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DYNAMIC MODELLING OF 3-RUS SPATIAL PARALLEL ROBOT MANIPULATOR

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ABSTRACT

Parallel manipulators are characterized as having closed-loop kinematic chains. Parallel robots have received increasing attention due to their inherent advantages over conventional serial mechanism, such as high rigidity, high load capacity, high velocity, and high precision. A definite advantage of parallel robots is the fact that, in most cases, actuators can be placed on the truss, thus achieving a limited weight for the moving parts, which makes it possible for parallel robots to move at a high speed. These advantages avoid the drawbacks on serial ones and make the mobile platforms of the parallel manipulators carry out higher performances. Therefore, parallel manipulators have been applied to the industrial manufacturing, flight simulation, medical resuscitation, and so on. The basis for model-based control of parallel manipulators is an efficient formulation of the motion equations. In this paper a formulation of the motion equations in redundant coordinates is presented for parallel manipulators. The fully coupled non-linear equations of motion of 3-RUS spatial parallel manipulator having 3 DOF with Revolute-Universal-Spherical joints are obtained by using the Lagrange equations with multipliers for constrained multibody systems.

Contribution/Originality: This paper presented the modelling of motion for 3-RUS spatial parallel manipulator by using the redundant generalized coordinates and the Lagrangian multipliers. A novel formulation of Coriolis/centrifugal matrix is constructed directly in matrix based manner by using Kronecker product. The skew symmetry property of dynamic model of robot manipulators is guaranteed by the proposed Coriolis/centrifugal matrix. Then, the equations of motion in DAEs form were transformed to ordinary differential equations.

1. INTRODUCTION

Parallel manipulators generally comprise two platforms which are connected by joints or legs acting in parallel. Over the past decades, parallel manipulators have received more and more attention from researches and industries [1, 2]. A spatial 3-DOF parallel manipulator, which can be used in several applications, including machine tools is proposed in this paper Figure 1.

In most applications, the robot must move rapidly from one position to another position or follow a desired trajectory in three dimensional spaces with high precision. In order to perform this task, recently, several control methods have been investigated such as computed torque with PD, PID controller [3-9]. The computed torque controller is easy to implement, but it cannot meet the control quality due to uncertainties in the system model and disturbances.

In this paper, mechanical models of 3-RUS spatial parallel robot are presented. The equations of motion for these cases are established by applying the Lagrangian equation with multipliers. The reaction force elimination method is used to determine the driving moments depending the given motion of the mobile platform.

The numerical simulations in Matlab are implemented to show the response of the robot.

2. KINEMATIC AND DYNAMIC MODELS OF A 3-RUS ROBOT

2.1. Kinematics and Dynamics of a 3-RUS Robot with Model of Rigid Links

The best model for 3-RUS robot is a system of rigid bodies connected by joints. The parallelogram mechanisms that connect the driving links to the mobile platform are modeled as homogeneous rods with universal and spherical joints at two ends. By using this model, the robot is seen as a multibody system with seven bodies: three legs each leg having two links and the mobile platform. The calculating models are shown in Figure 2.

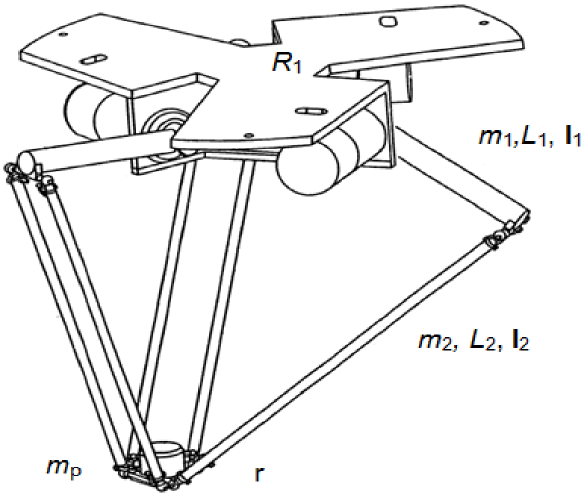

Figure-1. 3-RUS robot with three parallelogram mechanisms.

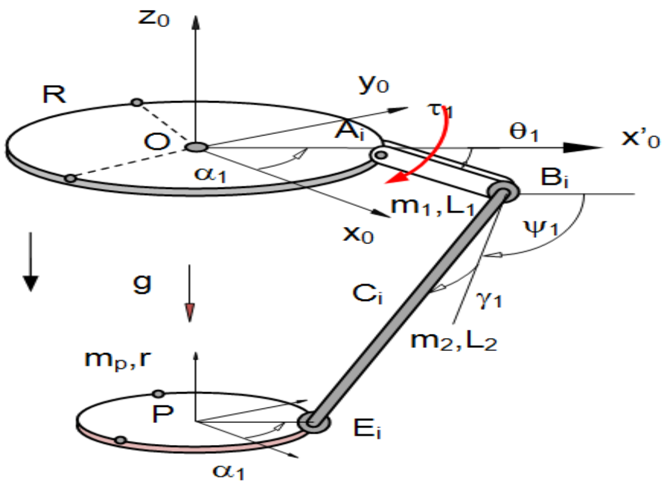

Figure-2. Calculating models of 3-RUS robot.

Putting θ_i, i = 1,2,3 be the driving angles of the actuated links; ψ_i, γ_i , i = 1,2,3 be the passive angles that determine the position of the connecting rods; and $\mathbf{x} = [x_p, y_p, z_p]^T$ be the position of the center of mass of the mobile platform. Hence, the position of robot is determined by the generalized coordinates:

$$\mathbf{q} = [\theta_1, \psi_1, \gamma_1, \theta_2, \psi_2, \gamma_2, \theta_3, \psi_3, \gamma_3, x_p, y_p, z_p]^T.$$

The position vectors of the points A_i, B_i, C_i, and E_i in the fixed frame, rotation matrices of driving and connecting links, and the mobile platform are given as follows:

$$\mathbf{r}_{Ai} = \mathbf{A}_z(\alpha_i)\mathbf{u}_A, \quad \mathbf{u}_A = [R, 0, 0]^T, \quad (i = 1,2,3) \tag{1}$$

$$\mathbf{r}_{C1,i} = \mathbf{r}_{A,i} + \mathbf{A}_z(\alpha_i)\mathbf{A}_y(\theta_i)\mathbf{u}_{C,1}, \qquad \mathbf{u}_{C1} = [l_1/2, 0, 0]^T, \qquad (i = 1,2,3) \tag{2}$$

$$\mathbf{r}_{Bi} = \mathbf{r}_{Ai} + \mathbf{A}_z(\alpha_i)\mathbf{A}_y(\theta_i)\mathbf{u}_B, \qquad \mathbf{u}_B = [l_1, 0, 0]^T, \quad (i = 1,2,3) \tag{3}$$

$$\mathbf{r}_{C2,i} = \mathbf{r}_{Bi} + \mathbf{A}_z(\alpha_i)\mathbf{A}_y(\psi_i)\mathbf{A}_z(\gamma_i)\mathbf{u}_{C2}, \qquad \mathbf{u}_{C2} = [l_2/2, 0, 0]^T, \qquad (i = 1,2,3) \tag{4}$$

$$\mathbf{r}_{Ei} = \mathbf{r}_{Bi} + \mathbf{A}_z(\alpha_i)\mathbf{A}_y(\psi_i)\mathbf{A}_z(\gamma_i)\mathbf{u}_E, \ \mathbf{u}_E = [l_2, 0, 0]^T, \quad (i = 1,2,3) \tag{5}$$

Where the rotation matrices are given as

$$\mathbf{A}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}, \ \mathbf{A}_y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}, \ \mathbf{A}_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{1,i} = \mathbf{A}_z(\alpha_i)\mathbf{A}_y(\theta_i), \ \mathbf{A}_{2,i} = \mathbf{A}_z(\alpha_i)\mathbf{A}_y(\psi_i)\mathbf{A}_z(\gamma_i).$$

On the other way, we can determine the position of the spherical joints E as

$$\mathbf{r}_{Ei}^p = \mathbf{r}_P + \mathbf{A}_z(\alpha_i)\mathbf{u}_E^p, \ \mathbf{u}_E^p = [r, 0, 0]^T, \ i = 1,2,3 \tag{6}$$

With $\mathbf{r}_P = \mathbf{x} = [x_p, y_p, z_p]^T$ being position of the mobile platform.

The constrained equations of the robot are obtained by setting (5) equal to (6), so we have

$$\mathbf{r}_{Ei} = \mathbf{r}_{Ei}^p,$$

or

$$\mathbf{A}_z(\alpha_i)\left(\mathbf{u}_A + \mathbf{A}_y(\theta_i)\mathbf{u}_B + \mathbf{A}_y(\psi_i)\mathbf{A}_z(\gamma_i)\mathbf{u}_E\right) = \mathbf{r}_P + \mathbf{A}_z(\alpha_i)\mathbf{u}_E^p, \qquad (i = 1,2,3) \tag{7}$$

The kinetic and potential energy of the robot are given as

$$T = \frac{1}{2}\sum_{i=1}^{3}\left(J_1\dot{\theta}_i^2 + m_2\mathbf{v}_{C2,i}^T\mathbf{v}_{C2,i} + \mathbf{w}_{2,i}^{(i)T}\mathbf{I}_{C2}^{(i)}\mathbf{w}_{2,i}^{(i)}\right) + \frac{1}{2}m_p(\dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2) = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \tag{8}$$

$$\Pi = -\sum_{i=1}^{3}\left(m_1 g l_{C,1}\sin\theta_i + m_2 g z_{C2,i}\right) + m_p g z_p \tag{9}$$

The generalized forces is

$$\mathbf{Q} = [\tau_1 - c\dot{\theta}_1, 0, 0, \ \tau_2 - c\dot{\theta}_2, 0, 0, \ \tau_3 - c\dot{\theta}_3, 0, 0, \ 0, 0, 0]^T = \mathbf{B}\boldsymbol{\tau} - \mathbf{D}\dot{\mathbf{q}}.$$

The motion equation is established by using the Lagrange formula [10-13]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right)^T - \left(\frac{\partial T}{\partial \mathbf{q}}\right)^T = -\left(\frac{\partial \Pi}{\partial \mathbf{q}}\right)^T + \mathbf{Q} - (\mathbf{\Phi}_q(\mathbf{q}))^T\boldsymbol{\lambda} \tag{10}$$

The Substituting (8) and (9) into Equation 10 and under consideration of constraint Equation 6, the equations of motion are obtained. These consist of twelve differential equations and nine algebraic equations (DAEs). These equations are written in details as follows

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{B}\boldsymbol{\tau} + (\boldsymbol{\Phi}_{q}(\mathbf{q}))^{T}\boldsymbol{\lambda} \\ \boldsymbol{\phi}(\mathbf{q}) = \mathbf{0} \end{cases} \tag{11}$$

Where

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} \mathbf{M}_1(\mathbf{q}_1) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2(\mathbf{q}_2) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_3(\mathbf{q}_3) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}_4(\mathbf{q}_4) \end{bmatrix}, \quad \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} \mathbf{C}_1(\mathbf{q}_1,\dot{\mathbf{q}}_1) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2(\mathbf{q}_2,\dot{\mathbf{q}}_2) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_3(\mathbf{q}_3,\dot{\mathbf{q}}_3) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{D} = \mathrm{diag}([\mathbf{D}_1,\mathbf{D}_2,\mathbf{D}_3,\mathbf{0}]), \quad \mathbf{D}_1 = \mathbf{D}_2 = \mathbf{D}_3 = \mathrm{diag}([c, 0, 0])$$

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} \mathbf{g}_1(\mathbf{q}) \\ \mathbf{g}_2(\mathbf{q}) \\ \mathbf{g}_3(\mathbf{q}) \\ \mathbf{g}_4(\mathbf{q}) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \mathbf{B}_3 \\ \mathbf{0}_{3\times 3} \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{B}_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix},$$

$$\mathbf{M}_1(\mathbf{q}) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ & m_{22} & m_{23} \\ \mathrm{sym.} & & m_{33} \end{bmatrix}, \text{with} \begin{array}{l} m_{11} = \frac{1}{3}m_1 l_1^2 + m_2 l_1^2, \qquad m_{23} = 0 \\ m_{12} = \frac{1}{2}m_2 l_1 l_2 \cos(q_1 - q_2)\cos q_3 \\ m_{13} = \frac{1}{2}m_2 l_1 l_2 \sin(q_1 - q_2)\sin q_3 \\ m_{22} = \frac{1}{3}m_2 l_2^2 \cos^2 q_3, m_{33} = \frac{1}{3}m_2 l_2^2, \end{array}$$

And

$$\mathbf{C}_1(\mathbf{q}_1,\dot{\mathbf{q}}_1) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

with $q_{1-2} = q_1 - q_2$, $c = \cos(\times)$, $s = \sin(\times)$,

$$\begin{aligned} C_{11} &= 0 \\ C_{12} &= -\tfrac{1}{2}m_2 l_1 l_2 \left[\dot{q}_2 s q_{1-2} c q_3 + \dot{q}_3 c q_{1-2} s q_3\right] \\ C_{13} &= -\tfrac{1}{2}m_2 l_1 l_2 \left[\dot{q}_2 c q_{1-2} s q_3 - \dot{q}_3 s q_{1-2} c q_3\right], \end{aligned}$$

$$\begin{aligned} C_{21} &= -\tfrac{1}{2}m_2 l_1 l_2 \dot{q}_1 \sin(q_1 - q_2)\cos q_3 \\ C_{22} &= -\tfrac{1}{3}m_2 l_2^2 \dot{q}_3 \cos q_3 \sin q_3 \end{aligned},$$

$$\begin{aligned} C_{23} &= -\tfrac{1}{3}m_2 l_2^2 \dot{q}_2 \cos q_3 \sin q_3 \\ C_{31} &= \tfrac{1}{2}m_2 l_1 l_2 \dot{q}_1 \cos(q_1 - q_2)\sin q_3 \\ C_{32} &= \tfrac{1}{3}m_2 l_2^2 \dot{q}_2 \cos q_3 \sin q_3; \\ C_{33} &= 0 \end{aligned}$$

$$\mathbf{g}_1(\mathbf{q}) = \begin{bmatrix} -(\frac{1}{2}m_1 + m_2)l_1 g \cos q_1 \\ -\frac{1}{2}m_2 l_2 g \cos q_2 \cos q_3 \\ \frac{1}{2}m_2 l_2 g \sin q_2 \sin q_3 \end{bmatrix}, \mathbf{M}_4 = \begin{bmatrix} m_p & 0 & 0 \\ 0 & m_p & 0 \\ 0 & 0 & m_p \end{bmatrix}, \mathbf{g}_4(\mathbf{q}) = \begin{bmatrix} 0 & 0 & m_p g \end{bmatrix}^T.$$

The matrices $\mathbf{M}_2(\mathbf{q}_2)$, $\mathbf{M}_3(\mathbf{q}_3)$, $\mathbf{C}_2(\mathbf{q}_2, \dot{\mathbf{q}}_2)$, $\mathbf{C}_3(\mathbf{q}_3, \dot{\mathbf{q}}_3)$, $\mathbf{g}_2(\mathbf{q}_2)$, $\mathbf{g}_3(\mathbf{q}_3)$ have the same structures as matrices $\mathbf{M}_1(\mathbf{q}_1)$, $\mathbf{C}_1(\mathbf{q}_1, \dot{\mathbf{q}}_1)$ and $\mathbf{g}_1(\mathbf{q}_1)$, where the generalized vector $\mathbf{q}_1 = [q_1, q_2, q_3]^T$ is replaced by $\mathbf{q}_2 = [q_4, q_5, q_6]^T$ and $\mathbf{q}_3 = [q_7, q_8, q_9]^T$, respectively.

The constrained equations are given details as follows

$$\mathbf{\Phi}_1(\mathbf{q}_1, \mathbf{q}_4) = [\phi_x \quad \phi_y \quad \phi_z]^T = 0 \tag{12}$$

with

$$f_x = \cos a_1 \left(R + l_1 \cos q_1 + l_2 \cos q_2 \cos q_3\right) - l_2 \sin a_1 \sin q_3 - x_p - r\cos a_1 = 0, \quad f_y = \sin a_1 \left(R + l_1 \cos q_1 + l_2 \cos q_2 \cos q_3\right) + l_2 \cos a_1 \sin q_3 - y_p - r \sin a_1 = 0$$

$$f_z = -l_1 \sin q_1 - l_2 \sin q_2 \cos q_3 - z_p = 0.$$

And the other constrained equations $\mathbf{\Phi}_2(\mathbf{q}_2, \mathbf{q}_4) = 0$, $\mathbf{\Phi}_3(\mathbf{q}_3, \mathbf{q}_4) = 0$ are obtained by replaced $a_1, \mathbf{q}_1$ in Equation 12 by $a_2, \mathbf{q}_2$ and $a_3, \mathbf{q}_3$.

2.2. Equations of Motion in Minimal Coordinates

In order to determine the driving torques and to design a controller for the robot, the Equation of motion in DAEs form will be transformed to ODEs form by using the jacobian matrix. Assuming that the jacobian matrix $\boldsymbol{F}_q(\mathbf{q}) = \partial \boldsymbol{f} / \partial \mathbf{q}$ is non-singular. Putting a matrix $\mathbf{R}(\mathbf{q}) \in \mathrm{R}^{m \times f}$ defined by Nguyen, et al. [7]:

$$\mathbf{R}(\mathbf{q}) = \begin{bmatrix} \mathbf{E} \\ -\mathbf{J}_p^{-1}\mathbf{J}_\theta \end{bmatrix} \tag{13}$$

with a unit matrix $\mathbf{E}$ of size $f \times f$ and $\mathbf{J}_\theta = \partial \boldsymbol{f} / \partial \boldsymbol{q}$, $\mathbf{J}_p = \partial \boldsymbol{f} / \partial \mathbf{p} = [\mathbf{J}_\psi, \mathbf{J}_x]$. The generalized velocity $\dot{\mathbf{q}}$ will be driven from velocity of the actuated coordinates as Blajer, et al. [14]

$$\dot{\mathbf{q}} = \mathbf{R}(\mathbf{q})\dot{\boldsymbol{q}} \tag{14}$$

Differentiating (14) with respect to time ones gets

$$\ddot{\mathbf{q}} = \mathbf{R}(\mathbf{q})\ddot{\boldsymbol{q}} + \dot{\mathbf{R}}(\mathbf{q})\dot{\boldsymbol{q}} \tag{15}$$

In order to eliminate Lagrangian multipliers $\boldsymbol{l}$ in Equation 11, the matrix $\mathbf{R}^T$ will be multiplied from link one gets

$$\mathbf{R}^T \left[\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q})\right] + \mathbf{R}^T \boldsymbol{F}_q^T(\mathbf{q})\boldsymbol{\lambda} = \mathbf{R}^T \mathbf{B}\boldsymbol{\tau} \tag{16}$$

By substituting (14) and (15) into(16), ones obtains

$$\mathbf{R}^T\left[\mathbf{M}(\mathbf{q})\mathbf{R}\ddot{q}+\mathbf{M}(\mathbf{q})\dot{\mathbf{R}}\dot{q}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{R}\dot{q}+\mathbf{D}\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})\right]=\mathbf{R}^T\mathbf{B}\boldsymbol{\tau} \tag{17}$$

By defining the following matrices

$$\mathbf{M}_\theta(q)=\mathbf{R}^T\mathbf{M}(\mathbf{q})\mathbf{R},\ \ \mathbf{M}_\theta\in\mathrm{R}^{f\times f},\ \mathbf{C}_\theta(q,\dot{q})=\mathbf{R}^T\left[\mathbf{M}(\mathbf{q})\dot{\mathbf{R}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{R}\right],\mathbf{D}_\theta=\mathbf{R}^T\mathbf{D}\mathbf{R},\mathbf{g}_\theta(q)=\mathbf{R}^T\mathbf{g}(\mathbf{q}),\ \ \mathbf{g}_\theta\in\mathrm{R}^f$$

The equation of motion in actuated coordinates is written as

$$\mathbf{M}_\theta(q)\ddot{q}+\mathbf{C}_\theta(q,\dot{q})\dot{q}+\mathbf{D}_\theta\dot{q}+\mathbf{g}_\theta(q)=\mathbf{R}^T\mathbf{B}_s\mathbf{u}=:\tau_\theta \tag{18}$$

Noting that, in Equation 18 the following properties are hold: $\mathbf{M}_\theta(q)$ is positive definite symmetric matrix, and

$\mathbf{N}_\theta=[\dot{\mathbf{M}}_\theta(q)-2\mathbf{C}_\theta(q,\dot{q})]$ is a skew-symmetric ones.

3. DRIVEN TORQUES OF 3-RUS PARALLEL ROBOT MANIPULATOR

In this section, some simulations in universal software Matlab are implemented. For simulation, the numerical methods are applied to obtain the actuated coordinates. The model of the 3-RUS robot is given in Figure 2 and the robot parameters as following.

$l_1=0.3; l_2=0.8; R=0.266; r=0.04;$ [m], $\alpha_A=0; \alpha_B=2\pi/3; \alpha_C=4\pi/3$, [rad]

$m_1=0.42; m_2=2\times0.20; m_P=0.75$, [kg], $\mathbf{I}_1=\frac{1}{12}m_1l_1^2\,\mathrm{diag}([0,1,1])$ [kg m²], $\mathbf{I}_2=\frac{1}{12}m_2l_2^2\,\mathrm{diag}([0,1,1])$ [kg m²].

In simulations, the motion of the mobile platform is defined by

$$x_p=0.3\cos(2\pi t_f^{-1}t), y_p=0.3\sin(2\pi t_f^{-1}t), z_p=-0.7\ \text{[m]}$$

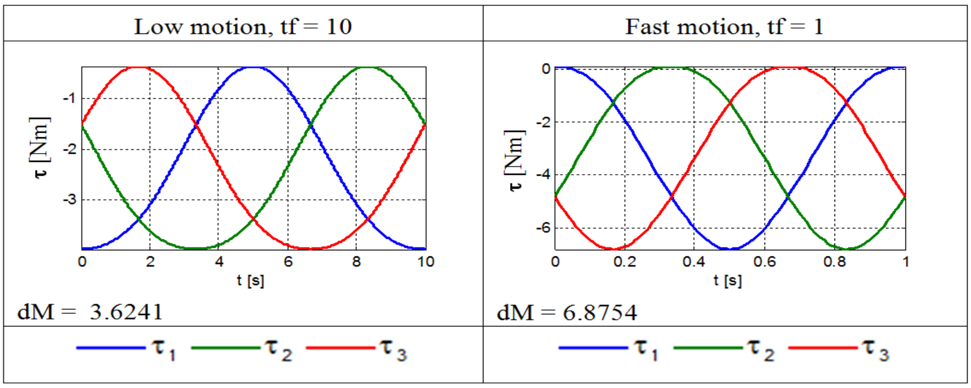

Figure-3. Computed torques.

The driving torques for low and fast motion corresponding is shown in Figure 3. The differences between max and min values of torques, dM = max(τ) – min(τ).

4. CONCLUSION

This paper presented the modelling of motion for 3-RUS spatial parallel manipulator by using the redundant generalized coordinates and the Lagrangian multipliers. Then, the equations of motion in DAEs form were transformed to ordinary differential form for the controller design.

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