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STEADY STATE ACCELERATION MODEL OF COSMIC RAY IN THE ATMOSPHERE

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ABSTRACT

Hydrodynamic equations were simplified to a layman understanding together with the diffusion-convection transport equation for the cosmic-ray distribution function $f(t,r,p)$, *which depends on time* t, *radial distance from the point of supernova explosion* r *and the particle momentum* p. *From the results, gas density* ρ *is inversely proportional to the gas velocity* u; *flux tubes have a tendency to rise at an appreciable speed; pressure of the atmospheric gas* P_g *depends on the cosmic-ray pressure* P_c *and the internal energy of the electron* U *is directly proportional to the gas density* ρ. *Based on what Vladimir, et al. [1] said: "More work is needed to understand how robust our results are", this has helped in making their bulky equations clearer and showing how robust their work are. In the cause of breaking down or simplifying the high energy equations, it was found that cosmic rays move with Alfven speed.*

Keywords: Hydrodynamic, Cosmic ray, Atmosphere, Acceleration, High energy, Particles, Density, Steady state.

Contribution/ Originality

With the use of steady state assumption, cosmic ray acceleration within the atmospheric region was examined using high energy equations. The formulas obtained are; $\rho = \frac{constant}{r^2 u}$, $P_g = -P_c, U = \frac{2l}{p}$. This paper, primarily showed that cosmic ray moves with Alfven speed and its acceleration rate is affected by atmospheric density, and other parameters.

1. INTRODUCTION

Cosmic rays are high energy sub-atomic particles from outside of the solar system which contains mostly protons and alpha particles [2]. Cosmic rays are associated with electromagnetic radiations and can travel at nearly the speed of light with enormous energy in the range 0.1-15Gev [3]. These cosmic rays believed to have come outside our solar system. And they produces ionization within the atmosphere. When cosmic rays enter the earth's atmosphere, they collide with ambient atmospheric gas molecules thereby ionizing them. This process produces more energetic cosmic ray particles which bring about an ionized cascade known as shower. The intensity and penetration dept of the cascade depends on the energy of the primary cosmic particles. Cascade of particles with several hundred MeV of kinetic energy may reach the ground. However, due to their charges, cosmic ray particles are additionally deflected by the geomagnetic field. Almost all particles can penetrate into the polar region, where the magnetic field lines are perpendicular to the ground, whereas only the rare most highly energetic particles with energy above 15Gev are able to penetrate the lower atmosphere near the equator.

It is interesting to note that solar energy flux reaching the earth's orbit $F_s = 1.36 \times 10^3 wm^{-2}$ whereas the cosmic energy flux (particles with energy > 0.1Gev) $F_{CR} = 10^{-5} Wm^{-2}$[4]. Thus, energy input by cosmic rays in the Earth's atmosphere is about 10^{-9} times that of solar energy and hence it is unlikely that cosmic rays could influence the atmosphere processes. However, cosmic rays are the only source of ion production in the lower atmosphere [5], which is confirmed from the measurements of Ermakov and Komotskov Ermakov, et al. [6]. The rate of this ion production by cosmic rays are highly affected by atmospheric electrical properties such as thunder cloud formation, lightening production etc. [7].Cosmic rays enters the atmosphere with high energy, and moves fast close to the speed of light. Since the discovery of the cosmic rays, scientists have measured a great number of cosmic rays with different energies with detectors all over the world [8]. It is believed that some cosmic rays are accelerated within the galaxy and some are accelerated outside the galaxy. Supernova remnants in the Milky Way galaxy are most promising accelerators of cosmic rays showering on the Earth [9].

In a paper presented by Yamazaki [10], he reviewed current unresolved problems and recent progress of both observational and theoretical works on the cosmic-ray acceleration at supernova remnants. He focus on the fact that recent X-ray and very-high-energy (TeV) gamma-ray observations tell us important information on this issue, especially on acceleration efficiency, evidence for proton acceleration, and so on. Here we show how robust the systems of equations governing the acceleration of cosmic rays in the atmosphere are. Also simplify elementarily, to a layman understanding the bulky relations accompanying cosmic ray acceleration. To show how the physical parameters vary or relate to one another. This work is limited to the acceleration of cosmic rays within the atmosphere, using hydrodynamic equation.

2. METHODOLOGY

Hydrodynamic equations laid by Vladimir, et al. [1] was employed. The equations were accompanied with the diffusion-convection transport equation for the cosmic-ray distribution function $f(t, r, p)$, which depends on time t, radial distance from the point of supernova explosion r and the particle momentum p. The full bulky system of governing equations described by Vladimir, et al. [1]:

$$\frac{\partial \rho}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}r^2 u\rho \tag{A}$$

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial r} - \frac{1}{\rho}\left(\frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r}\right) \tag{B}$$

$$\frac{\partial P_g}{\partial t} = -u\frac{\partial P_g}{\partial r} - \frac{\gamma_g P_g}{r^2}\frac{\partial r^2 u}{\partial r} - (\gamma_g - 1)(w - u)\frac{\partial P_c}{\partial r} \tag{C}$$

$$\begin{aligned}\frac{\partial f}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\, D(p,r,t)\frac{\partial f}{\partial r} - w\frac{\partial f}{\partial r} + \frac{\partial f}{\partial p}\frac{p}{3r^2}\frac{\partial r^2 w}{\partial r} \\ + \frac{\varphi\delta(p - p_{inj})}{4\pi p_{inj}^2 m}\rho(R+0,t)\left(\dot{R} - u(R,0,t)\right) \\ \times\ \delta\left(r - R_{(t)}\right)\end{aligned} \tag{D}$$

The equations (A-D) above were simplified elementarily by: assuming a steady state condition in roughly all the equations (A-D), arriving at equation of a bulky fluid behavior of plasma where some interaction terms cancelled each other by Newton's 3rd law in equation (B), introducing Ampere's law: $\left\{\frac{1}{4\pi}(\nabla X B)XB = \nabla P - \rho g\right\}$ in the same equation (B), considering in equation (D) only the momentum distribution function of the particles $\left(f(p,t)\right)$. The end point of each of the equation was noted, to show how the physical parameters vary or relate with each other; thereby giving insight on the cosmic ray acceleration.

3. RESULT

The following workings summarize how the four (4) equations, (equations A to B), accompanying cosmic ray acceleration were broken down to a layman understanding; thus showing how robust the results of the initiators, Vladimir, et al. [1] are.

3.1 From the Governing Equation (A)

From the relation equation governing equation (A), we assume a steady state condition: $\frac{\partial \rho}{\partial t} = 0$, so that equation (A) becomes:

$$0 = -\frac{1}{r^2}\frac{\partial}{\partial r}r^2 u\rho \tag{Ai}$$

Then,

$$0 = \frac{\partial}{\partial r} r^2 u\rho \qquad (1)$$

$$Constant = \int \frac{\partial}{\partial r} r^2 u\rho . dv$$

$$Constant = \int 2ru\rho . dv$$

$$Constant = r^2 u\rho$$

$$\rho = \frac{constant}{r^2 u} \qquad (2)$$

3.2 From the Governing Equation (B)

In governing equation (B), we multiply ρ to both sides of the equation; equation (B) becomes:

$$\rho \frac{\partial u}{\partial t} = \rho \left[-u \frac{\partial u}{\partial r} - \frac{1}{\rho} \left(\frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) \right]$$

$$\rho \frac{\partial u}{\partial t} = -\rho u \frac{\partial u}{\partial r} - \left(\frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) \qquad (3)$$

Equation (3) gives the energy equation; where u is the internal heat energy (i.e. energy per unit mass). This equation (3) is similar to the energy equation stated by Einar and Gordon [11], which is,

$$\rho \frac{\partial u}{\partial t} = -\rho \nabla . v + \rho g \qquad (3i)$$

Comparing equation (3) and (3i), we obtain:

$$-\rho u \frac{\partial u}{\partial r} - \left(\frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) = -\rho \nabla . v + \rho g$$

Therefore,

$$-u^2 \frac{\partial \rho}{\partial r} = -P \nabla . v + \rho g + \left(\frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right). \qquad (4)$$

But from equation (1) above,

$$0 = \frac{\partial}{\partial r} r^2 u\rho$$

Therefore,

$$\frac{\partial \rho}{\partial r} = 0 \qquad (4i)$$

Equation (4i) is a steady state assumption. Equation (4) becomes:

$$0 = -P \nabla . v + \rho g + \left(\frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) \qquad (5)$$

$$\left(\frac{\partial P_g + \partial P_c}{\partial r} \right) - P \nabla . v + \rho g = 0$$

$$\left(\frac{\partial P_g + \partial P_c}{\partial r}\right) - \nabla.(Pv) + nmg = 0, \qquad (5i)$$

Where $\rho = nm$ and n is the particle density. But,

$\left(\frac{\partial P_g + \partial P_c}{\partial r}\right)$is the same ratio of force to volume, that is,

$$\frac{Force}{volume} = \frac{Force}{Area} \times \frac{1}{l} = \frac{Mass}{volume} \times a = \rho a = \rho \frac{\partial v}{\partial t} = nm\frac{\partial v}{\partial t} = \frac{\partial (nmv)}{\partial t}$$

This makes equation (5i) to be:

$$\frac{\partial (nmv)}{\partial t} - \nabla.(Pv) + nF = 0 \qquad (6)$$

$$nm\frac{\partial v}{\partial t} = nF - \nabla.\varphi + P \qquad (6i)$$

where φ, is the simple case of an isotropic distribution of the random velocities of the particles. With an electric field and magnetic field present in the atmosphere the force in equation (6i) becomes:

$$F = e\left(E + \left(\frac{v}{c}\right) \times B_o\right) + mg \qquad (7)$$

Substituting equation (7) into equation (6i), we have;

$$n_e m_e \frac{\partial v_e}{\partial t} = -n_e e\left(E\frac{v_e}{c} \times B_o\right) - \nabla.\varphi_e + n_e m_e g + P_{ei} \qquad (7i)$$

Equation (7i) is for the electrons, and

$$n_i m_i \frac{\partial v_i}{\partial t} = -n_i Ze\left(E\frac{v_i}{c} \times B_o\right) - \nabla.\varphi_i + n_i m_i g + P_{ie} \qquad (7ii)$$

Is for the ions.

In order to obtain the bulk fluid behavior of the atmosphere (or plasma), we add equations (7i) and (7ii), which gives us:

$$n_e m_e \frac{\partial v_e}{\partial t} + n_i m_i \frac{\partial v_i}{\partial t} = n_e e\frac{(v_i v_e) \times B_o}{c} - \nabla P + (n_i m_i + n_e m_e)g \qquad (8)$$

Considering Newton's third law,

$$\rho\left(\frac{\partial v}{\partial t}\right) = \frac{1}{c} j \times B_o - \nabla P + \rho g \qquad (9)$$

If we apply Ampere's law,

$$\frac{1}{4\pi}(\mathrm{B}.\nabla)B = \nabla\left(\frac{\mathrm{B}^2}{8\pi} + \mathrm{P}\right) - \rho g, \qquad (10)$$

Considering a flux tube of uniform field strength $B_o\hat{x}$, placed in a field-free atmosphere, plane stratified in the Z-direction. The left hand side of equation (10) vanishes in this situation.

If the tube is in pressure balance and thermal equilibrium with its surroundings then:

$$2n_I kT = P_I = P_O - \frac{B_0^2}{8\pi} = 2n_O kT - \frac{B_0^2}{8\pi} \qquad (11)$$

So that,

$$n_I = n_O - \frac{B_o^2}{16\pi kT}.$$

It therefore noted that the tube is lighter than its surroundings and hence experiences an upthrust force of:

$$F = (n_O - n_I)m_H gV = \frac{B_o^2 m_H gV}{16\pi kT} (12)$$

$$U_f = \left(\frac{B_o^2}{4\pi\rho}\right)^{1/2} = V_A (13)$$

that is, the Alfven speed for the field strength B_o.

3.3. Governing Equation (C)

From the relation governing equation (C), we have:

$$\frac{\partial P_g}{\partial t} = -u\frac{\partial P_g}{\partial r} - \frac{\gamma_g P_g}{r^2}\frac{\partial r^2 u}{\partial r} - (\gamma_g - 1)(w - u)\frac{\partial P_c}{\partial r} \qquad (C)$$

$$\frac{\partial P_g}{\partial t} + \frac{\partial}{\partial r}\left(uP_g + (\gamma_g - 1)(w - u).P_c\right) = -\frac{1}{r^2}\frac{\partial r^2}{\partial r}u(P_g\gamma_g) (14)$$

Equation (14) will be used to examine the effect of pressure on both cosmic ray particle and atmospheric particle.

$$\frac{\partial P_g}{\partial t} + u\frac{\partial P_g}{\partial r} + (\gamma_g - 1)(w - u)\frac{\partial P_c}{\partial r} = -\frac{\gamma_g P_g}{r^2}\frac{\partial r^2 u}{\partial r}$$

$$\frac{\partial P_g}{\partial t} + \frac{\partial}{\partial r}uP_g + \frac{\partial}{\partial r}(\gamma_g - 1)(w - u).P_c = -\frac{\gamma_g P_g}{r^2}\frac{\partial r^2 u}{\partial r}$$

$$\frac{\partial P_g}{\partial t} + \frac{\partial\left(uP_g + (\gamma_g - 1)(w - u).P_c\right)}{\partial r} = -\frac{\gamma_g P_g}{r^2}\frac{\partial r^2 u}{\partial r}$$

$$\frac{\partial P_g}{\partial t} + \frac{\partial}{\partial r}\left(uP_g + (\gamma_g - 1)(w - u).P_c\right) = -\frac{1}{r^2}\frac{\partial r^2}{\partial r}u(P_g\gamma_g). \qquad (14i)$$

Recall that, in equation (Ai)

$$0 = -\frac{1}{r^2}\frac{\partial}{\partial r}r^2 u\rho;$$

then, equation (14i) becomes:

$$\frac{\partial P_g}{\partial t} + \frac{\partial}{\partial r}\left(uP_g + (\gamma_g - 1)(w - u).P_c\right) = 0$$

Recall also that in steady state assumption,

$$\frac{\partial P_g}{\partial t} = 0.$$

$$0 + P_g\frac{\partial u}{\partial r} + P_c\frac{\partial}{\partial r}(\gamma_g - 1)(w - u) = 0$$

Then,

$$P_g \frac{\partial u}{\partial r} = -P_c \frac{\partial}{\partial r}(\gamma_g - 1)(w - u)$$

$$P_g = -P_c, \tag{15}$$

Equation (15) shows that what affects atmospheric pressure also affects the pressure of the cosmic ray.

3.4 Governing Equation (D)

From the relation governing equation (D),

$$\frac{\partial f}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 D(p,r,t)\frac{\partial f}{\partial r} - w\frac{\partial f}{\partial r} + \frac{\partial f}{\partial p}\frac{p}{3r^2}\frac{\partial r^2 w}{\partial r} + \frac{\varphi\delta(p - p_{inj})}{4\pi p_{inj}^2 m}\rho(R+0,t)\left(\dot{R} - u(R,0,t)\right) \times \delta\left(r - R_{(t)}\right) \tag{D}$$

According to Ramaty [12], if we consider only the $f(p,,t)$, momentum distribution function of the particles, we have:

$$D(p) = \frac{1}{3}\left(\frac{U}{lv}\right)p^2$$

Then,

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D(p)\frac{\partial f}{\partial p}\right]. \tag{16}$$

From steady state assumption,

$$\frac{\partial f}{\partial t} = 0,$$

Then equation (16) becomes:

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D(p)\frac{\partial f}{\partial p}\right] = 0.$$

This implies that,

$$\frac{\partial\left(\frac{p^4U^2}{3lv}\right)}{\partial p} = 0$$

But by quotient rule,

$$\frac{\partial\left(\frac{p^4U^2}{3lv}\right)}{\partial p} = \frac{(3l^4.8p^3U) - (p^4U^2.12l^3)}{9l^8} = 0$$

$$3l^4.8p^3 - p^4U^2.12l^3 = 0$$

$$l^4.2p^3U = p^4U^2.l^3$$

$$l.2p^3U = p^4U^2$$

$$l.2p^3 = p^4U$$

$$l.2 = pU$$

$2l = pU$

Therefore,

$$U = \frac{2l}{p} \tag{17}$$

Where U the Internal energy of the particle is, p is the particle momentum, l is the length.

4. DISCUSSION

In discussing the acceleration of cosmic ray particles in the atmosphere, hydrodynamic equations were simplified to a layman understanding together with the diffusion-convection transport equation for the cosmic-ray distribution function $f(t, r, p)$, which depends on time t, radial distance from the point of supernova explosion r and the particle momentum p.The governing equations (A-D) above were simplified elementarily by: assuming a steady state condition. From the expression governing equation (A), $\rho = \frac{Constant}{r^2 u}$ reveals that gas density ρ is inversely proportional to the gas velocity u in equation (3); though it can also be constant only if r^2u is constant. According to Osterbrock [13], the magnetic field at this region may play some guiding and controlling role. Hence, in equation (7) it can be seen that if the component of the equation is parallel to the magnetic field, then there will be a uniform acceleration along the magnetic field-lines.

Further decomposition of the expression governing equation (B) leads to an equation of a bulky particle behavior of the atmosphere, where some Ampere's law was introduced, where the atmosphere was assumed to be in thermal equilibrium (11). From the expression governing equation (C): it entails that effect of pressure of the atmospheric gas P_g is also the effect of cosmic ray pressure P_c i.e. $P_g = -P_c$ (15).

Based on what Vladimir, et al. [1] said: "More work is needed to understand how robust our results are", this has helped in making their bulky equations clearer and showing how robust their work are by noting the end point of each of the four equations, how the physical parameters vary or relate with each other. This gives insight on the cosmic ray acceleration and behavior within the atmosphere.

5. CONCLUSION

In the result of this work, the acceleration of cosmic ray particles in the atmosphere has been resolved and simplified using high energy equation. These high energy equations are associated with some physical parameter. How these physical parameters vary or relate with each other were clearly shown. This gives insight on the cosmic ray acceleration and behaviours. Atmospheric density is inversely proportional to the particle velocity from the expression governing equation (A). From the expression governing equation (B), cosmic ray flux has a tendency to rise at an appreciable speed. Pressure of the atmospheric gas P_g depends on the cosmic-ray pressure P_c as seen from the relation governing equation (C). From the diffusion-convection transport equation for the cosmic-ray distribution function $f(t,r,p)$, (equation D), the internal energy of the electron U is directly proportional to the gas density ρ. It also shows that cosmic rays particle undergoes Alfven speed in the cause of acceleration within the atmosphere.

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