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DEVELOPMENT OF A MATHEMATICAL MODEL FOR ANGLE OF SOIL FAILURE PLANE IN CASE OF 3-DIMENSSIONAL CUTTING

Moayad B. Zaied[1] --- Mohammed H. Dahab[2] --- Ahmed M. El Naim[3]
[1]*Department of Agric. Eng., Faculty of Natural Resources, University of Kordofan, Elobied, Sudan*
[2]*Department of Agric. Eng., Faculty of Agriculture, University of Khartoum, Sudan*
[3]*Department of crop Sciences, Faculty of Natural Resources, University of Kordofan, Elobied, Sudan*

ABSTRACT

The interaction between tillage tools and soil is of a primary interest to the design and use of these tools for soil manipulation. A new explicit mathematical model to calculate the angle of soil failure plane when soil is cut with narrow tine was developed. Equations of Soil cohesion and soil adhesion cutting factors were partially differentiated with respect to angle of soil external friction, were maximized; Values of angle of soil failure plane were calculated by the model for tine rake angle range from 00 to 900 operated at 0.3 m depth and 0.15 m width in soils with different mechanical properties. It was found that angle of soil failure plane is acute and its values were used for calculating soil frictional cutting factor, soil overburden cutting factor, soil cohesion cutting factor, soil adhesion cutting factor, rupture distance, width of side crescent and soil resistance force. It was found that the force values were realistic. Therefore, the model is valid.

Keywords: Mathematical modeling, Soil failure plane, 3-D soil cutting.

Contribution/ Originality

The study originates new explicit formula for angle of soil failure plane (β) with respect to tool rake angle (α), external frictional angle at a soil–tool interface (δ), soil internal angle of friction (ϕ), tool operating depth (d) and tool operating width (w) in case of 3-D soil cutting

1. INTRODUCTION

The dynamic response of soil to farm implements is a main factor in determining their performance. The interaction between tillage tools and soil is of a primary interest to the design and use of these tools for soil manipulation. The definition of notations is as a follow:

C	Soil cohesion, *kPa*
C_a	Adhesion force at a soil–metal interface, *kPa*
d	Tool working depth, *m*
F_s	Soil resistance force, *kN*

† Corresponding author

g	Acceleration due togravity, m/s^2
K_p	Soil pressure cutting factor, dimensionless
K_c	Soil cohesion cutting factor, dimensionless
K_{ca}	Soil adhesion cutting factor, dimensionless
N_γ	Soil frictional cutting coefficient, dimensionless
N_q	Soil overburden cutting coefficient, dimensionless
N_c	Soil cohesion cutting coefficient, dimensionless
N_{ca}	Soil adhesion cutting coefficient, dimensionless
r	Rupture distance, m
w	Tool working width, m
W_s	Width of side crescent, m
α	Tool rake angle, deg.
β	Angle of soil failure plane, deg.
δ	External frictional angle at a soil–tool interface, deg.
ϕ	Internal frictional angle, deg.
ρ_d	Soil bulk density, g/cm^3
σ_n	Normal stress, kPa
τ	Shear strength, kPa

The force acting on a failure surface in the soil body can be determined by Mohr–Coulomb equation as follows

$$\tau = C + \sigma_n \tan\phi \qquad (1)$$

The forces acting at a metal-soil interface are determined by the following equation

$$\tau = C_a + \sigma_n \tan\delta \qquad (2)$$

Limit equilibrium is one of the most important approaches used to analyze soil–tool systems. Two most important factors in the approach are the shape of soil failure surface, and equilibrium equations, which are two or three dimensional cases. Grisso and Perumpral [1] reviewed four narrow tillage tool models, discussed assumptions, capabilities and limitations associated with each model, predicted tillage tool performance under two different soil conditions using the four models and compared simulated results with the experimental results.

Many models for prediction of soil pressure coefficients, cutting factors, soil forces and draft were developed. Terzaghi [2] and Hettiaratchi and Reece [3] established a two-dimensional model and evaluated soil loads and soil resistance to tillage tools. They reported that the soil in front of tool and above the failure surface is assumed to consist of Rankine passive zone and a complex shear zone bounded by part of logarithmic spiral as shown in Fig. 1.

Payne [4] developed a three-dimensional soil failure model depending on the upward movement of soil in front of the tool during tillage. In this model a failure zone includes a triangular center wedge, a center crescent and two side blocks called wings of the crescent (Fig. 2). It is proposed that failure wedge ahead of a cutting blade and the failure wedge consists of center wedge, two side crescents and straight rupture plane at the bottom [5], [6] as shown in Fig. 3. Zeng and Yao [7] developed a dynamic soil cutting model included the acceleration and strain-rate effects. Kuczewsk and Piotrowska [8] introduced a new model for forces on narrow soil cutting tines taking into account variability of the inclination angle of bottom failure surface in the side segment and inertial forces for different side segments. Draft force and power requirement for tillage implements were considerably affected by implement design and conditions of soil. In terms of effects on draft force and soil disturbance, Kheiralla, et al. [9] formulated a draft force model for ploughs based on traveling speed and tillage depth. Abo-Elnor, et al. [10]concluded that the blade cutting width had a significant effect on cutting forces so that the cutting forces increased but not in linear proportion as the cutting width increased. McKyes and Maswaure [11] demonstrated that designing a tillage tool for minimum draft requirement and high soil cutting efficiency called for a shallow operating depth and rake angle of 30^0. Chung and Sudduth [12] developed a model for soil failure caused by a vertically operating conical tool and concluded that the angle of soil failure plane and angle of internal soil friction are negatively correlated. Kasisira and DuPlessis [13] developed mathematical force models employing limit equilibrium analysis based on the soil-volume tilled to predict the draft requirements of tillage tool and reported that such models require a preliminary assumption of the soil failure pattern ahead of the tool.

Two-dimensional soil cutting model can be valid if the tool working width is larger than its depth. McKyes [14] mentioned two-dimensional models to calculate soil cutting factors and angle of soil failure plane as follow:

$$K_P = \frac{(\cot\alpha + \cot\beta)\sin(\beta + \phi)}{2\sin(\alpha + \beta + \delta + \phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$K_C = \frac{\cos\phi}{\sin\beta\sin(\alpha + \beta + \delta + \phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$K_{Ca} = \frac{-\cos(\alpha + \beta + \phi)}{\sin\alpha\sin(\alpha + \beta + \delta + \phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\beta = \cot^{-1}\left[\frac{\sqrt{\dfrac{\sin(\alpha + \delta)\sin(\delta + \phi)}{\sin\alpha\sin\phi}} - \cos(\alpha + \delta + \phi)}{\sin(\alpha + \delta + \phi)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Three-dimensional soil cutting model can be applied if the tool working width is smaller than its depth. Zhang and Kushwaha [15] mentioned three-dimensional models to predict soil pressure factors and cutting resistance force as follow

$$N_\gamma = \frac{0.5(\cot\alpha + \cot\beta)(1 + \frac{2d}{3w}\sqrt{\cot^2\beta + 2\cot\alpha\cot\beta})\sin(\beta + \phi)}{\sin(\alpha + \beta + \delta + \phi)} \ldots\ldots\ldots\ldots (7)$$

$$N_q = \frac{(\cot\alpha + \cot\beta)(1 + \frac{d}{w}\sqrt{\cot^2\beta + 2\cot\alpha\cot\beta})\sin(\beta + \phi)}{\sin(\alpha + \beta + \delta + \phi)} \ldots\ldots\ldots\ldots (8)$$

$$N_C = \frac{\cos\phi(1 + \frac{d}{w}\sqrt{\cot^2\beta + 2\cot\alpha\cot\beta}) \ldots\ldots.}{\sin(\alpha + \beta + \delta + \phi)} \ldots\ldots\ldots\ldots (9)$$

$$N_{Ca} = \frac{-\cos(\alpha + \beta + \phi)}{\sin\alpha\sin(\alpha + \beta + \delta + \phi)} \ldots\ldots\ldots\ldots (10)$$

$$r = d(\cot\alpha + \cot\beta) \ldots\ldots\ldots\ldots (11)$$

$$W_S = d\sqrt{\cot^2\alpha + 2\cot\alpha\cot\beta} \ldots\ldots\ldots\ldots (12)$$

$$F_S = (\rho_d g d^2 N_\gamma + CdN_C + C_a dN_{Ca})w \ldots\ldots\ldots\ldots (13)$$

2. MODEL DEVELOPMENT METHOD

2.1. Problem

In case of two-dimensional soil cutting, the angle of soil failure plane β is given explicitly as a function of α, δ and ϕ (equation 6). In three- dimensional soil cutting case, in equations [8-11] all parameters except angle of soil failure plane *(β)* are known. The soil failure plane angle can be identified by solving any of the following equations:

$$\frac{\partial N_\gamma}{\partial \beta} = 0 \ldots\ldots\ldots\ldots (11)$$

$$\frac{\partial N_q}{\partial \beta} = 0 \ldots\ldots\ldots\ldots (12)$$

$$\frac{\partial N_c}{\partial \beta} = 0 \ldots\ldots\ldots\ldots (13)$$

$$\frac{\partial N_{ca}}{\partial \beta} = 0 \ldots\ldots\ldots\ldots (14)$$

Since differentiations of these equations are rather complex and result formidable terms, thus, there is no explicit function for β with respect to α, δ, ϕ, d and w, therefore a numerical procedure was used to determine the failure plane angle (β). To this regard, the present work aims to look for a simple explicit model to calculate β whenever α, δ, ϕ, working depth (d) and working width (w) are known.

2.2. Solution for Model Development

The developed model will be in the form of $\beta = f(\alpha,\delta,\phi,d,w)$. To get that, some selections should be considered. Equation (7) contains constant values, so, it is excluded while Equation (8) and Equation (9) have no constant values in their terms, therefore any of them with Equation (10) can be chosen to conduct the partial derivative. Now Equation (9) is selected:

$$N_C = \frac{-\cos\phi(1+\frac{d}{w}\sqrt{\cot^2\beta + 2\cot\alpha\cot\beta})}{\sin(\alpha+\beta+\delta+\phi)}$$

The Partial derivative of N_c with respect to δ and maximization will be as follow

$$\frac{\partial N_C}{\partial \delta} = \frac{\cos\phi(1+\frac{d}{w}\sqrt{\cot^2\beta + 2\cot\alpha\cot\beta})\cos(\alpha+\beta+\delta+\phi)}{\sin^2(\alpha+\beta+\delta+\phi)} = 0 \qquad (15)$$

$$1+\frac{d}{w}\sqrt{\cot^2\beta + 2\cot\alpha\cot\beta} = 0 \qquad (16)$$

$$\frac{d^2}{w^2}\cot^2\beta + 2\frac{d^2}{w^2}\cot\alpha\cot\beta - 1 = 0 \qquad (17)$$

The resulting function is in the form of a quadratic where $\cot\beta$ is in the positive root of equation:

$$\cot\beta = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad (18)$$

Where,

$$a = \frac{d^2}{w^2}$$

$$b = 2\frac{d^2}{w^2}\cot\alpha$$

$$c = -1$$

Solving with respect to β will result the following relationship:

$$\cot\beta = \frac{w}{d}\sqrt{\left(\frac{d}{w}\cot\alpha\right)^2 + 1} - \cot\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

From equation (10):

$$N_{Ca} = \frac{-\cos(\alpha+\beta+\phi)}{\sin\alpha\sin(\alpha+\beta+\delta+\phi)}$$

Partial derivative of N_{ca} with respect to δ and maximization will be as follow:

$$\frac{\partial N_{Ca}}{\partial\delta} = \frac{\cos(\alpha+\beta+\phi)\sin\alpha\cos(\alpha+\beta+\delta+\phi)}{\sin^2\alpha\sin^2(\alpha+\beta+\delta+\phi)} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Let,

$$\cos(\alpha+\beta+\delta+\phi) = 0$$

Then,

$$\cos\beta\cos(\alpha+\delta+\phi) - \sin\beta\sin(\alpha+\delta+\phi) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Dividing by $\sin\beta\cos(\alpha+\delta+\phi)$

$$\cot\beta - \tan(\alpha+\delta+\phi) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Solving with respect to β will result the following relationship:

$$\cot\beta = \tan(\alpha+\delta+\phi) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

If $(\alpha+\delta+\phi) = \varepsilon$ or $(\alpha+\delta+\phi) = 90^0 - \varepsilon$, where ε is very small number then,

$$\tan(\alpha+\delta+\phi) \approx (\alpha+\delta+\phi)$$

Then β as function of α, δ and ϕ was given as follow:

$$\cot\beta = (\alpha+\delta+\phi) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Adding equation (18) to equation (24):

$$2\cot\beta = (\alpha+\delta+\phi) + \left(\frac{w}{d}\sqrt{\left(\frac{d}{w}\cot\alpha\right)^{2}+1} - \cot\alpha \right) \dots\dots\dots\dots\dots\dots (25)$$

Then,

$$\cot\beta = \frac{\left(\alpha+\delta+\phi+\left(\frac{w}{d}\sqrt{\left(\frac{d}{w}\cot\alpha\right)^{2}+1} - \cot\alpha\right)\right)}{2} \dots\dots\dots\dots\dots\dots (26)$$

Accordingly,

$$\beta = \cot^{-1}\left(\frac{\alpha+\delta+\phi+\frac{w}{d}\sqrt{\left(\frac{d}{w}\cot\alpha\right)^{2}+1} - \cot\alpha}{2}\right) \dots\dots\dots\dots\dots\dots (27)$$

Therefore, equation (27) is the developed model for calculating the angle of soil failure plane in case of three-dimensional soil cutting.

2.3. Model Validation Test

Values of angle of soil failure plane (β) are calculated using the developed model for different tine rake angles and varied soil mechanical properties for narrow tine operated at 0.3 m depth and 0.15 m width. The validity of the model depends on realistic of the results, that, (β) should be acute angle and real positive number. The resultant values of β were used to calculate soil frictional coefficient (N_{γ}), soil overburden coefficient (N_q), soil cohesion coefficient (N_c), soil adhesion coefficient (N_{ca}) as well as rupture distance, width of side crescent and soil resistance force.

3. RESULTS AND DISCUSSION

Table 1, Table 2 and Table 3 demonstrated the values of angle of soil failure plane, resultant soil coefficients, rupture distance, width of side crescent and soil resistance force at different soil mechanical properties and rake angles.

It was found that angle of soil failure plane (β) is acute and its values are real positive number. Moreover, it was shown that as the angle of internal soil friction (ϕ) increased, the value of β decreased and vice versa which agreed with Chung and Sudduth [12].

When values of β were used for calculating rupture distance, width of side crescent, soil frictional coefficient (N_{γ}), soil overburden coefficient (N_q), soil cohesion coefficient (N_c) and soil adhesion coefficient *(N_{Ca})*, the resultant values of these variables were found to be defined and determined. The values of coefficients were applied to calculate soil resistance force, it was found that the force values were realistic.

4. CONCLUSIONS

In case of three dimensional soil cutting, there is no explicit function for angle of soil failure plane (β) with respect to rake angle (α), external frictional angle at a soil–tool interface (δ), angle of soil internal friction (ϕ), tool operating depth (d) and tool operating width (w), therefore, a numerical procedures were used to determine the value of angle of soil failure plane.

A new explicit mathematical model to calculate angle of soil failure in case of three-dimensional cutting was developed. The model was developed by partially differentiating of soil cohesion and soil adhesion cutting coefficients with respect external frictional angle at a soil–tool interface and the two resultant equations were maximized and were solved simultaneously for angle of soil failure plane with respect to rake angle, external frictional angle at a soil–tool interface, angle of soil internal friction, tool depth and tool width,.

The resultant values of the angle of soil failure plane, correspondent soil cutting coefficients, rupture distances, width of side crescent and soil resistance forces obtained by the developed model at different tool rake angles were found to be realistic. Therefore, the model is valid.

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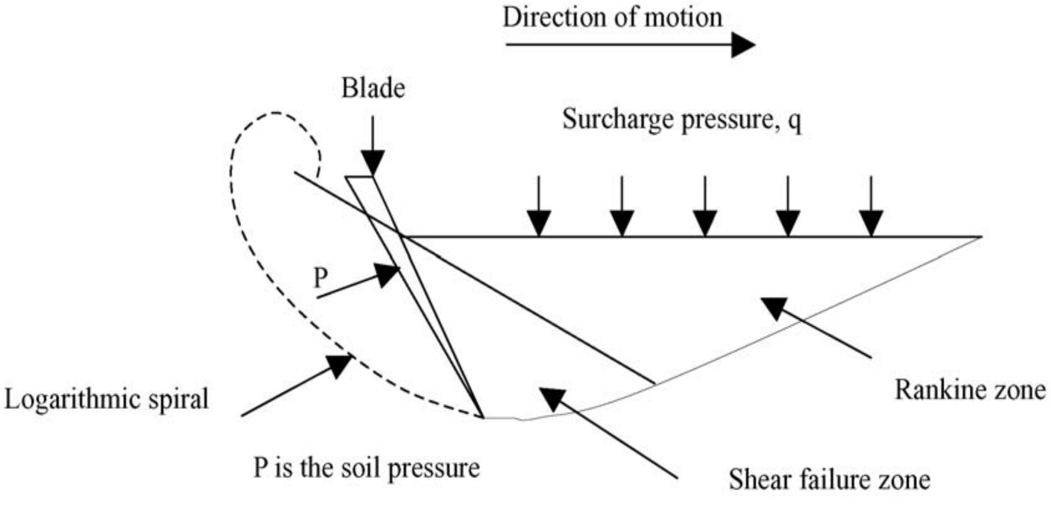
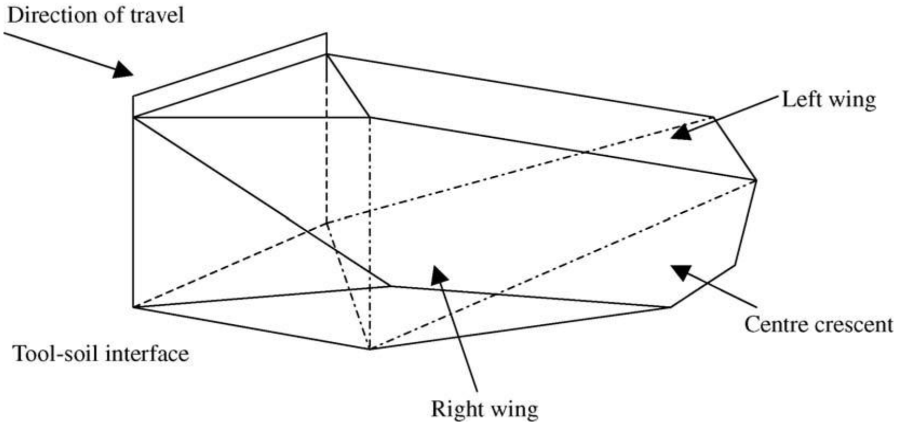
Fig-1. Logarithmic spiral failure zone, [3].



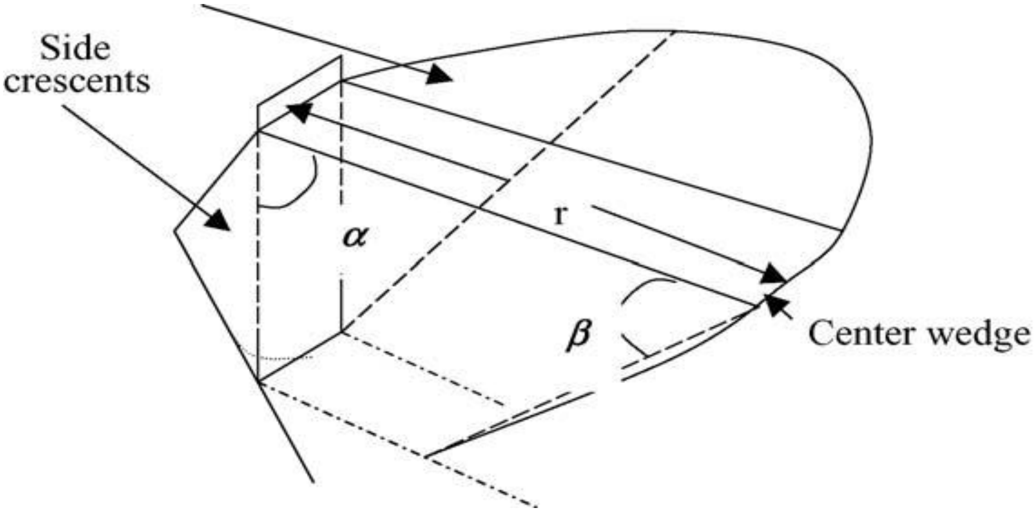
Source: [3].

Fig-2. Failure zone of the Payne model [4].



Source: [4].

Fig-3. Double-wedge failure zone of the McKyes-Ali model (a-rake angle; b-angle of soil failure plane; r-rupture distance).[5].



Source: [5].

Table-1. First Soil Sample Mechanical Properties, $\phi = 20^0$, $\delta = 10^0$, $\rho_d = 1.77$ g / cm^3, C = 3 kPa, $C_a = 1.62$ kPa

α^o	β^o	N_γ	N_q	N_c	N_{ca}	r (m)	Ws (m)	Fs (kN)
5	72.8	29.55	23.73	6.28	1.62	3.52	1.29	7.89
10	70.2	12.88	11.11	5.11	1.09	1.81	0.94	3.79
15	67.8	8.27	7.60	4.68	0.92	1.24	0.78	2.64
20	65.4	6.20	5.99	4.48	0.86	0.96	0.69	2.12
25	63.1	5.05	5.06	4.39	0.83	0.80	0.62	1.84
30	60.8	4.34	4.45	4.37	0.83	0.69	0.57	1.67
35	58.7	3.87	4.01	4.41	0.84	0.61	0.53	1.56
40	56.6	3.55	3.66	4.49	0.86	0.56	0.50	1.50
45	54.5	3.33	3.37	4.60	0.90	0.51	0.47	1.47
50	52.6	3.18	3.11	4.76	0.95	0.48	0.44	1.46
55	50.6	3.09	2.87	4.97	1.01	0.46	0.41	1.47
60	48.7	3.05	2.63	5.23	1.09	0.44	0.38	1.50
65	46.9	3.04	2.37	5.55	1.19	0.42	0.35	1.55
70	45.0	3.08	2.09	5.94	1.31	0.41	0.31	1.62
75	43.2	3.15	1.75	6.43	1.46	0.40	0.28	1.71
80	41.4	3.26	1.32	7.05	1.65	0.39	0.23	1.84
85	39.5	3.42	0.77	7.84	1.89	0.39	0.16	2.00
90	37.7	3.62	0.01	8.86	2.21	0.39	0.02	2.21

Table-2. Second Soil Sample Mechanical Properties, $\phi = 30^0$, $\delta = 20^0$, $\rho_d = 1.51$ g / cm^3, C = 2.5 kPa, $C_a = 1.4$ kPa

α^o	β^o	N_γ	N_q	N_c	N_{ca}	r (m)	Ws (m)	Fs (kN)
5	64.2	38.37	48.97	7.66	2.08	3.58	1.42	8.67
10	61.9	16.60	21.55	6.16	1.39	1.86	1.03	4.10
15	59.8	10.73	14.14	5.62	1.19	1.30	0.85	2.85
20	57.7	8.16	10.85	5.40	1.12	1.01	0.75	2.31
25	55.7	6.78	9.02	5.35	1.10	0.85	0.68	2.03
30	53.9	5.96	7.88	5.40	1.11	0.74	0.62	1.87
35	51.9	5.46	7.10	5.55	1.15	0.66	0.57	1.79
40	50.2	5.17	6.55	5.78	1.22	0.61	0.54	1.76
45	48.4	5.03	6.13	6.10	1.30	0.57	0.50	1.77
50	46.8	5.00	5.80	6.53	1.42	0.53	0.47	1.82
55	45.1	5.09	5.53	7.10	1.58	0.51	0.43	1.92
60	43.6	5.30	5.29	7.85	1.78	0.49	0.40	2.06
65	42.0	5.66	5.06	8.87	2.06	0.47	0.37	2.26
70	40.5	6.21	4.79	10.28	2.44	0.46	0.33	2.55
75	38.9	7.05	4.43	12.34	3.00	0.45	0.29	2.99
80	37.4	8.41	3.84	15.55	3.88	0.45	0.24	3.68
85	35.8	10.78	2.73	21.19	5.43	0.44	0.17	4.88
90	34.2	15.87	0.04	33.46	8.82	0.44	0.02	7.49

Table-3. Third Soil Sample Mechanical Properties, $\phi = 32^0$, $\delta = 23^0$, $\rho_d = 1.42$ g / cm^3, C = 2.4 kPa, $C_a = 1.14$ kPa

α^o	β^o	N_γ	N_q	N_c	N_{ca}	r (m)	Ws (m)	Fs (kN)
5	62.2	41.05	57.21	8.05	2.15	3.59	1.46	8.70
10	60.0	17.78	24.99	6.47	1.46	1.88	1.05	4.12
15	57.9	11.54	16.34	5.92	1.26	1.31	0.87	2.87
20	55.9	8.82	12.51	5.71	1.19	1.03	0.76	2.34
25	54.0	7.38	10.41	5.68	1.18	0.86	0.69	2.06
30	52.2	6.54	9.11	5.77	1.20	0.75	0.63	1.92
35	50.4	6.05	8.25	5.97	1.26	0.68	0.58	1.85
40	48.7	5.79	7.65	6.28	1.34	0.62	0.54	1.84
45	47.1	5.70	7.23	6.70	1.45	0.58	0.51	1.87
50	45.5	5.76	6.93	7.28	1.61	0.55	0.47	1.95
55	43.9	5.97	6.71	8.05	1.81	0.52	0.44	2.09
60	42.4	6.37	6.55	9.11	2.09	0.50	0.41	2.29
65	40.9	7.00	6.44	10.60	2.49	0.49	0.37	2.59
70	39.4	8.02	6.36	12.82	3.08	0.47	0.34	3.05
75	37.9	9.70	6.25	16.36	4.03	0.47	0.29	3.80
80	36.5	12.81	6.01	22.86	5.77	0.46	0.24	5.17
85	35.0	20.19	5.25	38.32	9.92	0.46	0.18	8.44
90	33.5	58.81	0.15	119.86	31.87	0.45	0.02	25.64