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EVALUATION AND QUANTIFICATION OF ELECTROMAGNETIC FIELD DISTRIBUTION FOR DIFFERENT CONFIGURATIONS OF AERONAUTICAL MATERIALS

Salim Bennoud[1]† --- Mourad Zergoug[2]
[1]*Laboratory of Aircrafts, University of Blida 1 (Saad Dahlab), Blida, Algeria*
[2]*Research Center in Industrial Technologies C.R.T.I (ex C.S.C), Cheraga, Algeria*

ABSTRACT

The electromagnetic field distributions are of great importance in various engineering applications, especially in aeronautical industry due to the presence of the communication systems, the embarked radars and antennas as well as the electric circuits and components on aircraft that requires to take account of the influence of these devices on their environment and in particular of their interactions with the material. This paper shows how electromagnetic field distributes across different aeronautical materials. This problem is solved using computational electromagnetic, such as the finite elements method, to resolve Maxwell's equations through the problem geometry using an in-house code developed during this work. Obtained results enable to calculate the field values in any place of the studied geometry and determinate the related physical parameters. Results show that parameters such as material properties, used frequency and sample dimensions have a strong influence on the field distribution. This dependence could have important consequences to characterize and optimize conditions to choose materials used in electromagnetic applications. For particular cases these results are compared with open-source codes. The results are very similar with a good precision which enables to use the developed code to carry out simulations for other geometries of materials with different proprieties.

Keywords: Aeronautical materials, Computational electromagnetic CEM, Field distributions, Finite element method FEM, Maxwell's equations, Magnetic vector potential, Electric scalar potential.

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Contribution/ Originality

The paper's primary contribution is presentation of new and developed CEM code characterized by capability to calculate and quantify the electromagnetic fields in various regimes. This study contributes in the existing literature by increasing the advantages to apply FEM method analysis to more comprehend electromagnetic problems.

1. INTRODUCTION

The structure defects are often detected, mainly by the formation of discontinuities due to an inhomogeneous electromagnetic field distribution [1].

Electromagnetic field can be presented in three main classes of application: magnetic fields, electric fields and electromagnetic waves [2]. To calculate the different distributions is then crucial to understand the complex

† Corresponding author

phenomenon of the interaction between materials and electromagnetic field. Several recent applications are studied and can be consulted in the literature [3-9].

The resolution of electromagnetic problems returns in fact to the resolution of Maxwell's equations that provide a fundamental tool for governing physical aspects of electromagnetic interactions [10].

These problems are solved using computational electromagnetic (CEM) which can solves typically the electromagnetic fields problem across the studied domain which has complex geometries.

Recent special studies focused on CEM modeling, including electromagnetic modeling by the Finite Element Method (FEM), are presented in various papers [11-15] and many authors have mentioned that the application of the FEM is suitable to investigate electromagnetic problems [16-20].

To demonstrate the useful of the developed code, the electromagnetic field actions, in conducting materials samples of selected shapes and geometries, were simulated. This case study presents a challenge for various engineering applications especially those exploited by the aeronautical industry such as the development and characterization of materials, the positioning and establishment of electric and magnetic devices, and the control and detection of defects using the electromagnetic non destructive techniques.

CEM modeling was provided using FEM method. First, the electric and/or magnetic fields are calculated for each of smaller part across the material and combined to give a solution for the entire configuration, and once this global field is given, others physical parameters are deduced from it.

Code validation was made for simple cases and for the particular cases these results are compared with FEMLAB code. Simulations of various cases will be made using developed code. This will enable to interpret the results, to present them graphically and to use simulations for various applications.

2. PROBLEM MODELING AND COMPUTATIONAL FORMULATION

The distribution of electromagnetic fields is described by Maxwell's equations with appropriate boundary conditions. The basic equations system of this problem consists of the Maxwell's equations and the electric and magnetic constitutive relations.

The governing equations are presented, for conducting, homogeneous and isotropic materials, as:

$$\text{curl } E = - \partial B / \partial t \tag{1}$$

$$\text{div. } B = 0 \tag{2}$$

$$\text{curl } B = \mu\,(J + \partial D / \partial t) \tag{3}$$

$$\text{div. } E = \rho/\varepsilon \tag{4}$$

$$B = \mu\, H \tag{5}$$

$$D = \varepsilon\, E \tag{6}$$

$$j = \sigma\, E \tag{7}$$

where ***E*** is the electric field intensity, ***B*** is the magnetic induction, ***D*** is the electric displacement, ***J*** is the current density, $\boldsymbol{\mu}$ is the permeability of the material, $\boldsymbol{\rho}$ and $\boldsymbol{\varepsilon}$ are respectively the electric charges density and the permittivity of the chosen material, ***H*** is the magnetic field intensity and $\boldsymbol{\sigma}$ is the conductivity of the material.

Substituting equations (5 to 7) into equations (1 and 3), equations (8 and 9) give the determinate form of Maxwell's partial differential equations:

$$\text{curl } H = \sigma\, \mathbf{E} + \varepsilon\, \partial\, \mathbf{E} / \partial t, \tag{8}$$

$$\text{curl } E = - \mu\, \partial H / \partial t. \tag{9}$$

However, in numerical analysis some difficulties arise from the nonlinearity of equations. Potentials must be introduced to simplify mathematical formulations. Thus, the flaw field can be constructed by introducing two potentials, the magnetic vector potential ***A*** and the electric scalar potential ***V.***

The magnetic vector potential ***A*** and the electric scalar potential ***V*** are defined so that electric and magnetic fields are obtained using the following relationships:

$$B = \text{curl } A \tag{10}$$

$$E= -\nabla V- \partial A / \partial t \quad (11)$$

Substituting equations (10 and 11) in the equations (3 and 4), equation (3) becomes:

$$\text{curl } B - \mu (\sigma (-\nabla V- \partial A / \partial t) = \mu J \quad (12)$$

Equation (13) is obtained by using equation (10):

$$\text{curl}(\text{curl}A) = \mu\,\sigma\left(-\nabla V - \frac{\partial A}{\partial t}\right) + \mu\,\epsilon\left(\frac{\partial\left(-\nabla V - \frac{\partial A}{\partial t}\right)}{\partial t}\right) \quad (13)$$

Using the curl-curl relationship (i.e: curl (curl A) =-div (∇A) + ∇ (div A)), Equation (14) is given by:

$$-\text{div}\,(\nabla A) + \nabla\,(\text{div}\,A) = \mu\,\sigma\left(-\nabla V - \frac{\partial A}{\partial t}\right) + \mu\,\epsilon\left(\frac{\partial\left(-\nabla V - \frac{\partial A}{\partial t}\right)}{\partial t}\right) \quad (14)$$

Or

$$\text{div.}\,(\nabla A) -\nabla\,(\text{div } A) + \mu\,\sigma\,(\nabla V+ \partial A / \partial t) = \mu J \quad (15)$$

Coulomb gauge (div A = 0), (∇ V=0) (there is no gradient of a scalar potential in equation (15) if there is no initial static charge on the medium) and J the current source (the gradient of ***V*** is implicit in J) are used to simplify equation (15) which is written as:

$$\text{div.}\,(\nabla A) = \mu\,\sigma\,(\partial A / \partial t) - \mu J \quad (16)$$

From equation (4), the second expression can be written:

$$\text{div.}\,(-\nabla V- \partial A / \partial t) = \rho / \varepsilon \quad (17)$$

In the static case, the term (∂A / ∂t) is negligible. So, equations (16 and 17) can be rewritten by employing the identity in the static case as:

$$\nabla^2 A = \mu J \quad (18)$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon} \quad (19)$$

with (∇^2) being the Laplace operator.

However, it is clear that in the static case, equations (18 and 19) reduce to the familiar form of ***Poisson's*** equation.

In the case of time varying magnetic field (for example: eddy current problem), the problem governing by equation (16) can be described mathematically by the following equation in terms of the magnetic vector potential:

$$\nabla^2 A + K^2 A = -\mu J \quad (20)$$

where ***A*** represents the magnetic vector potential, J is the excitation current source density, $K^2=-\omega\mu(j\sigma+\omega\varepsilon)$ and ω is the angular frequency of the excitation current.

The electromagnetic field problem can be solved by calculating the magnetic vector potential ***A***; from which the interest parameters can be deduced.

Different real electromagnetic problems are not analytically calculable. Indeed, the complexity of the studied problems section induced that the analytical solution exists only for limited simple cases, and obviously, our interest is directed towards the CEM techniques to search approximate numerical solutions.

The FEM method is a computational technique that is suitable for analysis of electromagnetic problems. It is used to find approximate solution of the partial derivative equations like equation (16).

After the domain discretization based on definition of geometrical dimensions of the problem and definition of mesh elements, the next step is to input material properties, physical constraints and boundary conditions which are necessary to obtain problem formulation by the FEM method.

To study problem governing by equations (16 and 17) mathematically, the main defy is to create an equation which approximates the equation to be studied. It is more convenient to use a discrete formulation (variational formulation) of problem. Several discrete formulations can be used for the same problem. In this paper, discrete formulation through the weak formulation of ***Galerkin*** is used.

The finite element formulation renders the governing algebraic equations into an equivalent matrix forms and computes the unknown values. The global system to solve is given by:

$$[k]\{W\}=\{F\} \quad (21)$$

where [K] is the (N × N) symmetric and banded global matrix, (W) is the (N x 1) vector of unknowns and (F) is the (N × 1) source vector (N is the total number of nodes in the studied domain).

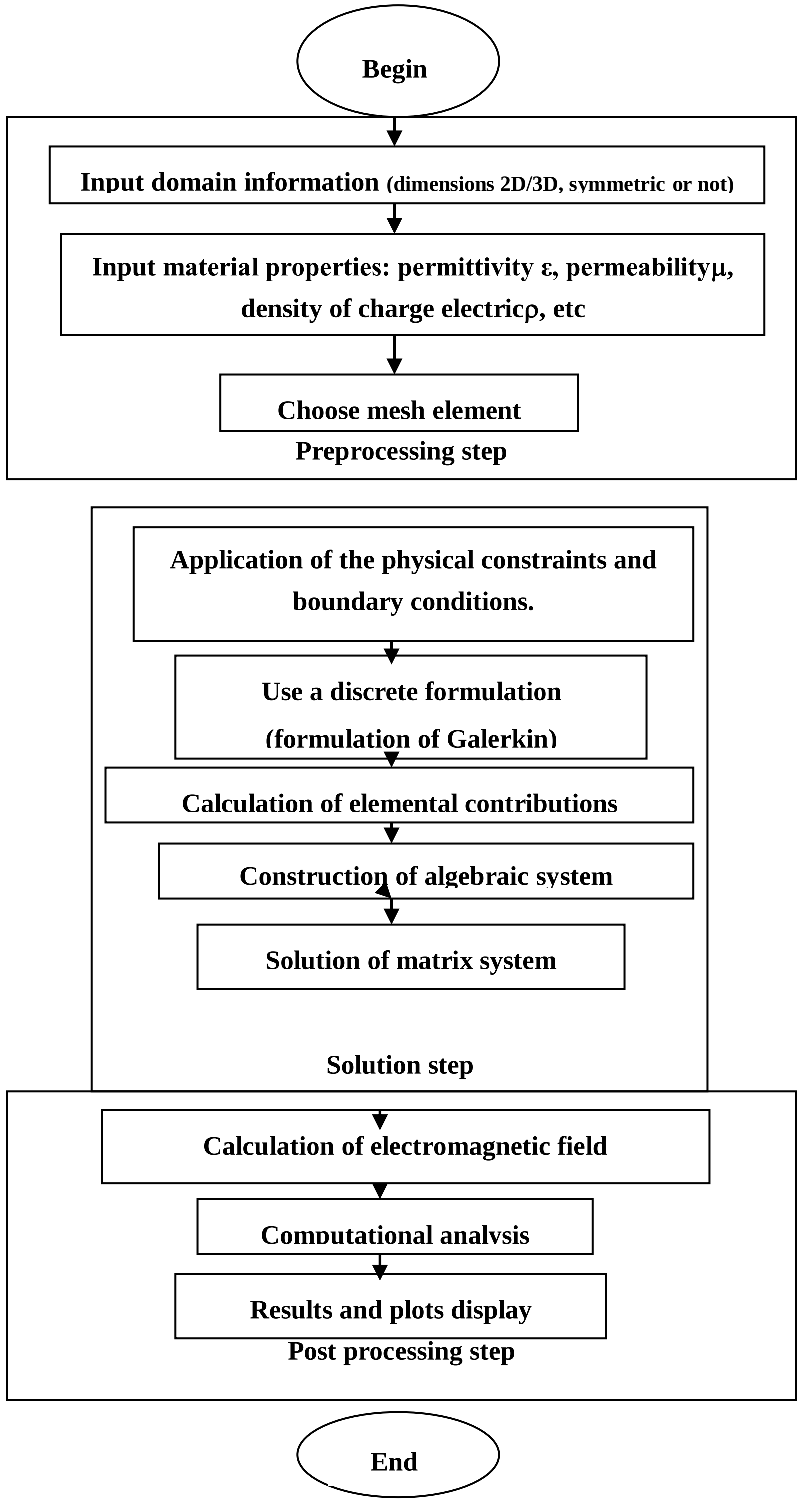

Fig-1. Flowchart of developed code.

Figure 1 presents the flowchart that includes the principal steps used to provide CEM application in order to obtain the electromagnetic interaction simulations.

The system is solved using a developed code. And the *Khalestki's* algorithm is chosen to resolve this system.

The last step of the CEM by the FEM method is the analysis and evaluation of the solution results. The distribution of magnetic (or electric) field through the studied domain is determinate, and once this field is quantified, others physical parameters can be deduced from it. So, CEM analysis provides solutions for field parameters such as force, torque, capacitance, inductance, resistance and impedance along with the ability to visualize electromagnetic fields.

3. CODE VALIDATION

In this section, the code validation is discussed and performed to express the useful of the developed code.

The calculations were done by using a full circular geometry. The mesh is realized with 225 nodes and 392 triangular elements, the relative permittivity of the section is $\varepsilon_r=1$, the relative permeability is $\mu_r=1$, the current density is J=1A and the circle radius is 10 cm and comparison with analytical values is made. Figure (2) gives some comparative values between the simulated and analytical results when the code applies to electrostatic case. Showed results are given across a scan in x-axis direction for specific coordinates (x [-5, 5] cm, y=0 cm).

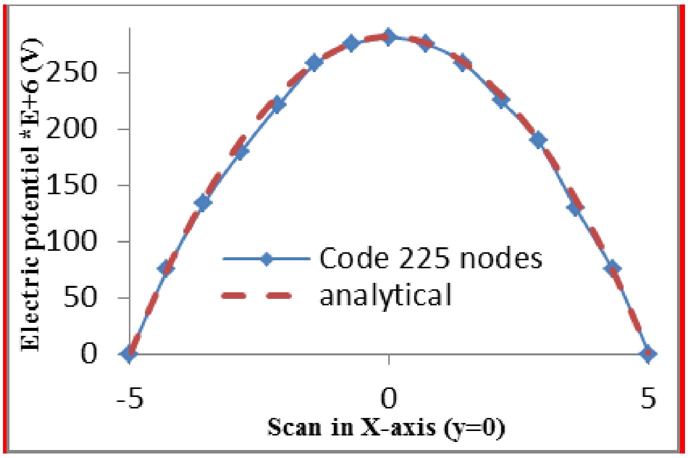

Fig-2. Comparison applied to simulated results and analytical results.

The code results and analytical results show an almost full agreement with an average error of 0.008 in the case of electrostatic field.

Another way to get the code validation is performed. Code results are compared with those given by the FEMLAB code. Table 1 gives some values specific to selected nodes.

It can be observed that the developed code results agree with the FEMLAB code results; the average error rate is 0.0125.

Table-1. Comparison with FEMLAB code for the electric field

NODES (cm)		ELECTRIC POTENTIAL (V)		ERR
x(i)	y(i)	Code(225)	FEMLAB	
10	0	0.00E+00	-9.33E-17	00
8	0	1.008E+08	1.015E+08	0.007
6	0	1.793E+08	1.806E+08	0.013
4	0	2.354E+08	2.370E+08	0.016
2	0	2.691E+08	2.709E+08	0.018
0	0	2.803E+08	2.824E+08	0.021

The analytical values and also simulations results (developed and FEMLAB) present an approximately full agreement. The maximum value of the electric potential is 2.823E+8 V for analytical values. FEMLAB code gives a value of 2.824E+8 V (simulation with 3697 nodes and 7032 elements) and developed code value is 2.803E+8 V (simulation with 225 nodes and 392 elements). These results are reasonably good and confirm the validation.

4. SIMULATIONS AND APPLICATIONS

The code described and validated previously can apply for various geometries, it can be a helpful tool to analyze and calculate the electromagnetic field distributions.

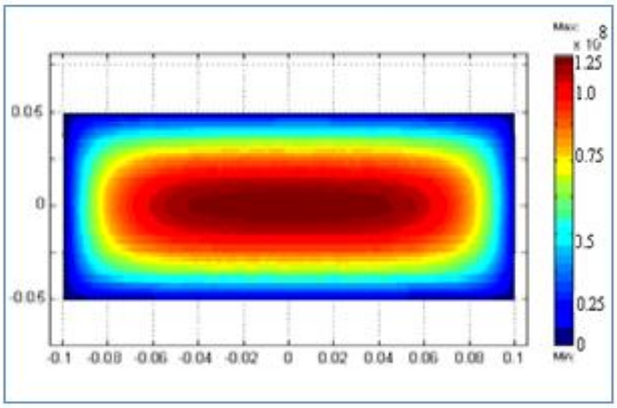

Fig-(3a). Electric field distributions obtained for a simple geometry (rectangular geometry 10cm x 20cm). Application with 3312 nodes and 6389 elements, ε_r=1, μ_r=1, and J=1A.

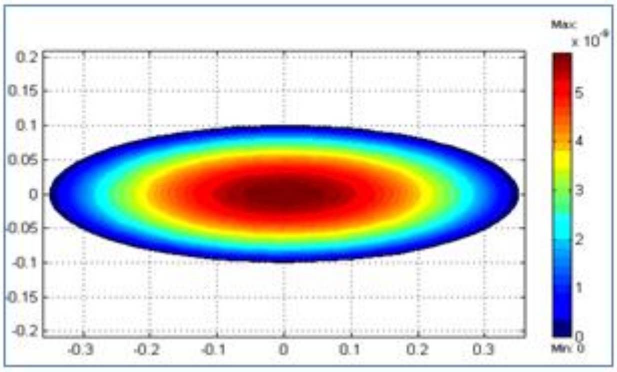

Fig-(3b). Magnetic field distributions obtained for an elliptic geometry. Application with 2615 nodes and 4815 elements, σ =1 MS/m, μ_r=1, and J=1A.

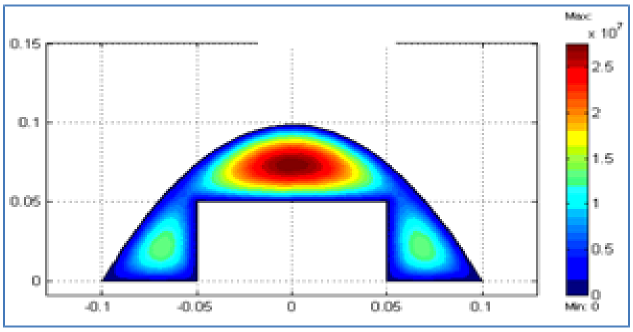

Fig-(3c). Magnetic field distributions obtained for complex geometry. Application with 2260 nodes and 4188 elements, σ =17 MS/m, μ_r=1, and J=1A.

Fig-3. Simulations applied for various sections of different materials.

Figure (3) shows the simulation of the electromagnetic field distribution in various cases. Each case displays the result of solving the field distribution problem on chosen material section using the developed code.

Figure (3a) displays the electric field distribution applied for a simple geometry (a rectangular geometry) and figure (3b) displays the magnetic field distribution applied for an elliptic geometry.

The uniformity of the electric field across the simple geometries is observed and this uniformity is directly affected in the case of complex geometry (figure 3c).

The static analysis is used to compute parameters such as magnetic flux, self and mutual inductances, forces, torques, current density, electric field and AC loss, etc.

By using this code, various meshes with triangular or quadratic elements can be obtained with different number of nodes, which gives the possibility to apply diverse interpolation functions forms.

Table (2) gives results on the importance of the optimization of a nodes and/or elements number to be used to increase the precision and to reach the best approximate solution.

The code applies for same circular geometry used for validation, changing the nodes and/or elements number used for each application and using Intel CPU (1.73 GHz).

Table-2. Precision evolution established to the nodes and/or elements increasing.(case of electrostatic problem).

Solution	Nodes	Elements	Maximal value (V)	Relative error	CPU time (ms)
Analytic			2.8230E+8	0	0
Code	221	200	2.8025E+8	7.2E-3	130
Code	221	100	2.8236E+8	2.1E-4	130
Code	225	392	2.8030E+8	7.0E-3	135
Code	225	196	2.8236E+8	2.1E-4	132
Code	441	800	2.8184E+8	1.6E-3	560
Code	441	400	2.8235 E+8	1.7E-4	500
Code	3721	7200	2.8229E+8	3.5E-5	1400
Code	3721	3600	2.8235 E+8	1.7E-4	1250

If the number of used nodes and/or elements increases, the approach to the analytical solution will be good. But this advantage must be controlled to limit run time cost (table (2) shows that increasing the number of used nodes and/or elements hasn't any important acts on precision evolution after a desirable value of this number, it only increases CPU run time).

The simulations, obtained using the developed code, showed that the number of nodes, the number of elements and shape function have an important influence on the field calculation and affect the precision and the reliability of the code results.

A good optimization of a number of the used nodes and/or elements is required to obtain good convergence, (find the solution by using the necessary number of nodes).

5. CONCLUSION

This paper shows how electromagnetic field distributes across different metallic aeronautical materials. This problem is solved using computational electromagnetic, such as the finite elements method, in solving Maxwell's equations through the problem domain using a code developed during this work which presents an efficiency way for numerical simulations and measurement problems of electromagnetic field computing and evolution.

The finite element method is successfully applied since it adapts for CEM problems. It is a good choice for solving electromagnetic field problems.

The numerical results enable to calculate the electric and/or magnetic field values in any point of the chosen geometry and to deduce thereafter additional system parameters like distribution of energy, power flow direction, resistance, reactance and impedance, etc.

The code validation is made for simple two-dimensional cases and for the special cases these results are compared with FEMLAB code.

The extended application of the model can be developed to investigate the possibility to simulate similar applications applied to non destructive testing (NDT) methods used for characterizing materials and for evaluating residual stress using magnetic methods such as eddy current and magnetic particle.

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