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ECCENTRIC CONNECTIVITY INDEX OF SOME SPECIAL MOLECULAR GRAPHS AND THEIR *R*-CORONA GRAPHS

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ABSTRACT

In this paper, we determine the eccentric connectivity index and augmented eccentric connectivity index of fan graph, wheel graph, gear fan graph, gear wheel graph and their r-corona graphs.

Keywords: Chemical graph theory, Organic molecules, Eccentric connectivity index, Augmented eccentric connectivity index, Fan graph, Wheel graph, Gear fan graph, Gear wheel graph, *r*-corona graph.

1. INTRODUCTION

Wiener index, edge Wiener index, Hyper-wiener index and eccentric connectivity index are introduced to reflect certain structural features of organic molecules. Several papers contributed to determine the Wiener index or Hyper-wiener index of special graphs (See Yan, et al. [1], Gao and Shi [2] and Yan, et al. [3] for more detail). Let P_n and C_n be path and cycle with n vertices. The graph $F_n=\{v\} \vee P_n$is called a fan graph and the graph $W_n=\{v\} \vee C_n$is called a wheel graph. Graph $I_r(G)$ is called r- crown graph of G which splicing r hang edges for every vertex in G. By adding one vertex in every two adjacent vertices of the fan path $P_n$of fan graph F_n, the resulting graph is a subdivision graph called gear fan graph, denote as $\tilde{F}_n$. By adding one vertex in every two adjacent vertices of the wheel cycle $C_n$of wheel graph W_n, The resulting graph is a subdivision graph, called gear wheel graph, denoted as $\tilde{W}_n$.

The graphs considered in this paper are simple and connected. The eccentricity $ec(u)$ of vertex $u \in V(G)$ is the maximum distance between u and any other vertex in G. Then the eccentric connectivity index (ECI) of G is defined as

$$\xi^c(G) = \sum_{v \in V(G)} ec(v)\deg(v).$$

Ranjini and Lokesha [4] determined the eccentric connectivity index of the subdivision graph of the complete graphs, tadpole graphs and the wheel graphs. Morgan, et al. [5] obtained an exact lower bound on $\xi^c(G)$ in terms of order, and showed that this bound is sharp. An asymptotically sharp upper bound was also derived. In addition, for trees of given order, when the diameter was also prescribed, precise upper and lower bounds are provided. Hua and Das [6] studied the relationship between the eccentric connectivity index and Zagreb indices. De [7] presented the explicit generalized expressions for the eccentric connectivity index and polynomial of the thorn graphs, and then considered some particular cases. Eskender and Vumar [8] calculated the eccentric connectivity index and eccentric distance sum of generalized hierarchical product of graphs. Moreover, they presented the exact formulaefor the eccentric connectivity index of *F*-sum graphs in terms of some invariants of the factors. Iiic and Gutman [9] proved that the broom has maximum $\xi^c(G)$ among trees with a fixed maximum vertex degree, and characterized such trees with minimum $\xi^c(G)$. Iranmanesh and Hafezieh [10] presented the eccentric connectivity index of some graph families. Dankelmann, et al. [11] proved the upper bound for eccentric connectivity index and constructed graphs which asymptotically attain the bound. Morgan, et al. [12] showed that a known tight lower bound on the eccentric connectivity index for a tree, in terms of order and diameter, was also valid for a general graph, of given order and diameter. Rao and Lakshmi [13] obtained explicit formulas for eccentric connectivity index of phenylenic nanotubes.

The augmented eccentric connectivity index (AECI) $\xi^A(G)$ of a molecular graph G is defined as

$$\xi^A(G) = \sum_{v \in V(G)} \frac{M(v)}{ec(v)},$$

where $M(v)$ denotes the product of degrees of all neighbors of vertex *v*.

Doslic and Saheli [14] explored the asymptotic behavior of augmented eccentric connectivity index and the compression ratios are computed for some special polymers. Sedlar [15] established all extremal graphs with respect to augmented eccentric connectivity index among all simple connected graphs, among trees and among trees with perfect matching. Yarahmadi, et al. [16] obtained the exact formulas for the augmented eccentric connectivity index of *V*-phenylenic nanotorus together with its extremal values. Doslic and Saheli [17] presented explicit formulas for the values of augmented eccentric connectivity indices of single-defect nanocones. Ediz [18] determined the exact formulas for the augmented eccentric connectivity index of some special nanotube and nanotorus. Ediz [19] obtained an exact formula for the augmented eccentric connectivity index of an infinite class of nanostar dendrimers.

In this paper, we present the eccentric connectivity index of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$. Also, the augmented eccentric connectivity index of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$ are derived.

2. ECCENTRIC CONNECTIVITY INDEX

Theorem 1. $\xi^c(I_r(F_n)) = r(7n+5)+(11n-6)$.

Proof. Let $P_n = v_1 v_2 \dots v_n$ and the r hanging vertices of v_i be v_i^1, v_i^2,…, v_i^r ($1 \le i \le n$). Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By the definition of eccentric connectivity index, we have

$$\xi^c(I_r(F_n)) = ec(v)\deg(v) + \sum_{i=1}^{n} ec(v_i)\deg(v_i) + \sum_{i=1}^{r} ec(v^i)\deg(v^i) + \sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_i^j)\deg(v_i^j)$$

$$= 2(r+n) + (3nr + (9n-6)) + 3r + 4nr$$

$$= r(7n+5)+(11n-6). \square$$

Corollary 1. $\xi^c(F_n) = 7n-4$.

Theorem 2. $\xi^c(I_r(W_n)) = r(7n+5)+11n$.

Proof. Let $C_n = v_1 v_2 \dots v_n$ and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i ($1 \le i \le n$). Let v be a vertex in W_n beside C_n, and v^1, v^2, …, v^r be the r hanging vertices of v. By the definition of eccentric connectivity index, we have

$$\xi^c(I_r(W_n)) = ec(v)\deg(v) + \sum_{i=1}^{n} ec(v_i)\deg(v_i) + \sum_{i=1}^{r} ec(v^i)\deg(v^i) + \sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_i^j)\deg(v_i^j)$$

$$= 2(r+n) + (3nr + 9n) + 3r + 4nr$$

$$= r(7n+5)+11n.$$

Corollary 2. $\xi^c(W_n) = 7n$.

Theorem 3. $\xi^c(I_r(\tilde{F}_n)) = r(20n-4)+(25n-18)$.

Proof. Let $P_n=v_1v_2\ldots v_n$ and $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i($1\le i\le n$). Let $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ ($1\le i$ $\le n$-1). Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r.By virtue ofthe definition of eccentric connectivity index, we get

$$\xi^c(I_r(\tilde{F}_n)) = ec(v)\deg(v) + \sum_{i=1}^{n} ec(v_i)\deg(v_i) + \sum_{i=1}^{r} ec(v^i)\deg(v^i) + \sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_i^j)\deg(v_i^j)$$

$$+\sum_{i=1}^{n-1} ec(v_{i,i+1})\deg(v_{i,i+1}) + \sum_{i=1}^{n-1}\sum_{j=1}^{r} ec(v_{i,i+1}^j)\deg(v_{i,i+1}^j)$$

$$=3(r+n)+(4nr+(12n-8))+4r+5nr+5(n-1)(2+r)+6r(n-1)$$

$$=r(20n-4)+(25n-18).$$

Corollary3. $\xi^c(\tilde{F}_n) = 19n-14$.

Theorem 4. $\xi^c(I_r(\tilde{W}_n)) = r(20n+7)+25n$.

Proof. Let $C_n=v_1v_2\ldots v_n$ and v be a vertex in W_n beside C_n. $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v^1, v^2, …, v^r be the r hanging vertices of v and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i($1\le i\le n$). Let $v_{n,n+1}=v_{1,n}$ and $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ ($1\le i\le n$). In view of the definition of eccentric connectivity index, we deduce

$$\xi^c(I_r(\tilde{W}_n)) = ec(v)\deg(v) + \sum_{i=1}^{n} ec(v_i)\deg(v_i) + \sum_{i=1}^{r} ec(v^i)\deg(v^i) + \sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_i^j)\deg(v_i^j) +$$

$$\sum_{i=1}^{n} ec(v_{i,i+1})\deg(v_{i,i+1}) + \sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_{i,i+1}^j)\deg(v_{i,i+1}^j)$$

$$=3(r+n)+(4nr+12n)+4r+5nr+5n(2+r)+6nr$$

$= r(20n+7)+25n$.

Corollary 4. $\xi^c(\tilde{W}_n)=19n$.

3. AUGMENTED ECCENTRIC CONNECTIVITY INDEX

Theorem 5. $\xi^A(I_r(F_n))=\dfrac{r^2(3n+4)+r(35n-8)+(4n^2+42n-44)}{12}$.

Proof. By the definition of augmented eccentric connectivity index, we have

$$\xi^A(I_r(F_n))=\frac{M(v)}{ec(v)}+\sum_{i=1}^{n}\frac{M(v_i)}{ec(v_i)}+\sum_{i=1}^{r}\frac{M(v^i)}{ec(v^i)}+\sum_{i=1}^{n}\sum_{j=1}^{r}\frac{M(v_i^j)}{ec(v_i^j)}$$

$$=\frac{r(n+1)+(3n-2)}{2}+\frac{r(4n-2)+(n^2+6n-8)}{3}+\frac{r(n+r)}{3}+\frac{n(3r+r^2)-2r}{4}$$

$$=\frac{r^2(3n+4)+r(35n-8)+(4n^2+42n-44)}{12}.$$

Corollary 5. $\xi^A(F_n)=\dfrac{n^2+12n-12}{2}$.

Theorem 6. $\xi^A(I_r(W_n))=\dfrac{r^2(3n+4)+r(35n+6)+(4n^2+42n)}{12}$.

Proof. By the definition of augmentedeccentric connectivity index, we have

$$\xi^A(I_r(W_n))=\frac{M(v)}{ec(v)}+\sum_{i=1}^{n}\frac{M(v_i)}{ec(v_i)}+\sum_{i=1}^{r}\frac{M(v^i)}{ec(v^i)}+\sum_{i=1}^{n}\sum_{j=1}^{r}\frac{M(v_i^j)}{ec(v_i^j)}$$

$$=\frac{r(n+1)+3n}{2}+\frac{4nr+(n^2+6n)}{3}+\frac{r(n+r)}{3}+\frac{n(3r+r^2)}{4}$$

$$=\frac{r^2(3n+4)+r(35n+6)+(4n^2+42n)}{12}.$$

Corollary 6. $\xi^A(W_n)=\dfrac{n^2+12n}{2}$.

Theorem 7. $\xi^A(I_r(\tilde{F}_n))=\dfrac{r^2(22n+5)+r(187n-90)+(15n^2+192n-196)}{60}$.

Proof. By virtue ofthe definition of augmented eccentric connectivity index, we get

$$\xi^A(I_r(\tilde{F}_n)) = \frac{M(v)}{ec(v)}+\sum_{i=1}^{n}\frac{M(v_i)}{ec(v_i)}+\sum_{i=1}^{r}\frac{M(v^i)}{ec(v^i)}+\sum_{i=1}^{n}\sum_{j=1}^{r}\frac{M(v_i^j)}{ec(v_i^j)}+\sum_{i=1}^{n-1}\frac{M(v_{i,i+1})}{ec(v_{i,i+1})}+$$

$$\sum_{i=1}^{n-1}\sum_{j=1}^{r}\frac{M(v_{i,i+1}^j)}{ec(v_{i,i+1}^j)}$$

$$= \frac{r(n+1)+(3n-2)}{3} + \frac{r(4n-2)+(n^2+4n-4)}{4} + \frac{r(n+r)}{4} + \frac{r^2 n + r(3n-2)}{5} + \frac{r(3n-3)+(6n-8)}{5} + \frac{r(n-1)(2+r)}{6}$$

$$= \frac{r^2(22n+5)+r(187n-90)+(15n^2+192n-196)}{60}.$$

Corollary 7. $\xi^A(\tilde{F}_n) = \frac{n^2+13n-13}{3}$.

Theorem 8. $\xi^A(I_r(\tilde{W}_n)) = \frac{r^2(22n+15)+r(187n+20)+(15n^2+192n)}{60}$.

Proof. In view of the definition of augmented eccentric connectivity index, we deduce

$$\xi^A(I_r(\tilde{W}_n)) = \frac{M(v)}{ec(v)} + \sum_{i=1}^{n}\frac{M(v_i)}{ec(v_i)} + \sum_{i=1}^{r}\frac{M(v^i)}{ec(v^i)} + \sum_{i=1}^{n}\sum_{j=1}^{r}\frac{M(v_i^j)}{ec(v_i^j)} + \sum_{i=1}^{n}\frac{M(v_{i,i+1})}{ec(v_{i,i+1})} + \sum_{i=1}^{n}\sum_{j=1}^{r}\frac{M(v_{i,i+1}^j)}{ec(v_{i,i+1}^j)}$$

$$= \frac{r(n+1)+3n}{3} + \frac{4nr+(n^2+4n)}{4} + \frac{r(n+r)}{4} + \frac{n(3r+r^2)}{5} + \frac{3nr+6n}{5} + \frac{rn(2+r)}{6}$$

$$= \frac{r^2(22n+15)+r(187n+20)+(15n^2+192n)}{60}.$$

Corollary 8. $\xi^A(\tilde{W}_n) = \frac{n^2+13n}{3}$.

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