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IMPACT OF FORECASTING TECHNIQUES AND MARKET DEMAND SCENARIOS ON MULTI-PRODUCT, MULTI-PERIOD AGGREGATE PRODUCTION PLANNING

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ABSTRACT

Customer demands fluctuate over a different time horizon and if forecasted with same forecasting method shows errors in production planning. A linear programming mathematical model is reformulated in this paper for aggregate production planning (APP) to find the best-suited forecasting techniques for different market demand scenarios. The model is reformulated as a linear programming model and solved using excel solver to minimize relevant costs (backorder cost, inventory cost, and regular time production cost) while meeting the forecasted demand. The system performance is evaluated on the basis of service level (SL) and inventory level (IL). A case studied from a silk industry of Bangladesh used here to define three demand scenario High, Peak and Few. For each of them, the service level and inventory level was compared with the inclusion of simple moving average (SMA), weighted moving average (WMA) and simple exponential smoothing (SES) forecasting methods in the APP model. We found from the computations that for High and Few scenarios SMA is best in terms of SL and IL but for Peak scenario, WMA is best in terms of IL.

Contribution/Originality: The paper's primary contribution is reformulating the linear programming mathematical model including regular time production cost in the objective function, using the WMA and SES as forecasting techniques, and using the Excel Solver for solving the APP model which are different from other works.

1. INTRODUCTION

Aggregate production planning (APP) is followed by capacity planning using a medium range of demand forecast. Where the relationship between products and facilities are so many that lead to complex scheduling and production planning. APP will give a plan for maximizing the facilities and minimizing cost [1]. Utilizing an organization's resources is the main goal of the APP for satisfying customer demand. It determines both output levels to be planned and appropriate resource mix that can be used [2]. In APP different forecasting techniques along with different demand scenario will help to take a managerial decision with changing demand values to determine which forecasting method will be suitable for future forecasting and with changing demand pattern, to get acquainted with market demand, forecasting techniques should be changed. Needs of customers fluctuates over different planning horizon ranging from half yearly to long full year. In this period, every detail for further processing in production is difficult to calculate. Aggregate production planning considers this situation and determines inventory required, production level, and work force level over that horizon. After the calculation

applying constraints, specific quantities that need to be produced is obtained from the aggregate production plan [3].

Ho and Ireland [4] found that forecasting error does not cause a higher degree of scheduling instability suggesting that if the schedulers find a good forecasting method, they don't need to be much more concerned with forecasting error. Appropri ate lot-size will cope with the error. Enns [5] investigated forecast bias and demand uncertainty in terms of material production schedule and delivery performance. The result showed increasing planned lead or safety stock will improve performance along with less finished goods inventory. Aardal, et al. [6] considered one product, continuous review inventory system where the relationship between service level and shortage cost is studied. As most companies desired service level is known, so from the proposed objective function the value of shortage cost can be obtained. Mula, et al. [7] reviewed 87 number of literature about production planning under uncertain environment. In their opinion, artificial intelligence based models and fuzzy set theory is an appropriate methodology in recent production planning.

A study focused on the control decisions in the area of multi-period, aggregate production planning where the goal was to minimize the costs associated with overtime, over-and-under production and productivity cost. The presented solution was generally not of base-stock type, a correspondence between it and the solution of the classical newsboy problem was revealed [8]. In a case study [9] multiple conflict objectives were studied along with market demand uncertainties. The uncertainties were modeled as discrete scenarios and different probabilities for different expected outcomes. The researchers found that the linear degree of satisfaction can be used to find the best result for multiple objectives. In another study, a mixed integer linear programming model was developed by Silva, et al. [10] maximizing profit and minimizing late orders and workforce level changes were the objective function. They proposed a decision support system so that the best solution can be obtained by a practitioner without mathematical complexities of the model. San-José, et al. [11] studied a single-item inventory model where the shortages are allowed. They developed a new approach where backlogging unit cost is continuous, positive and non-decreasing with time. They developed an effective solution procedure to determine the optimal policy and the maximum average profit. On the other hand, a heuristic method is introduced by Pradenas, et al. [12] which can handle problems formulated without any simplification of assumptions as well as can solve more larger and complex problems than exact methods. The main goal of APP is to meet customer demand which is dependent on the forecasting and fluctuation in the demand during a certain period by minimizing the cost related to inventory, backorders, regular time production, sub-contracting, backlog, payroll and overtime [13]. APP strategies effectiveness was evaluated [14] by modeling a practical APP decision-making problem. The model was simulated using a hybrid discrete event simulation (DES) and system dynamics methodology. They used a total profit criterion to compare with the system's performance measure. The simulation result showed prioritized APP strategies by the pure chase strategy, the modified chase strategy, the pure level strategy, the modified level strategy, the mixed strategy, and the demand management strategy, respectively. Al-e, et al. [15] presented a model for a medium-term planning horizon where their first objective attempts to minimize the sum of the expected value and second objective function highlighted the concept of customer service level. They found the practicability of the proposed multi-objective stochastic model as well as the proposed algorithm.

Ning, et al. [16] presented a multiproduct APP model where uncertain variables are taken into account. Their objective was to obtain the belief degree of profit rather than the predetermined rate of profit. They found that if the uncertain variables are linear they can be solved by intelligent algorithm i.e. genetic algorithm otherwise the model cannot be converted to crisp equivalent. Dai, et al. [17] presented a computer simulation for such APP model with fuzzy linear programming. A fuzzy multi-objective linear programming model (FMOLP) was developed by Wang and Liang [18] for minimizing total production, carrying, and back order costs in a multi-product aggregate production model. The model yielded a compromised solution giving the decision maker overall level of satisfaction and freedom to interactively modifying the membership functions. The same researchers proposed linear approach

(PLP) for solving a multi-product APP problem in another study (2005). The model minimized the costs associated with inventory, labor, machine, overtime, subcontracting, backorder level, and warehouse capacity. But Aliev, et al. [19] mentioned the drawbacks of existing fuzzy models that only deals with separate APP without considering the interrelated nature of distribution and production system. For the solution of such similar model, Fung, et al. [20] proposed a parametric programming based fuzzy solution and an interactive procedure. A scheme of a multi-period and multi-product APP which was formulated as an integer linear programming model that uses a triangular possibility distribution for handling the imprecise operating costs, demands, and also for the capacity data where the researchers used particle swarm optimization (PE-PSO) approach to solving the APP model. The experimental results demonstrate that the PE-PSO variant provides better qualities in the aspects of its accuracy when compared to the other two algorithms genetic algorithm (GA) and a fuzzy based genetic algorithm (FBGA) [21]. A similar type of APP model is introduced by Khalili-Damghani and Shahrokh [22] considering three objective functions, including maximizing customer service level, quality of end product and minimizing total cost. They solved the approach using LINGO software and the results were compared with the existing methods of a company. Mirzapour, et al. [23] proposed the same type of APP nonlinear model but first transformed into a multi-objective linear one and solved applying linear programming metrics method. Gholamian, et al. [24] formulated fuzzy multi-objective mixed integer nonlinear programming where they focused on four conflicting objectives such as minimizing total cost and fluctuations in the rate of changes of the workforce and maximizing customer satisfaction and the total value of purchasing. Another hybrid fuzzy interference system is proposed by Fiasché, et al. [25] where they computed Pareto solutions optimization with two different techniques.

Two optimization techniques, Genetic Algorithm Optimization (GAO) approach and Big M method used for solving a real-time multi-product, multi-period aggregate production planning (APP) decision problem in another study. The proposed model used in solving an APP decision problem and attempted to bridge the gap of not including the waste cost, workforce incentive in an industrial case study. In that research, only one meta-heuristic algorithm (that is Genetic Algorithm) was compared with the linear programming method named Big-M technique Hossain, et al. [26]. Ramezanian, et al. [27] developed a mixed integer linear program for multi-product, multi-period, multi-machine, and two-stage system where the problem was solved using the genetic algorithm and tabu search. Their computational result showed that the two implemented algorithms provide good solutions for APP. Gansterer [28] defined demand scenarios as when average demand per period is 10, utilizes 95% capacity then the demand scenario is called *High*. When Average order size (including periods in which no orders arrive) is 4, utilizes 60% machine capacity then it is called *Few* demand scenarios. And finally, *Peak* can be defined when customer demand is highly volatile. This results in an average order amount of 10.

In this paper, we reformulated an APP linear programming model to minimize cost considering inventory, backorder, and regular time production cost. Which will satisfy customer demand as per the forecasted demand by three forecasting methods simple moving average, weighted moving average and simple exponential smoothing for three different market demand scenarios *High*, *Few* and *Peak*. We have included regular time production cost in the objective function along with backorder and inventory cost and WMA and SES forecasting techniques along with SMA and used *Excel Solver* for solving the APP model which differentiate our work from previous literature. Then the service level and inventory level are calculated, for each demand forecasting techniques and different demand scenarios the result is analysed to find out the best forecasting technique in each market demand scenarios so that the production can choose best forecasting method for different market demand situations.

2. MODEL FORMULATION

2.1. Problem Description

It is an unfortunate reality that some critical parameters such as customers demand, price are uncertain in most cases, and manufacturing capacity is limited by the available resources. The impact of performance efficiency can be

devastating if the decision maker will not get the necessary production planning. For our investigation, we reformulated an APP model by adding decision variable (regular time production, inventory level and backorder level) because the previous literature only included backorder and inventory level not the regular time production as a decision variable as well as constraints which is important in APP for future planning of production. Then we investigated the impact of three different forecasting techniques in three different demand scenarios. In our study demand data is forecasted on the basis of the past demand data and is used in the APP linear programming model. Capacity restrictions used here refer to available machine hours in the production floor. The result will be evaluated based on the customer service level (SL) and minimized inventory level (IL). For calculating the service level we sum up the number of backorders which is divided by the total amount of demand and then the percentage of the reverse of this value is taken. And the IL is calculated as a percentage of inventory of all products which is divided by the total number of products produced in a specific time horizon [28]. It was assumed that a company manufactures P kinds of products (two products in our case) to meet market demand over a planning horizon T.

2.2. Assumptions

1. The values of all parameters are certain over the next T planning horizon except demand.
2. Maximum machine capacity cannot exceed to the maximum levels.
3. The machine capacity was calculated for machine groups for a product rather individual machines.

2.3. Notations

The following notation is used after reviewing the literature and considering practical situations [29].

p=Product type & t= period

Rcpt=Regular time production cost per unit for pth product in period t (Tk./unit)

Rxpt=Regular time production of pth product in period t (unit)

Icpt=Inventory carrying cost per unit of pth product in period t (Tk./unit)

Ixpt=Inventory level of pth product in period t (unit)

Bcpt=Backorder cost per unit of pth product in period t (Tk./unit)

Bxpt=Backorder level of pth product in period t (unit)

Dpt=Forecasted demand for pth product in period t (unit)

Mpt=hours of labor usage per unit of pth product in period t (machine-hour/unit)

Mtmax=Maximum machine capacity available in period t (ft2)

Iptmin=Minimum inventory in period t.

Decision Variable:

Rxpt=Regular time production of pth product in period t (unit)

Ixpt=Inventory level of pth product in period t (unit)

Bxpt=Backorder level of pth product in period t (unit)

2.4. Objective Function

Total cost is usually included in solving APP problems in most practical decision making. The proposed linear programming consists of a single objective function aiming to minimize the total cost of APP including regular time production cost, backorder cost, inventory cost. The objective function is given below:

$$\text{Min, Z} = \sum_{p \in P} \sum_{t \in T} (Ix_{pt} * Ic_{pt}) + \sum_{p \in P} \sum_{t \in T} (Bx_{pt} * Bc_{pt}) + \sum_{p \in P} \sum_{t \in T} (Rx_{pt} * Rc_{pt})$$

2.4.1. Constraints

Inventory and Backorder Balancing Constraint: This equation ensures that production amounts meets the forecasted customer demand.

$$Ix_{pt} - Bx_{pt} = Ix_{p(t-1)} + Rx_{pt} - D_{pt} - Bx_{p(t-1)}$$

Constraint on Machine Capacity: This constraint ensures that machine capacities are not exceeded.

$$Rx_{pt} M_{pt} \leq M_{tmax}$$

Constraint on Carrying Inventory:

$Ix_{pt} \geq I_{ptmin}$

Non-negativity Constraints on decision variables are:

$Ix_{pt}, Bx_{pt}, Rx_{pt} \geq 0$

Where,

t= 1,2,3, ... T;

and   P= 1,2,3, ... P;

Where, in the first constraint (1) D_{pt} denotes forecasted customer demand of the pth product in period t and represents the sum of regular time production, inventory levels, backorder levels essentially should equal market demand. The second equation ensures that the production amount does not exceed machine capacity and the third constraint determine the level of inventory. Model is expanded and solved for 2 products (P=2) and three periods (T=3). The expansion is given in the Appendix.

3. CASE DESCRIPTION

The APP decision problem for manufacturing plant presented here focuses on developing an interactive LP approach for minimizing total costs. In our study, we used secondary data from a journal [29] and demand forecasting scenarios are derived from the data provided by a silk industry of Bangladesh. The demand follows a seasonal demand pattern. The planning horizon is three months (t=3) long, including April, May, and June. The model includes two types of items namely the Tosor Silk Shari (product 1) and Panjabi (product 2).

3.1. Data Description

1. Initial inventory in period 1 is 500 units of product 1 and 200 units of product 2.
2. Hours of machine usage per unit for each of the two planning periods are 0.1 machine hours for product 1 and 0.08 machine hours for product 2.
3. The previous year demand was assumed on the basis of data provided by a silk industry of Bangladesh.
4. The demand scenarios are considered, April as Few, May as High, and June as Peaks on the basis of demand fluctuations.

Table-1. Initial inventory, hours of machine usages per unit of product (Tosor Silk Shari and Panjabi).

Item	Product 1	Product 2
Initial Inventory	500 units	200 units
Hours of machine usage per unit of product.	0.1 machine-hour	0.08 machine-hour

Source: Chakrabortty and Hasin [29].

Table-2. Related operating cost.

Product	Rc_{pt}(Tk./unit)	Ic_{pt}(Tk./unit)	Bc_{pt}(Tk./unit)
1	22	3.5	42
2	20	4	47

Source: Chakrabortty and Hasin [29].

Table-3. Maximum machine capacity and minimum inventory.

Item(unit)	Period 1	Period 2	Period 3
I_{1tmin}	300	450	500
I_{2tmin}	480	580	470
M_{1max}	300	350	360
M_{2max}	160	180	170

Source: Data collected from a silk industry in Bangladesh and the table is prepared by authors.

Table-4. Previous year demand of product 1 (Tosor Silk Shari).

Year	Period 1	Period 2	Period 3
2015	3000	3450	3580
2016	3200	3550	3600
2017	3300	3700	3700

Source: Data collected from a silk industry in Bangladesh and the table is prepared by authors.

Table-5. Previous year demand of product 2 (Panjabi).

Year	Period 1	Period 2	Period 3
2015	1800	1880	2200
2016	2000	2000	2150
2017	2100	1900	2300

Source: Data collected from a silk industry in Bangladesh and the table is prepared by authors.

4. RESULT

Decision variable and objective function values of the APP model using different forecasting methods are solved here by using Microsoft Excel (2013) Solver Add-Inns which are given in Table 6. Objective value for SMA= 410009 Tk., for WMA= 451655 Tk., for SES= 434655.99 Tk. Comparison using a bar chart is shown in Figure 1. Comparing the total cost for three for the methods it can be seen that SMA shows the minimum cost. The total cost for SMA is less than others because both the backorders and inventory level is minimum comparing the other two. The result of our computational study is presented in Table 7. We report the average service level (SL) and inventory level (IP) percentage for each demand and APP scenario. The best results are in the blocked font.

For three of the forecasting techniques SMA, WMA and SES different market demand scenarios are presented with SL and INV. The highest the SL and lowest the INV is considered a good result. In the *Few* demand scenario, SL is highest 91.57% and INV is lowest 15.60% for SMA than other two APP scenario such as SL 91.51% and INV 17.96% for WMA and SL 86.88% and INV 16.73 % for SES. For a *High* market demand scenario, SL is highest 94.36% and lowest 17.89% than the other two forecasting techniques like SL 81.17% and INV 17.91% for WMA and SL 90.73% and INV 17.96% for SES. But the *Peaks* demand scenario shows a different result in INV than *Few* and *High* market demand scenario. The lowest INV is 16.92% which is for WMA than the other two forecasting techniques which are INV 16.94% for SMA and INV 17.31% for SES.
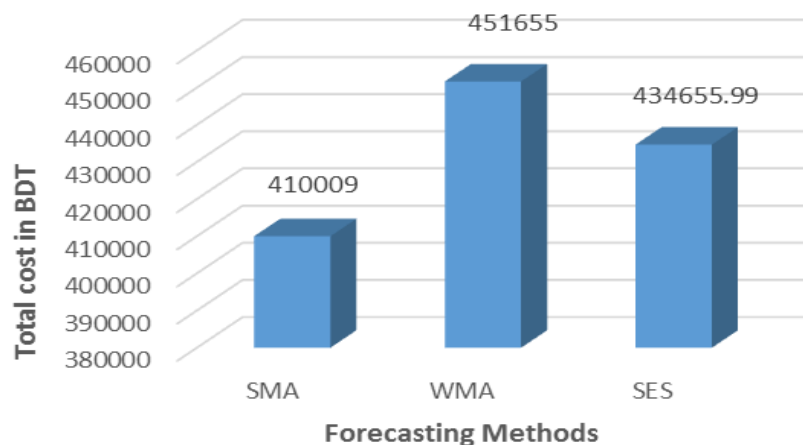

Figure-1. Comparison between SMA, WMA, SES on the basis of total cost.

Table-6. Value of decision variables.

Variable	Value (Units) (MA)	Value (Units) (WMA)	Value (Units) (SES)
Ix_{11}	300	400	360
Bx_{11}	87	210	183
Rx_{11}	3000	2900	3020
Ix_{12}	450	450	450
Bx_{12}	304	465	335
Rx_{12}	3500	3500	3500
Ix_{13}	500	500	500
Bx_{13}	381	525	413
Rx_{13}	3600	3600	3600
Ix_{21}	480	480	480
Bx_{21}	278	465	414
Rx_{21}	2000	2000	2000
Ix_{22}	580	580	582
Bx_{22}	55	241	175
Rx_{22}	2250	2250	2248
Ix_{23}	470	470	493
Bx_{23}	37	204	142
Rx_{23}	2125	2132	2138

Source: The table is prepared by authors using MS-Excel 2013.

Table-7. Final result of service level and inventory percentage.

Demand	APP Scenario					
	SMA		WMA		SES	
	SL	IL	SL	IL	SL	IL
Few	91.57%	15.60%	91.51%	17.96%	86.88%	16.73%
High	94.36%	17.89%	81.17%	17.91%	90.73%	17.96%
Peak	93.95%	16.94%	80.92%	16.92%	91.73%	17.31%

Source: The table is prepared by authors using MS-Excel 2013.

5. DISCUSSIONS

It can be seen from the computational result that in the case of well-utilized capacities (demand scenario High) the service level is maximum among all other demand scenarios. This is because in the case of High demand scenarios when the capacity is scarce APP is the best choice for satisfying customer demand. Here SMA setting gives the best result for both High and Few demand scenario in terms of SL and IL.

In demand scenario *Few* the maximum SL found here is 91.57% and minimum IL is 15.60% which is the best result in the entire APP scenario for SMA. WMA also shows a near result in terms of SL but IL is high compared to SMA and IL. The best SL in the entire APP scenario is obtained applying SMA for *High*. Here also SMA shows the best result in terms of SL and IL than the other two techniques. The *Peak* is the most challenging one in terms of planning. The best result is found for SMA with a 93.95% service level. WMA shows the worst SL for *Peak* but minimized IL. The worst result of applying WMA is because it is very important to estimate accordingly as an overestimated *Peak* season will automatically lead to a huge allocation of resources. Therefore there will be no free capacity resources to react to unexpected customer orders.

So, we can state that aggregate planning with the simple moving average for every mentioned demand scenario seems to be a very good choice here. It dominates the other two forecasting methods. As the trade-off between service and inventory level will depend on direct and indirect costs for holding products on stock and delivery time according to the privilege of a company, so the best decision will be chosen accordingly.

6. CONCLUSIONS AND FUTURE WORK

For forecasting demand, Linear Algorithm is used to solve Aggregate production planning problem to optimize the production cost for multiple products in multiple time horizon. This study is based on situation, where a manufacturer produces multiple products in multiple time period. This complex situation needs to decide which

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forecasting method is best for being flexible with market demand variations. For that reason, we chose three forecasting techniques (simple moving average, weighted moving average, simple exponential smoothing) for three assumed market demand scenarios (*High, Few and Peak*) which were used as an input to obtain the optimum decision on the reformulated LP model. By analyzing all the factors above, inventory level and service level is calculated in this study. Comparing among the cost calculated using simple moving average, weighted moving average, and simple exponential smoothing, we found the values of the simple moving average is better than others. The major limitation of our study is that all parameters related to APP were not included in the proposed model and the results are influenced by typical situations faced and provided data by the silk industry, so we are not able to derive general statements about the best forecasting technique suitable for different demand scenarios. But the model and the procedures for finding the best forecasting techniques in maximizing service level and minimizing inventory level can be applied to any other type of environments for decision making.

We used Excel solver to solve the APP model and find the inventory, backorder and regular time production amount minimizing cost. The APP problem can be solved by other optimization techniques. Different APP strategies can be applied and standard optimization methods such as Genetic Algorithm (GA), fuzzy method, mixed integer optimization programming (MIOP), Big M method may also be applied in the model.

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APPENDIX

Objective function,

$$\text{Min, Z} = \sum_{p\in P}\sum_{t\in T}(Ix_{pt}*Ic_{pt}) + \sum_{p\in P}\sum_{t\in T}(Bx_{pt}*Bc_{pt}) + \sum_{p\in P}\sum_{t\in T}(Rx_{pt}*Rc_{pt})$$

For P=2 Products and T=3 periods

Minimize   Z=

$$(Ix_{11}*Ic_{11}) + (Bx_{11}*Bc_{11}) + (Rx_{11}*Rc_{11}) + (Ix_{12}*Ic_{12})+$$

$$(Bx_{12}*Bc_{12}) + (Rx_{12}*Rc_{12}) + (Ix_{13}*Ic_{13}) + (Bx_{13}*Bc_{13}) + (Rx_{13}*Rc_{13}) + (Ix_{21}*Ic_{21}) + (Bx_{21}*Bc_{21}) +$$

$$(Rx_{21}*Rc_{21}) + (Ix_{22}*Ic_{22}) + (Bx_{22}*Bc_{22}) + (Rx_{22}*Rc_{22}) + (Ix_{23}*Ic_{23}) + (Bx_{23}*Bc_{23}) + (Rx_{23}*Rc_{23})$$

CONSTRAINTS

1. Inventory and Backorder Balancing Constraint

$$Ix_{pt} - Bx_{pt} = Ix_{p(t-1)} + Rx_{pt} - D_{pt} - Bx_{p(t-1)}$$

For P=2 Products and T=3 periods

$$Ix_{11} - Bx_{11} = Ix_{10} + Rx_{11} - D_{11} - Bx_{10} \quad Ix_{12} - Bx_{12} = Ix_{11} + Rx_{12} - D_{12} - Bx_{11}$$

$$Ix_{13} - Bx_{13} = Ix_{12} + Rx_{13} - D_{13} - Bx_{12}$$

$$Ix_{21} - Bx_{21} = Ix_{20} + Rx_{21} - D_{21} - Bx_{20}$$

$$Ix_{22} - Bx_{22} = Ix_{21} + Rx_{22} - D_{22} - Bx_{21}$$

$$Ix_{23} - Bx_{23} = Ix_{22} + Rx_{23} - D_{22} - Bx_{22}$$

2. Constraint on Machine Capacity:

$$Rx_{pt}M_{pt} \leq M_{tmax}$$

For P=2 Products and T=3 periods

$$Ix_{11} \geq I_{11min}$$

$$Ix_{11} \geq I_{11min}$$

$$Ix_{12} \geq I_{12min}$$

$$Ix_{13} \geq I_{13min}$$

$$Ix_{21} \geq I_{21min}$$

$$Ix_{22} \geq I_{22min}$$

$$Ix_{23} \geq I_{23min}$$

3. Constraint on Carrying Inventory:

$Ix_{pt} \geq I_{ptmin}$

For P=2 Products and T=3 periods

$$Rx_{11} * M_{11} \leq M_{1max}$$

$$Rx_{12} * M_{12} \leq M_{2max}$$

$$Rx_{13} * M_{13} \leq M_{3max}$$

$$Rx_{21} * M_{21} \leq M_{1max}$$

$$Rx_{22} * M_{22} \leq M_{2max}$$

$$Rx_{23} * M_{23} \leq M_{3max}$$