation 18, the resulting linear form is as follows:

$$\text{Log } D = \text{log } B + n \text{ log } (1-i) \quad (19)$$

Equation 19 can alternatively be given as:

$$\text{Log } D = A + \beta n \quad (20)$$

Where $A = \text{log } B$ and $\beta = 1-i$

Equation 20 is now a simple linear regression of log depreciated value on independent variable n, the number of time period. The intercept A is the logged book value of the assets and the slope parameter β , captures the information about the depreciation rate.

4.2.1. The Concept of Compounding

The basic compounding formula is given by:

$$S = P(1 + r)^n \quad (21)$$

Where

S = Sum arising in the future at the end of the nth-period.

P = Initial amount invested or deposited in the bank at the beginning of the period.

r = Rate of interest expressed as an interest rate per annum.

n = number of interest bearing periods, usually expressed in years.

Analogous to Equation 21 is the formulae for computing the future value of an investment. It is given by:

$$FV_n = V_0(1 + r)^n \quad (22)$$

Where

FV_n = Future value of an investment at the end of the nth-period

V_0 = Initial amount invested at the beginning of the period

r = Rate of return on investment

n = number of years for which the money is invested

We shall show that Equation 21 or its counterpart Equation 22 can be linearized, and subjected to regression analysis. On introducing logs to both sides of Equation 21:

$$\text{Log } S = \text{log } P + n \text{ log } (1+r) \quad (23)$$

Where $\text{log } P$ and $\text{log } (1+r)$ are constants, specifically, $\text{log } (1+r)$ is the slope parameter and $\text{log } P$ is the intercept of the regression model respectively, $\text{log } S$ and n are the usual y and x variables in the linear regression model. Equation 23 can be reduced to:

$$\text{Log } S = A + \beta n \quad (24)$$

Equation 24 is a linear regression equation of $\text{log } S$, on n where, $A = \text{log } P$, and $\beta = \text{log } (1+r)$

Similarly, the future value FV_n , of the present value of investment v_0 , after n period given in Equation 22 can be expressed as a linear function of time n given by:

$$\text{Log } FV_n = \log v_0 + n \log (1+r)$$

4.2.2. Exponential Growth Bias and Compounding

Empirical researches in behavioral finance have on recent questioned the basic compounding formulae given in Equation 21 because of what they termed as an exponential growth bias. Scholars such as Levy and Tasoff (2017) have argued that individuals have a tendency to partially neglect compounding of exponential growth. According to Almenberg and Gerdes (2011) this tendency of underestimating the value of the variable growing at exponential rate has been linked to house hold finance. As shown by Goda and Sojourner (2012) exponential growth bias influences individual's saving decision over a long time horizons, for example, saving for retirement. An individual cannot make optimal saving decision if he has not accurately estimated how that saving will be growing over time. Exponential growth bias has also been linked to financial literacy (Almenberg & Gerdes, 2011). To measure this bias, this paper follows papers such as Almenberg and Gerdes (2011) and Stango and Zinman (2009) by incorporating an exponential growth bias parameter, λ in the compounding formula in Equation 21 as follows:

$$S = P(1+r)^{(1-\lambda)n} \quad (25)$$

The parameter, λ lies in the interval, $0 \leq \lambda \leq 1$. The interpretation of equation 25 is straight forward. If $\lambda = 0$, there is no exponential growth bias and Equation 25 is analogous to compounding formula given by Equation 21. However, if $0 < \lambda < 1$, then exponential growth is negatively biased, and growth is under estimated. In another extreme case, if $\lambda = 1$, exponential bias is maximum and an individual does not perceive any growth in his saving.

$$\text{Log } S = \log P + (1-\lambda)n \log (1+r) \quad (26)$$

The linear transformation of Equation 25 yields Equation 26. From Equation 26, when we take $\log(1+r)$ to be the slope parameter of the regression equation and time n as independent variable, it is easy to see that as the size of λ increases, the term, $\log(1+r)$ will be understated and subsequently the sum, S arising in future after n th period will be understated thus leading to future value bias.

4.3. Discounting and the Present Value Concept

The concept of time value of money holds a very prominent position in finance and it forms the basis of major decisions such as; investment decision, dividend decision, capital structure decision etc. Since many financial decisions are made based on the present value of the future cash flows, future cash flows are converted to their approximate present values using appropriate discount factors. Specifically, the technique of discounting is always applied in project evaluations, bond pricing, dividend decisions, annuities etc. For those purposes, the basic discounting model is always given as a direct transformation of the future value formula given in equation 22. It is given by:

$$PV_0 = \frac{FV_n}{(1+r)^n} \quad (27)$$

Equation 27 is a non-linear function relating PV_0 , the present value of future cash flow to FV_n , is the future value of cash flow at the end of the n th-period, r = discount rate, n = the time period usually in years in

which the cash inflow will be earned. The logarithmic transformation of equation 27 yields the linear function of the form given by Equation 28.

$$\text{Log } PV_0 = \text{Log } FV_n - n \log (1+r) \quad (28)$$

Equation 28 demonstrates a possibility of the time series regression of logged present value of future cash flows on the time period, n . Depending on the type of data available, we shall show that the regression of the linear relationship in equation 28 can solve a number of practical problems in finance and accounting decisions.

4.4. Life Cycle Costing

According to Munteanu and Mehedintu (2016) the concept of Life Cycle Costing comes from the US and was introduced in 1960 by the Logistic Management Institute. It comes from the military field and the calculation methods have been developed and have become widespread by the Ministry of Defense of the USA. In the 1970s the concept was used in the public construction sector. ISO 15686-5: 2008 defines Life Cycle Costing as “a valuable technique that is used for predicting and assessing the cost performance of constructed assets”. This ISO also specifies that Life Costing is one form of analysis for determining whether a project meets the client’s performance requirements. As argued by Savić, Milojević, and Petrovic (2019) Life Cycle Costing has become a very essential costing method applied in the planning, design, and construction of buildings as well as various infrastructure projects and in public sector. The basic idea of a life cycle costing (LCC) is to capture all the costs that arise from the creation of an idea, through the development of products, its production, and post-sales services, up to the withdrawal of the product from use. As such, the concept should provide a picture of overall costs over the life of a product, which is at the same time the starting point for assessing the viability of the product being monitored (Savić et al., 2019).

Very few parametric models exist in literature of cost and management accounting to model the behavior of total cost over time based on life cycle costing. However, the model given by Equation 29, that was proposed by Lapašinskaitė and Boguslauskas (2006) seems to be very useful. It is given by:

$$Y = c \cdot e^{dx} \quad (29)$$

Where Y = Operational and Maintenance costs, x = time, c & d are parameters and e = the base of the natural log system. The model shown in equation 29 assumes that operational and maintenance costs increase exponentially over time. That is, as you continue to use the product, for example a non-current asset like plant and machinery for a longer time period, more and more operational and maintenance costs will be incurred. This assumption makes sense in real life and may form the basis for making useful decisions such as: disposal of existing equipment, acquisition of new equipment at a certain point in time and other short run tactical decisions. One big difficulty faced by managers in applying Life Cycle Costing is estimation of future costs associated with a particular cost object. Initial costs incurred such as research and development costs and acquisition costs that are known with accuracy, may be classified as sunk costs and thus not relevant for decision making purposes. Conversely, costs such as operational and maintenance expenses are future costs incurred by an organization as it continuous to own that cost object such as plant and machinery. These future costs are more relevant for decision making purposes but very difficult to estimate. This paper proposes to apply regression technique to this estimation problem after transforming Equation 28 to a linear relationship. For simplicity, let’s consider costs classified as operational and maintenance. The corresponding linear form of the above non-linear model specification using natural logarithmic transformation is given by:

$$\ln Y = \ln c + dx \quad 30$$

Equation 30 shows that log operational and maintenance costs are a linear function of time, x with d , as the slope parameter. The intercept, $\ln c$ captures the component of total cost which is not dependent on time (for

example does not depend on the age of the equipment). The decision maker can regress historical cost data on time index, x in order to estimate the parameters d & c to make the regression equation 30 complete. Using the resulting regression equation, the method of linear extrapolation can then be applied to estimate the future costs for a specific future time.

4.5. The Learning Curve Model

The learning curves depicts the way people learn by doing a task and are therefore able to complete a task more quickly the next time they attempt it [Lucey \(2002\)](#); [Drury \(2008\)](#). During the early stages of producing a new part or carrying out a new process, experience and skill is gained, productivity increases and there is a reduction of time taken per unit. Empirical evidence shows that there is a tendency for the time per unit to reduce at some constant rate as production volume increases. Example as cumulative production quantities doubles, the cumulative average time per unit falls by 10%, therefore giving rise to a 90% learning curve. Conversely, an 80% learning curve means that doubling of production causes a 20% fall in the cumulative average time per unit. The linearization of the learning curve model and subsequent application of simple regression will generate interesting results.

$$y = ax^{\beta} \quad (31)$$

[Equation 31](#) gives the learning curve model. [Equation 31](#) is similar to equation 16 and its linear transformation is analogous to [Equation 17](#). From the above model:

y = cumulative average time per unit / marginal time for the x th unit.

x = cumulative number of units.

a = the number of labour hours for the first unit.

β = learning coefficient.

5. ILLUSTRATIONS AND INTERPRETATIONS OF REGRESSION RESULTS

5.1. Depreciation of Non-Current Assets

The following data was extracted by an auditor from an asset register of XY Company limited. The data relates to a certain non-current asset that was bought five years ago: The Company uses Reducing balance methods for charging depreciation.

Table-1. Depreciation of non-current assets using reducing balance method.

Year (n)	Depreciated amount at the year-end (D)
1	£ 11,250
2	8,437.5
3	6328.125
4	4,746.1
5	3559.57

[Table 1](#) shows hypothetical net book values of a non-current asset at the end of each year taken from the asset registry of a certain company. The depreciation rate is not known and the book value in year 0 is not known. The net book values are reducing in a non-linear fashion with increase in time in years that the company continuous to use the asset. We shall find an approximate linear relationship of the above exponential decay in net book value and use regression to tackle some accounting problems associated with depreciation of non-current assets. Specifically, the auditor may want to verify the following using regression technique:

- i. The depreciation rate that the company applies.
- ii. The book value of this asset.
- iii. The expected depreciated value (net book value) by the end of the tenth year.
- iv. The accumulated depreciation by the end of the tenth year.

- v. Give a brief discussion of this problem and results.

Solution

Consider the Table 2 for the detailed derivations of key input for regression model.

Table-2. Table of solution for the problem on depreciation of non-current assets.

x (n)	y (D)	Log y (log D)	x log y	x ²
1	11,250	4.0512	4.0512	1
2	8,437.5	3.9262	7.8524	4
3	6,328.125	3.8012	11.4036	9
4	4,746.1	3.6763	14.7052	16
5	3,559.57	3.5514	17.757	25
15		19.0063	55.7694	55

Table 2 generates key values that are necessary for regression model based on information given in Table 1. The variable Y has already been transformed to the logged variable but the basic principles of application of regression model will be followed. Specifically, Table 2 will help us to determine the slope of regression model and the intercept. The resulting regression equation will provide important insights to the above problems of accounting for depreciation using reducing balance.

Solution (i)

$$\beta = \frac{n\sum xlogy - \sum x \sum log y}{n\sum x^2 - (\sum x)^2} = \frac{5 \times 55.7694 - 15 \times 19.0063}{5 \times 55 - 15^2} = \frac{-6.2475}{50} = -0.12495$$

From $\beta = \log(1 - i)$

$\log(1 - i) = -0.12495$

$i = 1 - (\text{antilog } -0.12495) = 0.25 = 25\%$

Therefore the depreciation rate that the company applies is 25% on a reducing balance.

Solution (ii)

$$A = \frac{\sum log y}{n} - \frac{\beta \sum x}{n} = \frac{19.0063}{5} + 0.12495 \times \frac{15}{5} = 4.17611$$

From $A = \log B$

$\log B = 4.17611$

$B = \text{antilog } 4.17611 = 15,000$

Therefore the original book value is UGX 15,000

Solution (iii)

Depreciated value by the end of tenth year is as follows:

Using regression equation:

$\log D = A + \beta n$

$\log D = 4.17611 - 0.12495 n$

When $n = 10$

$\log D = 4.17611 - 0.12495 \times 10$

$\log D = 2.92661$

$D = 845$

Therefore, the depreciated value by the end of the tenth year is UGX 845

Solution IV

Accumulated depreciation by the end of the tenth year is the difference between the book value at the beginning of year one when the asset was first acquired and the net book value at the end year ten. Therefore accumulated depreciation at the end of the tenth year = UGX 15,000 – UGX 845 = UGX 14,155

Solution V: Brief Discussion of the Problem and Results

In the above problem, depreciated amounts at the end of every year were given for n consecutive years while the original book value and depreciation rate were not given. This is different from existing approaches in current literature on accounting for non- current assets where depreciated rates and the book value of assets are always given, and the task is to determine the depreciation charge for the year and the net book value at the end of that particular period. Relevant international accounting and reporting standards for instance; International Accounting Standard 16 (Property, Plants and Equipment), International Financial Reporting Standard 5 (Non-Current Assets held for resale), International Accounting Standard 36(Impairment of Non-Current Assets) have not approached depreciation problem in a manner demonstrated here. This particular approach of using regression is useful in a number of ways in practice. Firstly, in the absence of information about depreciation rate and book value, if year ends net book values are available for some n consecutive years, depreciation rate and book value can be determined. Secondly, even if the rate and the book value are given, regressing consecutive year ends net book values on time index will help to verify if the rate used throughout the period was right, or if the rate was correctly applied throughout the period. This kind of verification can be very useful especially if an auditor is conducting substantive tests on accuracy and correctness. Thirdly, through the method of linear extrapolation, the regression equation can be used to estimate the net book value, salvage value, accumulated depreciation at any time, n in future. Fourthly, the decision maker can easily generate regression equation using statistical software. The major limitation of this approach is that it can only be applied to reducing balance method of depreciation yet there are many methods of depreciation recognized in IAS 16.

5.2. Compounding

Mr. John is saving with an insurance company in which interest earned every year on the initial amount invested is ploughed back at the end of every year. He has received the financial statement below from the Insurance Company showing how the initial investment has been growing at the end of every year for five years:

Table-3. Growth in amount of investment when interest is compounded annually.

Year (n) x	Total Amount earned (S) y
1	105,000
2	110,250
3	115,763
4	121,551
5	127,623

Table 3 shows the total amount earned at the end of each year. The above amounts include both the principals and interest earned, however, the initial amount invested and interest rate is not known. The growth in amount invested is a non-linear exponential function of time. In this particular kind of problem, this current paper proposes that regression model can be used to tackle the following problems associated with compounding, using the data in Table 3. The following problems may be of interest to the decision maker:

- i. The interest rate that the company offers per annum.
- ii. The initial amount invested at the beginning of the year 1.

- iii. Use the above regression model to estimate how much John will earn at the end of the tenth year

Solution

Consider the table below:

Table-4. Table of solution for the problem on compounding.

x (n)	y (S)	Log y (log S)	x log y	x ²
1	105,000	5.0212	5.0212	1
2	110,000	5.0424	10.0848	4
3	115,763	5.0636	15.1908	9
4	121,551	5.0848	20.3392	16
5	127,628	5.1059	25.5295	25
15		25.3179	76.1655	55

Table 4 generates key values that are necessary for computing the regression model. The variable \mathcal{Y} has already been transformed to logged variable but the basic principles of application of regression model are still followed. Specifically, the above table will help us to determine the slope and intercept of the regression model. The resulting regression equation provides interesting solutions to the problem of compounding.

$$\beta = \frac{n\sum x \log y - \sum x \sum \log y}{n\sum x^2 - (\sum x)^2} = \frac{5 \times 76.1655 - 15 \times 25.3179}{5 \times 55 - 15^2} = \frac{1.059}{50} = 0.02118$$

From $\beta = \log(1 + i)$

$$\log(1 + i) = 0.02118$$

$$i = (\text{antilog } 0.02118) - 1 = 0.0499 = 5\% \text{ (1dp)}$$

Therefore the interest rate that the company offers is 5% per annum.

$$A = \frac{\sum \log y}{n} - \frac{\beta \sum x}{n} = \frac{25.3179}{5} - 0.02118 \times \frac{15}{5} = 5$$

From $A = \log P$

$$\log P = 5$$

$$P = \text{antilog } 5 = 100,000$$

Therefore the initial amount invested is UGX 100,000. By the end of the tenth year, the amount of this investment will be:

Using regression equation:

$$\log S = A + \beta n$$

$$\log S = 5 + 0.02118 n$$

When $n = 10$

$$\log S = 5 + 0.02118 \times 10$$

$$\log S = 5.2118$$

$$S = \text{antilog } 5.2118 = \text{UGX } 162,855$$

Therefore, this investment will be UGX 162,855 by the end of the tenth year

5.3. Exponential Growth Bias

As noted earlier, experimental results in behavioral finance have indicated that individuals have exponential growth bias about the growth of their savings and the amortization of their loan obligation. Using the above illustration on compounding, suppose John had an exponential bias given by $\lambda = 0.15$. We shall see the impact of

this bias on his perceived growth in saving. His initial deposit is UGX 100,000 (calculated from example 1 above) an insurance company offers compound interest of 5%.

Required:

- Determine the perceived growth in saving when exponential growth parameter, $\lambda = 0.15$
- By using regression, what percent rate of interest does this perceived growth in saving imply?
- Comment briefly on this problem and results.

Solution (i)

We shall use Equation 25 to determine the perceived growth with an exponential growth parameter $\lambda = 0.15$. Consider the table below for the summary of the workings.

Table-5a. Solution for problem on exponential growth bias.

Year end	Unbiased exponential growth	Biased exponential growth
1	105,000	$S = 100,000 \times 1.05^{(1-0.15) \times 1} = 104,234$
2	110,000	$S = 100,000 \times 1.05^{(1-0.15) \times 2} = 108,648$
3	115,763	$S = 100,000 \times 1.05^{(1-0.15) \times 3} = 113,249$
4	121,551	$S = 100,000 \times 1.05^{(1-0.15) \times 4} = 118,044$
5	127,628	$S = 100,000 \times 1.05^{(1-0.15) \times 5} = 123,042$

Table 5a shows the growth of an initial investment perceived by an investor with exponential growth bias. Compared to an unbiased exponential growth shown in Table 3, an investor with an exponential growth bias parameter, $\lambda = 0.15$ at an interest rate of 5% compounded annually, will perceive a lower growth in future value. For instance, without bias the future value at the end of year one is UGX 105,000 but an investor with exponential growth bias of $\lambda = 0.15$ will perceive a future value of only UGX 104,234 at the end of the first year. We shall regress exponential growth bias data in Table 5a on time as regressor. The detailed solution is given in Table 5b below:

Solution (ii)

Consider the table below:

Table-5b. Solution for problem on exponential growth bias.

X (Year)	Y (Saving)	logY	XlogY	X^2
1	104,234	5.0180	5.0180	1
2	108,648	5.0360	10.0720	4
3	113,249	5.0540	15.1620	9
4	118,044	5.0720	20.2880	16
5	123,042	5.0901	25.4505	25
15		25.2701	75.9905	55

Table 5b provides key values for the computation of linear regression equation for the problem of compounding when there is an exponential growth bias. As noted earlier, the values for the variable Y (Saving) are now understated because of exponential growth bias. The derivations of these values are illustrated in Table 5a.

$$t \quad \beta = \frac{n\sum x \log y - \sum x \sum \log y}{n\sum x^2 - (\sum x)^2} = \frac{5 \times 75.9905 - 15 \times 25.2701}{5 \times 55 - 15^2} = \frac{0.901}{50} = 0.01802$$

From $\beta = \log(1 + i)$

$$\log(1 + i) = 0.01802$$

$$i = (\text{antilog } 0.01802) - 1 = 0.0423 = 4.23\% \text{ (1dp)}$$

Therefore the interest rate implied as a result of exponential growth bias is 4.23% per annum.

Solution (iii): Brief comments

While the actual rate set by the fund manager (an insurance company) is 5% compounded annually, the rate earned per annum as perceived by an individual investor due to exponential growth bias of $\lambda = 0.15$ is only 4.23% per annum. As the value of the bias parameter increases, this perceived rate will continue to fall. This rate can easily be verified by looking at the percentage increase of initial deposit of UGX 100,000 to the perceived growth of UGX 104,234 at the end of year one. This biased perception may discourage saving and encourage current consumption instead. As discussed earlier, regression is very useful in this case because of its simplicity as compared to its counterpart, the non-linear exponential function. The decision maker in practice, with the aid of statistical software, would easily run these regressions using software and obtain useful results. Regression models can also be subjected to further statistical analysis like correlations, coefficients of determination, scatter plots etc to further permit more informed decision making.

5.4. Lifecycle Costing

A certain hotel has a delivery van which they have been using purposely for delivering meals to various construction sites in Lira town in Northern Uganda. The van was acquired five years ago at the cost of \$80,000. For the last five years, the repair and maintenance costs for this van recorded by the cost accountant are as follows:

Table-6. The yearly repair and maintenance costs for the delivery van.

Year	1	2	3	4	5
Costs(\$)	1,800	2,699	4,052	6,078	9,119

Table 6 shows hypothetical data for the yearly repair and maintenance costs for the hotel’s delivery van. The above costs pattern for the last five years conforms to an exponential model of the form:

$$Y = c \cdot e^{dx} \quad \text{refer to Equation 28}$$

Where Y = costs, x = time in years, c&d are parameters

Note: This is similar to the notion that the costs, Y depends on the age, x of the van

Required:

- i. Decompose the total costs for the year into its fixed and variable components using regression technique.
- ii. In the tenth year, a certain transport company has offered to hire its car at the cost of \$60,000 for one year to the hotel to do delivery instead of them using their old delivery van. Hotel management has the option of hiring the car or continues to use their old delivery van. If they hire the car, they will write off their delivery van as obsolete and sell it off at a scrap value of \$5,000. Based on the above cost pattern, advise management on which option they should pursue in the tenth year.
- iii. Comment briefly on this problem and results.

Solution

The linear form of the above model is given by

$$\ln Y = \ln c + dx$$

We shall estimate the slope parameter, d and the intercept, Inc. Table 7 generates key values for computation of linear regression equation for the non-linear exponential cost function. The resulting regression equation is used to decompose total costs into fixed and variable components. The fixed components are captured by the intercept of the regression equation and variable components are captured by the slope parameter of the regression equation.

Table-7. Table of solution for problem on life cycle costing.

X	Y (\$)	lnY	XlnY	x ²
1	1,800	7.4955	7.4955	1
2	2,699	7,9006	15.8012	4
3	4,052	8.3069	24.9207	9
4	6,078	8.7124	34.8496	16
5	9,118	9.1181	45.5905	25
15		41,5335	128.6575	55

$$d = \frac{n\sum x \ln y - \sum x \sum \ln y}{n\sum x^2 - (\sum x)^2} = \frac{5 \times 128.6575 - 15 \times 41.5335}{5 \times 55 - 15^2} = \frac{20.285}{50} = 0.4057$$

$$\text{Inc} = \frac{\sum \ln y}{n} - \frac{d \sum x}{n} = \frac{41.5335}{5} - 0.4057 \times \frac{15}{5} = 7.0896$$

$$c = e^{7.0896} = \$1,199 \text{ (fixed costs per year)}$$

Thus, the particular regression equation for this problem is given by:

$$\ln Y = 7.0896 + 0.4057x$$

Using the above regression equation, the yearly total costs are decomposed as below:

Table-8. Decomposition of total costs into fixed and variable components.

Year	Total Costs	Fixed Costs	Variable costs per year
1	1,800	1,199	601
2	2,699	1,199	1,500
3	4,052	1,199	2,853
4	6,078	1,199	4,879
5	9,118	1,199	7,919

Table 8 shows the decomposition of total costs into their fixed and variable components using the computed regression equation based on hypothetical cost data given in Table 6. Fixed costs are captured by the intercept of the regression equation, and they remain constant at \$1,199 every year. However, it is clear from Table 8 that variable repair and maintenance costs are increasing very rapidly with respect to time. The breakdown of total costs is shown in the working below:

In year 1, $\ln Y = 7.0896 + 0.4057x$

$$\ln Y = 7.0896 + 0.4057 \times 1$$

$$\ln Y = 7.4953$$

$$Y = e^{7.4953} = 1800$$

Also from: $\text{Inc} = 7.0896$, $c = 1199$ (fixed costs)

Hence Variable costs per year = $1800 - 1199 = 601$

The same procedures are followed for all the years

Solution (ii)

If they hire the car;

Table-9. Table of Solution for evaluating alternative decisions.

Descriptions	Amount (\$)
Cost Saving from disposing the old van (W1)	69,328
Resale value of scrap	5,000
Cost of hiring a delivery van	(60,000)
Net saving	14,328

Table 9 provides solution to problem (ii) of life cycle costing. The incremental costs of the two alternatives of disposing the van and hiring another van are presented in Table 9. When they dispose the van, they will save repair and maintenance costs which are estimated to be \$69,328 (refer to working 1 below). They will also realize a resale scrap value of \$5,000. Moreover, they will incur additional cost of \$60,000 for hiring a delivery van in that year. However, the incremental revenue from disposing the delivery van is higher than the incremental costs of hiring the van, leading to the net saving of \$14,328.

5.5. Decision and Brief Comments

It is advisable for them to dispose of their old car in the tenth year and accept to hire the car from the transport company and get the net gain of \$14,328. However, management should consider other factors which are not captured in this analysis. For instance, the alternative of buying the new delivery van if the current market price will be fair at that time. Another qualitative factor they should consider relates to the efficiency of the car to be hired otherwise, it remains unclear as to which party will be responsible for repairs and maintenance of the car under this new arrangement. The purchase cost of \$80,000 for the old delivery van is a sunk cost which remains irrelevant for this decision. As indicated in Table 8, the yearly incremental costs of continuing to use the delivery van is building up very rapidly and may completely erode the recoverable value of the van if it is kept beyond the period of ten years.

Working 1: Cost saving from disposing the old van:

$$\text{From: } \ln Y = 7.0896 + 0.4057x$$

$$\ln Y = 7.0896 + 0.4057 \times 10 = 11.1466$$

$$Y = e^{11.1466} = 69,328$$

5.6. The Learning Curve

Consider the following data for the illustration of an 80% learning curve where the first unit of production takes 50 hours. Using the regression model, determine the learning curve coefficient

Table-10. Hypothetical data for an 80% learning curve for the first 50 units.

Cumulative production (units) x	Cumulative average time per unit (hours) y
1	50
2	40
4	32
8	25.6
16	20.48
32	16.384

Table 10 shows hypothetical data for cumulative average time per unit and cumulative production units for an 80% learning curve.

As cumulative units doubles, cumulative average time reduces by 20%. However, the reduction in cumulative average time per unit is not linearly related cumulative production. Using data in Table 10, we shall obtain a linear equivalence relationship which permits the application of regression analysis as shown in the solution below:

Solution

Table 11 shows the derivation of key values for computing regression equation based on data in Table 10. Unlike the previous cases, it is easy to note that in Table 11, both X and Y variables have been transformed to log variables. The slope parameter of the resulting regression equation in this case is used to determine the learning curve coefficient. However, it can also be used for forecasting purposes through the technique of linear interpolations. The workings demonstrate how to compute the learning curve coefficient using the technique of regression.

Table-11. Solution for the problem on learning curve.

X	Y	Log x	Log y	log x log y	$(\log x)^2$
1	50	0	1.6989	0	0
2	40	0.3010	1.6020	0.4822	0.0906
4	32	0.6020	1.5051	0.9061	0.3624
8	25.6	0.9030	1.4082	1.2716	0.8154
16	20.48	1.2041	1.3113	1.5789	1.4499
		3.0101	7.5255	4.2388	2.7183

$$\beta = \frac{n\sum \log x \log y - \sum \log x \sum \log y}{n\sum \log x^2 - (\sum \log x)^2} = \frac{5 \times 4.2388 - 3.0101 \times 7.5255}{5 \times 2.7183 - 3.0101^2} = \frac{-1.4585}{4.5308} = -0.3219$$

Note: We shall realize that regression technique provides an alternative method of estimating the learning curve coefficient, β . Scholars such as Lucey (2002); Drury (2008) have computed this coefficient using the formulae:

$$\beta = \frac{\log(1 - \text{proportionate decrease})}{\log 2} = \frac{\log(1 - 0.2)}{\log 2} = -0.3219 \text{ (for an 80\% learning curve.)}$$

Now let's go back to our regression analysis

$$\text{The intercept, } \log a = \frac{\sum \log y}{n} - \frac{\beta \sum \log x}{n} = \frac{7.5255}{5} + 0.3219 \times \frac{3.0101}{5} = 1.6989$$

$$\log a = 1.6989$$

$$a = \text{antilog } 1.6989 = 50$$

Therefore our regression equation is:

$$\log y = \log a + \beta \log x$$

$$\log y = 1.6989 - 0.3219 \log x$$

But the original learning curve equation: $y = ax^\beta$ now becomes:

$$y = 50x^{-0.3219}$$

6. CONCLUSION

Recent studies in finance and accounting have indicated that many decision problems in finance and accounting can be better modeled by non-linear models in practice. However, existing literatures have also shown that managers are not very conversant with non-linear problems as compared to linear problems because of the

simplicity of linear models. In this paper, attempts are made to transform some non-linear models of exponential forms that decision makers are always confronted with, into their linear forms. The resulting estimated linear forms are subjected to regression analysis in order to estimate some key parameters and interpreted these parameters in a manner that aids management's decision making. The key non-linear problems that have been linearized in this paper includes; depreciation of non-current assets using reducing balance method, compounding, discounting, exponential growth bias, life cycle costing, and the learning curve model. Although these particular cases are not many, the paper proposes that the methods applied here can be generalized to all non-linear models in accounting and finance that are in exponential form. The paper ends by giving practical illustrations using hypothetical data.

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