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CHAOTIC PARTICLE SWARM OPTIMIZATION FOR IMPRECISE COMBINED ECONOMIC AND EMISSION DISPATCH PROBLEM

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ABSTRACT

Most of researchers presented a model to solve combined economic emission dispatch (CEED) problem in a precise formulation, but in reality data cannot be reported or collected preciously due to several reasons. The impreciseness of the mathematical model is occurring due to environmental fluctuations or instabilities in the global market which leads to the rapid fluctuations of prices. Therefore, in many cases, the various parameters of CEED model cannot be considered in a precise manner. So, in this paper, a new methodology is presented to solve imprecise CEED problem. In this methodology, we propose a chaos based enriched swarm optimization algorithm that relies on chaos in order to enhance its global search ability. The enriched swarm optimization algorithm combining two heuristic optimization techniques, particle swarm optimization (PSO) and genetic algorithm (GA) to integrate the merits of both them. Also, to improve the search engine visibility of the proposed approach, PSO has been enriched with a new evolution scheme; where a chaotic constriction factor is used to control the velocity of each particle in the swarm. Furthermore, local search (LS) technique is applied to improve the results quality; where it intends to scan the less-crowded region and obtain more nondominated solutions. Finally, the new methodology is carried out on the standard IEEE 30-bus 6-generator test system. From the results it is quite evident that our approach gives comparable minimum fuel cost and comparable minimum emission or better than those generated by other evolutionary algorithms (EAs). Also using the imprecise model enables us to predict the best cost and emission for any price fluctuation without solving the problem again.

Contribution/ Originality: This study presents a new methodology for solving imprecise combined economic emission dispatch using a chaos based enriched swarm optimization algorithm; where it integrates the main features of particle swarm optimization and genetic algorithm. The tests demonstrated that the proposed approach has a satisfactory performance compared to previous studies.

1. INTRODUCTION

In the past two decades optimal power flow (OPF) problem has received much attention, because of its capability to determine the dispatch of generators so as to meet the load demand while minimizing the total fuel cost, subject to the satisfaction of all constraints on the system. OPF [1] is considered highly nonlinear, large-scale, non-convex, static optimization problem restricted by inequality and equality constraints [2]. Recently more objective functions have been embedded into the formulation of OPF. These include optimization of reactive/active losses, power system stability, voltage profile, and plants emission. So, OPF definition has been extended from a single objective problem to a multiobjective one [3, 4] such as CEED multiobjective problem. The CEED problem seeks to minimize the fuel cost with the emissions produced by power system plants.

Various mathematical and optimization techniques, in the previous literatures, have been used to solve OPF. On the other hand, Traditional approaches such as linear programming method, gradient method, quadratic programming, nonlinear programming method, Lagrange relaxation method [5-10] etc. have been applied for solving the CEED problems. Also, in Liang and Glover [10] the authors proposed the dynamic programming as a new algorithm; where there are no restrictions on the nature of cost curves and hence it can solve CEED problems if it is convex or non-convex. But, in the solution procedure, dynamic programming has many limitations in the problems that have high dimensionality. Generators nonlinear features is the main reason that the classical optimization technique may not be able to find a solution in a suitable computational time and also, the restriction on these approaches leads to lack in their robustness and efficiency in a number of practical limitations.

According to these limitations EAs methods are proposed [11]. EAs are stochastic search algorithm that simulates the metaphor of natural biological evolution. Because of their universality, validity for parallel computing, and ease of implementation, EAs often take less computational time than the traditional methods to reach the optimal solution [12, 13]. In addition, due to the availability of high-speed technology, more interests have been focused on the application of EAs techniques for the solution of CEED problem.

Recently, there has been a boom in applying EAs to solve CEED problems. Several EAs methods, such as GA [12-15] artificial neural networks [16] Tabu search [17] evolutionary programming [18] PSO [19-21] ant colony optimization [22] differential evolution (DE) [23] and Hopfield neural networks [24] have been developed and applied successfully to CEED problems. Also, other powerful techniques called hybridization algorithms have been suggested. The hybrid approaches are using to deal with complicated problems such as: fuzzy adaptive hybrid PSO algorithm [25] hybrid PSO and sequential quadratic programming (PSO–SQP) [26] hybrid PSO and LS scheme (PSO–LS) [27] self-adaptive real-coded GA [28] hybrid chaotic DE and sequential quadratic programming (DE–SQP) [29] multiobjective EA based on decomposition (MOEA/D) [30] and combination between ACO and EA based on decomposition [31].

This paper intends to present a new optimization approach to solve imprecise CEED. The impreciseness of the mathematical model in CEED problem is occurring due to environmental fluctuations or instabilities in the global market which leads to the rapid fluctuations of prices. Therefore, the various parameters of CEED model cannot be considered in a precise manner. The new approach integrates the advantages of both PSO and GA. Also, to improve the search engine visibility of the proposed approach and control the velocity of each particle in the swarm; it has been enriched with a new evolution scheme (chaotic constriction factor). In addition, to control the velocity of each particle in the swarm, PSO has been enriched with a new evolution scheme (chaotic constriction factor). Furthermore, LS technique is applied to enhance the quality of the obtained solutions. The results demonstrate the abilities of our approach to generate well-distributed Pareto optimal front of the imprecise CEED problem and it can help us to predict what happens if there is a change in the system parameters.

The paper is structured as follows: Section 2 provides prerequisite mathematics on multiobjective optimization. Imprecise multiobjective optimization is presented in section 3. Multiobjective imprecise CEED problem is discussed in section 4. The proposed approach is described in section 5, while section 6 is introduced the implementation of the proposed approach. Results and discussion are given in section 7. Finally, the conclusions are drawn in Section 8.

2. PREREQUISITE MATHEMATICS

In this section we discuss some preliminary mathematics which we have used to study the imprecise CEED model. **Definition 1. (Interval number):** An interval number B is represented by a closed interval $[b_l, b_u]$ and defined by $B = [b_l, b_u] = \{y : b_l \le y \le b_u, y \in R\}$, where R is the set of real numbers and b_l, b_u are the lower and upper bounds of the interval number respectively.

Definition 2. (Interval-valued function): Let $c, d > 0$ and consider an interval of the form $[c, d]$, the interval-valued function of the interval $[c, d]$ is represented as $h(p) = c^{1-p} d^{p};\ p \in [0,1]$.

Now some arithmetic operations on interval valued functions is presented. Let $A = [a_l, a_u]$ and $B = [b_l, b_u]$ be two interval numbers so that $a_l, b_l > 0$.

Addition: $A + B = [a_l, a_u] + [b_l, b_u] = [a_l + b_l, a_u + b_u]$. The interval-valued function for the interval $A + B$ is given by $h(p) = a_L^{1-p} a_U^{p},\ p \in [0,1]$; where $a_L = a_l + b_l$ and $a_U = a_u + b_u$.

Subtraction: $A - B = [a_l, a_u] - [b_l, b_u] = [a_l - b_u, a_u - b_l]$. Provided $a_l - b_u > 0$. The interval-valued function for the interval $A - B$ is given by $h(p) = b_L^{1-p} b_U^{p},\ p \in [0,1]$ where $b_L = a_l - b_u$ and $b_U = a_u - b_l$.

Scalar multiplication: $\beta A = \beta[a_l, a_u] = \begin{cases} [\beta a_l, \beta a_u], & \text{if } \beta \ge 0 \\ [\beta a_u, \beta a_l], & \text{if } \beta < 0 \end{cases}$ provided $a_l > 0$. The interval-valued function for the interval βA is given by $h(p) = c_L^{1-p} c_U^{p},\ p \in [0,1]$ if $\beta \ge 0$ and $h(p) = -d_U^{1-p} d_L^{p},\ p \in [0,1]$ if $\beta < 0$, where $c_L = \beta a_l, c_U = \beta a_u, d_L = |\beta| a_l$ and $c_U = |\beta| a_u$.

3. IMPRECISE MULTIOBJECTIVE OPTIMIZATION

The following imprecise vector minimization problem (I-VMP) involving interval value parameters in the objective and constraints:

$$\left.\begin{array}{l} Min \ \ \{f_1(X, \tilde{a}), f_2(X, \tilde{a}), \ldots.., f_m(X, \tilde{a})\} \\ subject\ to\ g(X, \tilde{a}) \le 0 \end{array}\right\}; \tag{1}$$

where $f_i(X, \tilde{a})$ is the *ith* objective function; and $g(X, \tilde{a})$ is constraint vector, X is vector of decision variables; and $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \ldots \tilde{a}_n)$ represented a vector of interval numbers in the problem.

Definition 3. (*p*-level set): The *p*-level set or *p*-cut of the interval numbers $\tilde{a}$ is defined as $L_p(\tilde{a}) = \{a \mid a = a_l^{1-p} a_u^{p}\}$; where a_l, a_u gives the lower and upper bound for the parameter a.

For a certain degree *p*, the imprecise multiobjective problem can be represented as ordinary multiobjective problem as follows:

$$\left.\begin{array}{l} Min \ \ \{f_1(X, a), f_2(X, a), \ldots.., f_m(X, a)\} \\ subject\ to\ g(X, a) \le 0 \\ X = (x_1, x_2, \ldots x_n),\ a = (a_1, a_2, \ldots., a_n) \\ a_i = a_{il}^{1-p} a_{iu}^{p},\ p \in [0,1] \end{array}\right\}; \tag{2}$$

where constraint a_{il}, a_{iu} gives the lower and upper bound for the parameters a_i .

Definition 4.(*p*–Pareto optimal solution): $x^* \in X$ is said to be a *p*–Pareto optimal solution to the (I-VMP), if and only if there does not exist another $x \in X$, $a_i = a_{il}^{1-p} a_{iu}^{p}$, $p \in [0,1]$ such that $f_i(x,a) \le f_i(x^*, a^*)$ $\forall i = 1,2,...,m$,

with strictly inequality holding for at least one *i*, where the corresponding values of parameters a_i^* are called *p*-level optimal parameters.

In real life applications, input data or related parameters are frequently imprecise, so the concept of Pareto stability is introduced for the Pareto optimal solutions of a vector valued problem.

Definition 5. (Stable Pareto optimality): A Pareto optimal solution *x* of the I-VMP is said to be stable if and only if there exists a real number $p \in [0,1]$ such that *x* is still Pareto optimal if *a* is replaced by any a' satisfying the following requirement:

$$a' = \{a \mid a = a_l^{1-p} a_u^p\},\ p \in [0,1]. \tag{3}$$

Such a solution *x* is said to be a stable Pareto optimal solution.

4. MULTIOBJECTIVE IMPRECISE CEED PROBLEM

The imprecise CEED involves the simultaneous optimization of fuel cost and produced emission objectives, which is presented as follows.

4.1. Objectives

- **Fuel Cost Objective**.

The fuel cost objective can be described as follows:

$$f_1(\cdot) = C_t = \sum_{i=1}^{n} C_i(P_{Gi}) = \sum_{i=1}^{n} (\tilde{a}_i + \tilde{b}_i P_{Gi} + \tilde{c}_i P_{Gi}^2)\$/hr \tag{4}$$

where C_t : total fuel cost ($/hr), C_i : fuel cost of generator *i*, $\tilde{a}_i, \tilde{b}_i, \tilde{c}_i$: interval value fuel cost coefficients of generator *i*, P_{Gi} : power generated (p.u.) by generator *i* and *n*: number of generators.

- **Emission Objective**

The produced emission objective can be stated as the sum of produced emission, such as NO_x, SO_2, thermal emission, etc., with suitable pricing or weighting on each pollutant emitted. In this study, only emission NO_x is taken into account. The amount of NO_x emission is presented as a function of power generator output, that is, the sum of a quadratic and exponential function:

$$f_2(\cdot) = E_{NO_x} = \sum_{i=1}^{n} \left[10^{-2} \left(\tilde{\alpha}_i + \tilde{\beta}_i P_{Gi} + \tilde{\gamma}_i P_{Gi}^2 \right) + \tilde{\xi}_i \exp\left(\tilde{\lambda}_i P_{Gi} \right) \right] ton/hr \tag{5}$$

where $\tilde{\alpha}_i$, $\tilde{\beta}_i$, γ_i, $\tilde{\xi}_i$, $\tilde{\lambda}_i$: interval value coefficients of the *ith* generator's emission characteristic.

4.2. Nonlinear Constraints

There are many restrictions of the CEED problem which are described in the following:

- **Power Generation Balance Constraint:**

The total power generated must supply the total load demand and the transmission losses [32].

$$\sum_{i=1}^{n} P_{Gi} - P_D - P_{Loss} = 0; \tag{6}$$

where P_D : total load demand (p.u.), and P_{Loss} : transmission losses (p.u.).

The transmission losses are given by $P_{Loss} = \sum_{i=1}^{n}\sum_{i=1}^{n}\left[A_{ij}\left(P_iP_j + Q_iQ_j\right) + B_{ij}\left(Q_iP_j - P_iQ_j\right)\right]$:

where $P_i = P_{Gi} - P_{Di}$, $Q_i = Q_{Gi} - Q_{Di}$, $A_{ij} = \frac{R_{ij}}{V_iV_j}\cos\left(\delta_i - \delta_j\right)$, $B_{ij} = \frac{R_{ij}}{V_iV_j}\sin\left(\delta_i - \delta_j\right)$ and

n: Number of buses,
R_{ij} : Series-resistance connecting buses *i* and *j*,
V_i : Voltage magnitude at bus *i*,
δ_i : Voltage-angle at bus *i*,
P_i : Real power injection at bus *i*, and
Q_i : Reactive power injection bus *i*.

- **Minimum and Maximum Limits of Power Generation:**

The power generated P_{Gi} by each generator is constrained between its minimum and maximum limits, i.e.,

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max}, \qquad Q_{Gi\min} \le Q_{Gi} \le Q_{Gi\max},$$
$$V_{i\min} \le V_i \le V_{i\max}, \qquad i = 1,\ldots\ldots,n$$

where $P_{Gi\min}$: minimum power generated, and $P_{Gi\max}$: maximum power generated.

- **Line Security Constraints:**

The CEED problem should consider only the small proportion of lines in violation, or near violation of their respective security, which are marked as the critical lines. We consider only the critical lines that are binding in the optimal solution. The detection of such critical lines is assumed done by the experiences of the decision maker. An improvement in the security can be obtained by minimizing the following function.

$$S = f\,(P_{Gi}) = \sum_{j=1}^{k}(|T_j\,(P_G)\,|\,/T_j^{\max}); \tag{7}$$

where $T_j\left(P_G\right)$ is the real power flow $T_j^{\max}$ is the maximum limit of the real power flow of the *j*th line and *k* is the number of monitored lines. The line flow of the *jth* line is expressed in terms of the control variables, by utilizing the generalized generation distribution factors (GGDF) [3] and is given below.

$$T_J\,(P_G) = \sum_{i=1}^{n}(D_{ji}\,P_{Gi}); \tag{8}$$

where, D_{ji} is the generalized GGDF for line *j*, due to generator *i*.

For secure operation, the transmission line loading S_l is restricted by $S_\ell \le S_{\ell\max}, \ell = 1,\ldots,n_\ell$; where n_ℓ is the number of transmission line.

5. THE PROPOSED APPROACH

In this section, we propose a new methodology to solve imprecise CEED problem, which combining PSO and GA to integrate the merits of both them. In addition, to control the velocity of each particle in the swarm, PSO has been enriched with a new evolution scheme (chaotic constriction factor). Furthermore, LS technique is applied to enhance the results quality; where it intends to scan the less-crowded region and obtain more solutions. In the proposed approach, three phases (PSO, GA and LS) are described as follows:

Phase I: PSO

Step 1: Initialization: A population of N particles with random positions $X_i^{t=0}$ and velocities $V_i^{t=0}$ on n-dimensions is initialized in the problem space; where t is the time counter and $i = 1,\ldots,N$.

1- For each particle i, identify the local set $L_i^{t=0} = \left\{ X_i^{t=0} \middle| \, i = 1,\ldots,N \right\}$ and the local preferred element $LP_i^{t=0} = \left\{ X_i^{t=0} \right\} \subset L_i^{t=0}$. All local sets $\left\{ L_i^{t=0} \right\}$ for each particle are collected in a pool C such that $C = \bigcup_{i=1}^{N} L_i^{t=0}$.

2- A global set $G^{t=0} = ND(C)$ is defined as the nondominated solutions in the pool C; where $ND(\cdot)$ refers to the function which has the ability to determine the nondominated solutions.

3- For each particle, the distances between $X_i^{t=0}$ and the members in $G^{t=0}$ are measured using the L_2 as follows:

$$\mathrm{d}\left(\vec{x}_i, \vec{x}_j\right) = \left\| \vec{x}_i - \vec{x}_j \right\|_2 = \sqrt{\sum_{d=1}^{D} \left(x_{i,d} - x_{j,d} \right)^2}. \tag{9}$$

The preferred global element GP_i^t is defined as the nearest member in $G^{t=0}$ to the *i-th* particle. Set the external set $E^{t=0} = G^{t=0}$.

Step 2: Update particles: Update the particle velocity and position as follows:

$$V_i^{t+1} = wV_i^t + c_1 r_1 \left(LP_i^t - x_i^t \right) + c_2 r_2 \left(GP_i^t - x_i^t \right) \tag{10}$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{11}$$

Step 3: Velocity restriction: To restrict the velocity and control it during evolution of particles and enhance the performance of PSO, some authors [33-35] use a constant/dynamic constriction factor χ . In our algorithm, chaotic constriction factor is merged into the PSO to enrich the searching behaviour and avoid being trapped into the infeasible region. A well-known logistic equation is employed, where it exhibits chaotic dynamics.

$$\chi_{n+1} = \mu \cdot \chi_n (1 - \chi_n), \ \chi_0 = 10^{-6}, \mu = 4, \ n = 0,1,2\ldots; \tag{12}$$

where, n is the age of the infeasible particle (How long it's still unfeasible?).

The new position X_i^{t+1} depends on the velocity V_i^{t+1} as in equation (11). Then, V_i^{t+1} makes the particle i to lose its feasibility, so we introduce a chaotic constriction factor χ such that new modified position of the particle is computed as:

$$X_i^{t+1} = X_i^t + \chi \, V_i^{t+1} \tag{13}$$

Interested readers could refer to Liu, et al. [35] for more details. The pseudo code of the chaotic constriction factor is shown in Fig 1.

Procedure make $\left(POPULATION = \{P_i = \{X_i, V_i\} : (i = 1, 2, ..., N)\}\right)$

While $(i < N)$ **Do**

$\chi_0 = 10^{-6}$

While $P_i = \{X_i, V_i\}$ unfeasible

$X_i^{t+1} = X_i^t + \chi V_i^{t+1}$

Check feasibility

$\chi_{n+1} = \mu \cdot \chi_n (1 - \chi_n),$

End

End

Fig-1. Chaotic constriction factor Pseudo code.

Step 4: Update L_i^t : The new position of each particle X_i^{t+1} is added to L_i^t , to form L_i^{t+1} which is updated according to algorithm 2 in Fig. 2.

Step 5: Update G **:** Update G as $G^{t+1} = ND\left(\bigcup_{i=1}^{N} L_i^{t+1}\right)$.

Step 6: Update E^t : Update E^t by copying the elements of G^{t+1} to E^t and dominance criteria is applied to delete all dominated elements from E^t (i.e., three probabilities of each element in G^{t+1} according to algorithm 3 in Fig. 3).

Algorithm 2: Update local set L_i^t

Input$\left(L_i^t, X_i^{t+1}\right)$

If $\exists\ X \in L_i^t \mid X \succ X_i^{t+1}$ (X dominate X_i^{t+1}) then

$L_i^{t+1} = L_i^t$

Else if $\exists\, X \in L_i^t\ \wedge X_i^{t+1} \succ X$ then

$L_i^{t+1} = L_i^t \bigcup \{X_i^{t+1}\} / \{X\}$

Else if $\nexists\ X \in L_i^t \mid X \succ X_i^{t+1}$ then

$L_i^{t+1} = L_i^t \bigcup \{X_i^{t+1}\}$

End

Output$\left(L_i^{t+1}\right)$

Fig-2. Algorithm 2: Evolution of particles

Algorithm 3: Update external set E^t

Input$\left(E^t, X \in G^{t+1}\right)$

If $\exists\ Y \in E^t \mid Y \succ X$ then

$E^t = E^t$

Else if $\exists\, Y \in E^t\ \wedge X \succ Y$ then

$E^t = E^t \bigcup \{X\} / \{Y\}$

Else if $\nexists\ Y \in E^t \mid Y \succ X$ then

$E^t = E^t \bigcup \{X\}$

End

Output$\left(E^t\right)$

Fig-3. Algorithm 3: Update external set

Step 7: Updating LP_i^{t+1} and GP_i^{t+1}: For each particle *i*, the distances between X_i^{t+1} and members in L_i^{t+1} and G^{t+1} are measured using equation (9). The nearest member in L_i^{t+1} and G^{t+1} to the *i-th* particle is defined as LP_i^{t+1} and GP_i^{t+1} respectively.

Phase 2: GA

Step 8: Ranking: Ranks individuals (particles) in E^t according to their objectives value, and returns a column vector containing the corresponding individual fitness value, in order to establish the probabilities of survival which are necessary for the selection process.

Step 9: Selection: Two parents are selected to generate new strings (i.e., offspring). Parents are selected from the population based on its rank. The selected parents generate new offspring using GA operator [36].

Step 10: Crossover: In GAs, crossover is used to vary chromosomes from one generation to the next; where it combines two chromosomes to yield new offspring. The new offspring may be better than both of the parents if it takes the best features from both parents [37].
Step 11: Mutation: By using mutation, the solution is changed entirely from the previous solution; hence GA can go to better solution [38].
Step 12: Repairing: The infeasible individual is repaired to be feasible. The repairing approach is applied to the set of infeasible individuals up to they become feasible [39].
Step 13: Elitist strategy (Replacing): Since evolution in GAs depends on stochastic operators, GAs does not guarantee a monotonic improvement in the solutions of the problem unless deterministic overlapping systems are used. So, elitist strategy is applied; where some of the best individuals are copied into the next population without applying any GA operators.

Phase 3: LS

To improve the solution quality a modified local search (MLS) is implemented, where it aims to reconnoiter the solution space near the best population (particles) and discover the less-crowded areas in the external set to possibly obtain more solutions. In this subsection, the MLS is presented, which is a modification of Hooke and Jeeves [40] to handle MOP and it is described by the following steps:

Step 14: Start with the point $\left(X_m \in R^n\right) \in E^t$; where m is the size of E^t. Set the prescribed step lengths Δx_i in each of the coordinate directions u_i, $i = 1, 2, ..., n$ and $k = 1$ where k is number of trial (s.t., $k = 1, ..., k_{\max}$) to obtain preferred solution than X_m.

Step 15: A perturbation about X_m is implemented to obtain the new base point $X_m^{'}$ as:

$$X_m^{'} = \begin{Bmatrix} X_m + \Delta x_i u_i & if \quad f^{+}(\cdot) \succ f \\ X_m - \Delta x_i u_i & if \quad f^{-}(\cdot) \succ \left(f(\cdot) \wedge f^{+}(\cdot)\right) \\ X_m & if \quad f(\cdot) \succ \left(f^{+}(\cdot) \wedge f^{-}(\cdot)\right) \end{Bmatrix} \forall i = 1, 2, ..., n; \tag{14}$$

where, $f(\cdot) = f(X_m)$, $f^{+}(\cdot) = f(X_m + \Delta x_i u_i)$ and $f^{-}(\cdot) = f(X_m - \Delta x_i u_i)$. Assume $f(\cdot)$ is the evaluation of the objective functions at the new point.

Step 16: If the point X_m unchanged and the number of trial k not satisfied, reduce Δx_i using the dynamic equation $\Delta x_i = \Delta x_i \left(1 - (r)^{k/k_{\max}}\right)$; where r is a random number $r \in [0,1]$, and go to step 15.

Step 17: Else, if $X_m^{'}$ is better than X_m i.e., $f\left(X_m^{'}\right) \succ f\left(X_m\right)$, then the new base point is $X_m^{'}$ and go to step 18.

Step 18: With the help of the base points X_m and $X_m^{'}$, establish a pattern search direction S as $S = X_m^{'} - X_m$. Find a new point $X_m^{''}$ by the equation $X_m^{''} = X_m^{'} + \lambda S$; where λ is the step length, which can be taken as $\lambda = 1$.

Step 19: If $f\left(X_m^{''}\right) \succ f\left(X_m^{'}\right)$ set $X_m = X_m^{'}$, $X_m^{'} = X_m^{''}$, and go to step 18.

Step 20: If $f\left(X_m^{''}\right) \nsucc f\left(X_m^{'}\right)$ set $X_m = X_m^{'}$, and go to step 16.

These steps are applied to all solutions in E^t. Fig. 4 shows the pseudo code of the proposed algorithm.

Initialization: $POPULATION = \{P_i = \{X_i, V_i, L_i, LP_i, G, GP_i, E\} : (i = 1, 2, ..., pop\_size\ (N))\}$

While (navigation not completed) do

Phase 1:

While (swarm navigation< N1←PSO iteration) Do

Update $V_i^{t+1} = \psi(X_i^t, V_t^t, LP_i^t, GP_i^t), X_i^{t+1} = \xi(X_i^t, V_t^{t+1})$, for all particles

Velocity restriction

Update $(L_i^t, G, E^t, LP_i^t, GP_i^t)$

End

Phase 2:

While (GA generation< N2←GA generation) Do

Ranking

Selection

Recombine (crossover-mutation)

Repair

Elitist strategy (Replacing)

End

End

Phase 3:

Start with $X_m \in E^t \rightarrow$ Generate $X_m^{'}$

While ($f(X_m^{'}) \not\succ f(X_m) \vert$ stopping criterion satisfied) DO

If $X_m^{'} = X_m$

Reduce $\Delta x_i \rightarrow$ Generate $X_m^{'}$

End

Set a pattern direction $S \rightarrow$ Generate $X_m^{"}$

If $f(X_m^{"}) \succ f(X_m^{'})$, set $X_m = X_m^{'}$, $X_m^{'} = X_m^{"}$

Set $S \rightarrow$ Generate $X_m^{"}$

Else if $f(X_m^{"}) \not\succ f(X_m^{'})$

$X_m = X_m^{'}$

Fig-4. The pseudo code of the proposed algorithm

6. NUMERICAL SIMULATION

The described algorithm has been applied to the standard IEEE-30-bus-6-generator test system. The single-line diagram of this system is shown in Fig. 5, while the detailed data are given in [13, 41]. The values of fuel cost (\$/h) and emission (ton/h) coefficients are given in Table 1. The proposed algorithm used in this study were developed and implemented on dual-core processor PC using MATLAB environment. We have kept the parameters of the proposed approach as is shown in Table 2.

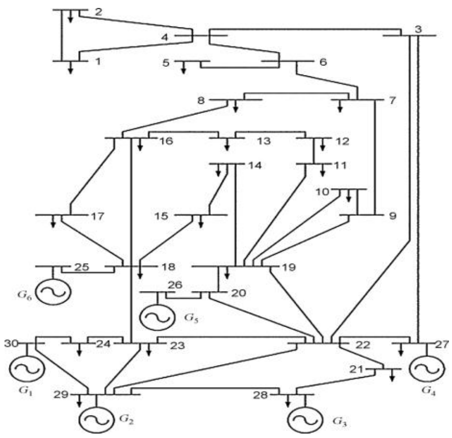

Fig-5. Single line diagram of IEEE 30 bus-6-generator test system

Naturally, the input data shown in Table 1 involve many controlled parameters whose values are vague and uncertain. Consequently each numerical value in the domain can be assigned by a specific "grade of membership" where 0 represents the smallest possible grade of membership, and 1 is the largest possible value. Thus every interval value parameter $\tilde{a}$ can be represented by *p*-grade membership function between 0 and 1 where $p \in [0,1]$ as follows:

$$(p-grade)_{\tilde{a}} = \begin{cases} 1, & \tilde{a} = a_u \\ (0,1), & \tilde{a} = a_l^{1-p} a_u^p \\ 0 & \tilde{a} = a_l \end{cases} \tag{15}$$

By using *p*-grade membership function, these interval parameters can be transformed to $\left(a_l^{1-p} a_u^p,\ p \in [0,1]\right)$ having lower bound a_l and upper bound a_u, which declared in equation (15). More clearly, if the parameter $a = 10$ and the variation of this parameter is about $\pm 10\%$ of its value, then its interval value is defined as $a_l = 0.9(10) = 9$, $a_u = 1.1(10) = 11$, by taking $p = 0$, its value equal to its lower bound a_l, $p = 1$, its value equal to its upper bound a_u and for $p \in (0,1)$, its value retain in $a_l^{1-p} a_u^p$, while by taking $p = (\ln a - \ln a_l)/(\ln a_u - \ln a_l)$ its value is changed to become *a* as in the following equation:

$$(p-grade)_{\tilde{a}} = \begin{cases} 1 & \tilde{a} = a_u = 1.1a \\ (0,1) & \tilde{a} = a_l^{1-p} a_u^p \\ (\ln a - \ln a_l)/(\ln a_u - \ln a_l) & \tilde{a} = (a_u + a_l)/2 \\ 0 & \tilde{a} = a_l = 0.9a \end{cases} \tag{16}$$

Table-1. Generator cost and emission coefficients

		G1	G2	G3	G4	G5	G6
Cost($/h)	a	10	10	20	10	20	10
	b	200	150	180	100	180	150
	c	100	120	40	60	40	100
Emission(ton/h)	α	4.091	2.543	4.258	5.426	4.258	6.131
	β	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
	γ	6.490	4.638	4.586	3.380	4.586	5.151
	ζ	2.0E-4	5.0E-4	1.0E-6	2.0E-3	1.0E-6	1.0E-5
	λ	2.857	3.333	8.000	2.000	8.000	6.667

Table-2. The parameter adopted in the implementation of the proposed algorithm

Parameters	Value
Cognitive parameter (c_1)	2.8
Social parameter (c_2)	1.3
Inertia weight (w)	0.6
Crossover probability (P_c)	0.9
Mutation probability (P_m)	0.02
Selection operator	roulette wheel selection
Crossover operator	Single point
Mutation operator	Real-value
Initial constriction factor χ_0 navigation	$\chi_0 = 10^{-3}$
PSO iteration (N1)	5
GA generation (N2)	5

7. RESULTS AND DISCUSSION

In order to efficiently obtain the results, the search process is carried out in two steps. Firstly, a set of eficient solutions (nondominated) is obtained at different values of the parameters by using different p cut level, all the range of the parameter fluctuation were scanned, two bounds of *p*-grade value have been considered $p = 0, p = 1$, and also we take some values between these bounds $p = 0.2, 0.4, 0.6, 0.8$. Secondly, the problem is solved at the standard value of the parameters at $p - \text{grade} = (\ln a - \ln a_l)/(\ln a_u - \ln a_l)$. Graphical presentations of the experimental results for seven instances problems are presented in Figs. 6-12.

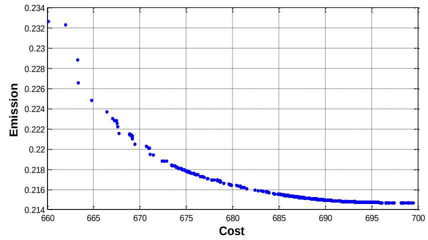

Fig-6-a. Pareto optimal set for *p*-grade = 1

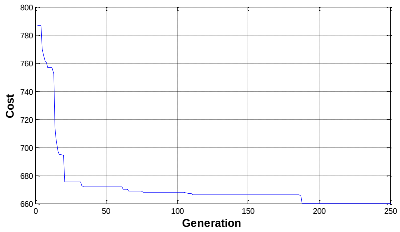

Fig-6-b. Best Cost versus generation for *p*-grade = 1

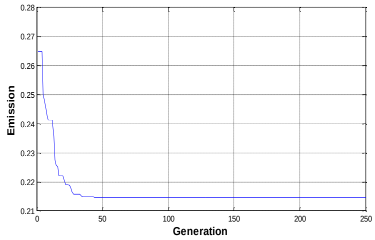

Fig. 6-c. Best Emission versus generation for *p*-grade = 1

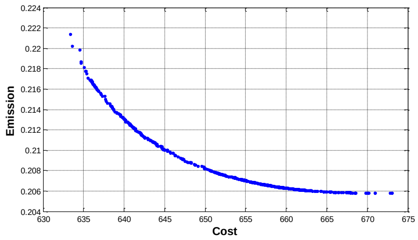

Fig-7-a. Pareto optimal set for *p*-grade = 0.8

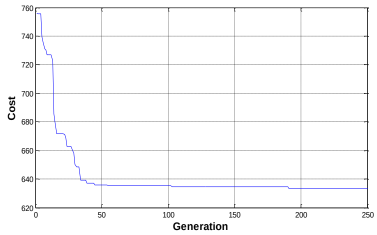

Fig-7-b. Best Cost versus generation for *p*-grade = 0.8

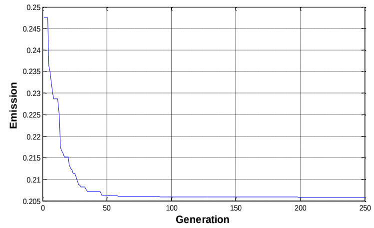

Fig-7-c. Best Emission versus generation for *p*-grade = 0.8

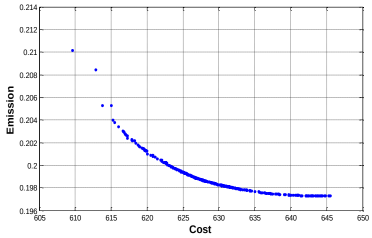

Fig-8-a. Pareto optimal set for *p*-grade = 0.6

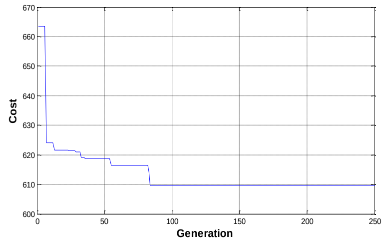

Fig-8-b. Best Cost versus generation for *p*-grade = 0.6

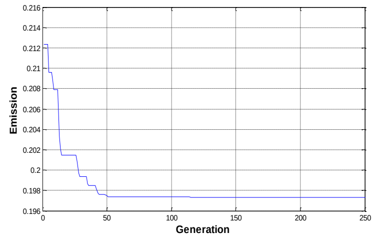

Fig-8-c. Best Emission versus generation for *p*-grade = 0.6

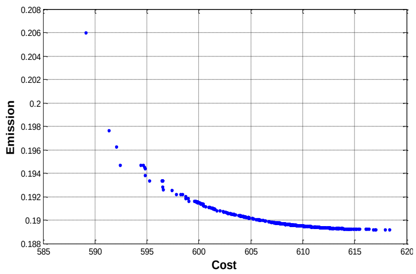

Fig-9-a. Pareto optimal set for *p*-grade = 0.4

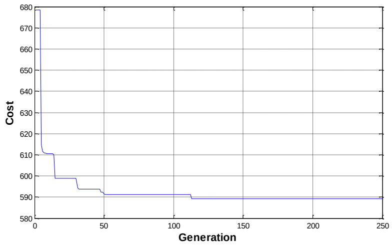

Fig-9-b. Best Cost versus generation for *p*-grade = 0.4

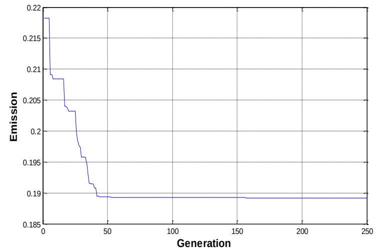

Fig-9-c. Best Emission versus generation for *p*-grade = 0.4

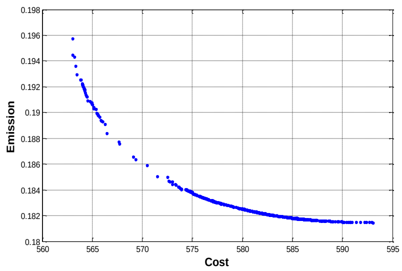

Fig-10-a. Pareto optimal set for *p*-grade = 0.2

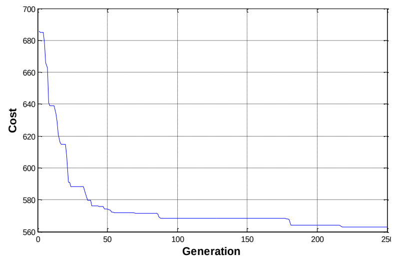

Fig-10-b. Best Cost versus generation for *p*-grade = 0.2

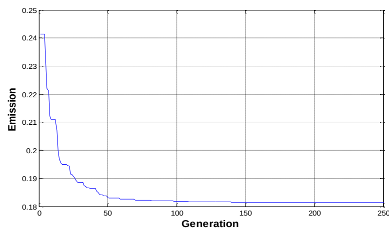

Fig-10-c. Best Emission versus generation for *p*-grade = 0.2

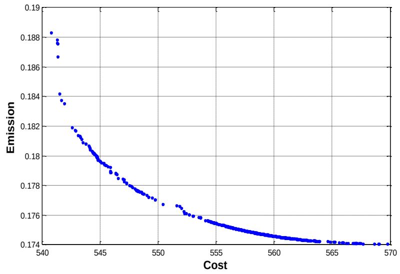

Fig-11-a. Pareto optimal set for *p*-grade = 0

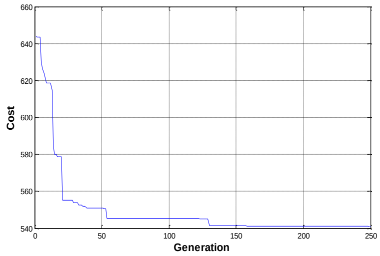

Fig-11-b. Best Cost versus generation for *p*-grade = 0

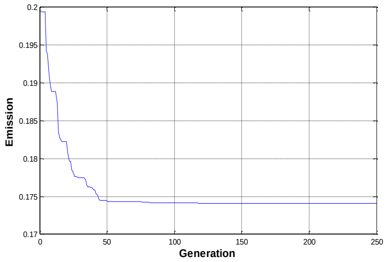

Fig-11-c. Best Emission versus generation for *p*-grade = 0

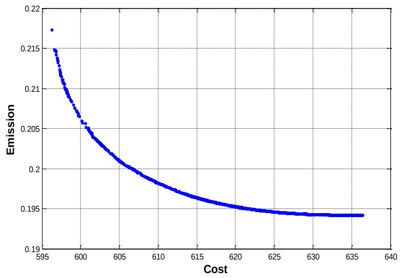

Fig-12-a. Pareto optimal set for *p*-grade $=(\ln a-\ln a_l)/(\ln a_u-\ln a_l)$

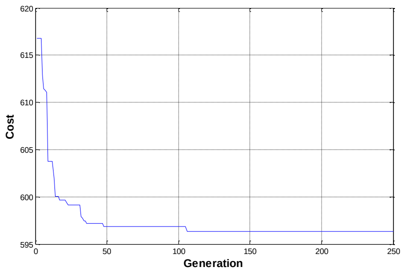

Fig-12-b. Best Cost versus generation for *p*-grade $=(\ln a-\ln a_l)/(\ln a_u-\ln a_l)$

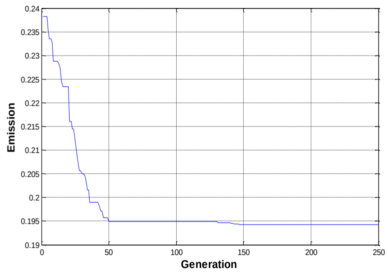

Fig-12-c. Best Emission versus generation for *p*-grade $=(\ln a-\ln a_l)/(\ln a_u-\ln a_l)$

The results show that our algorithm is effective to solve CEED optimization where in one run the Pareto optimal solutions can be found. In addition, the optimal Pareto front is well distributed and has satisfactory diversity features. The proposed algorithm does not impose any limitation on the number of objectives and it is can be extended to include more objectives is a straight forward process.

For comparison purposes with the recorder results, Table 3 and 4 show the best fuel cost and best NO_x emission obtained by proposed algorithm for the initial value of the parameters (i.e., $p=\dfrac{(\ln a-\ln a_l)}{(\ln a_u-\ln a_l)}=0.525041$) as compared to No NSGA [42] NPGA [43] SPEA [44] and epsilon dominance approach [13]. It is quite evident that our approach gives comparable minimum fuel cost and comparable minimum emission or better than those obtained by other EAs.

Also Figs.13, 14 give best cost and best emission versus *p*-grade. We concluded that the change of the best cost is linearly proportional with the *p*-grade; also the change of the best emission is linearly proportional with the *p*-grade which enables us to predict the best cost and emission for any price fluctuation without solving the problem again. By other words, if the value *p*-grade is increased the values of best cost and best emission are increased and vice versa. On the other hand, Fig. 15 declares all the Pareto set for all cases (different *p*-grade). From the figure, we can see that when *p* increased from 0 to 1 the Pareto curve is transformed in the direction of increasing the cost and emission.

Table-3. Best fuel cost

	NSGA [42]	NPGA [43]	SPEA [44]	ε-dominance [13]	Proposed
Best cost	608.245	608.147	607.807	606.4533	596.3087
Corresponding Emission	0.21664	0.22364	0.22015	0.2028	0.2173

Table-4. Best NO_x Emission

	NSGA [42]	NPGA [43]	SPEA [44]	ε-dominance [13]	Proposed
Best Emission.	0.19432	0.19424	0.19422	0.1882	0.19420
CorrespondingCost	647.251	645.984	642.603	642.8976	632.1844

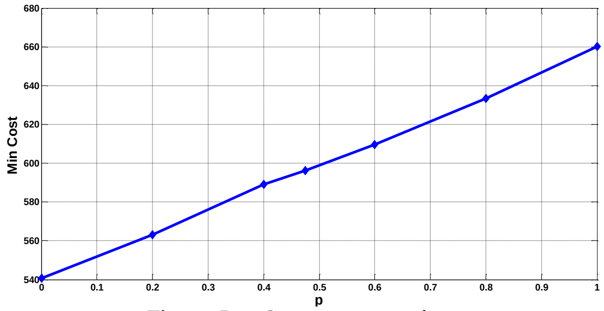

Fig-13. Best Cost versus p-grade

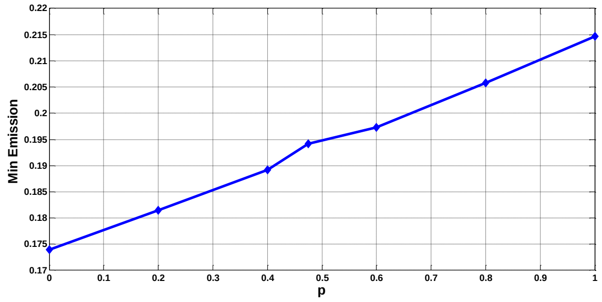

Fig-14. Best Emission versus p-grade

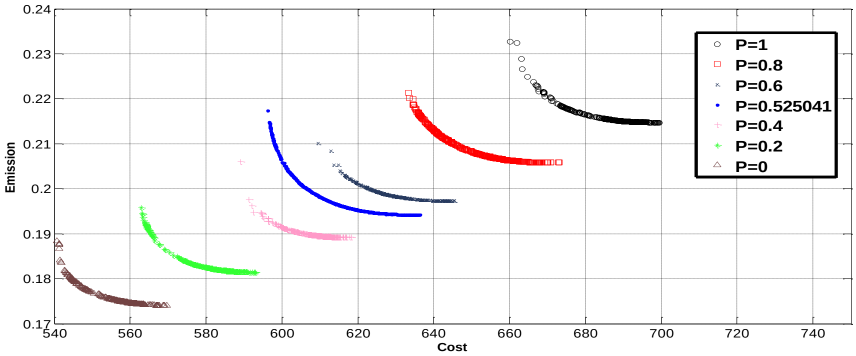

Fig-15. Pareto optimal set for different p-grade

8. CONCLUSIONS

In this paper we present a new methodology, chaos based enriched swarm optimization, for solving imprecise CEED. In the proposed approach, PSO has been enriched with a new evolution scheme, such that the movement of each particle is controlled using chaotic constriction factor to enhance the search engine visibility. In addition, the quality of the obtained solutions is improved by applying LS technique; where it aims to explore the less-crowded area and obtain more solutions. Also, we introduced p-grade function using to solve the CEED problem under imprecision. Moreover, the proposed approach is applied to the standard IEEE 30-bus 6-generator test system to illustrate its capability to generate true Pareto front of the CEED with well distribution. The main features of our approach could be summarized as follows:

(a) The results show that our approach is effective for solving CEED optimization where in one run the Pareto optimal solutions can be found.

(b) The obtained Pareto fronts have satisfactory diversity features with good distribution.

(c) Our algorithm does not levy any limitation on the number of objectives.

(d) the proposed approach gives comparable minimum fuel cost and comparable minimum NO_x emission or better than those obtained by other EAs that reported in the literature

(e) Implementation of chaotic constriction factor improve search engine visibility by controlled the movement velocity of each particle and accelerate the convergence of our approach.

(f) Using p-grade function concluded that the change of both the best cost and the best emission is linearly proportional with the p-grade; which enables us to predict the best cost and emission for any price fluctuation without solving the problem again.

(g) The change of the value of p from 0 to 1 show that the Pareto curve is transformed in the direction of increasing the cost and emission which enables us to forecast the place of Pareto curve for any changeable of the parameters.

Generally speaking, the improvement of our algorithm performance still remains in the experimental stage for lack of solid theoretical support; thus, for further work, we aim to test it on more real-life applications that have more than two objectives.

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Contributors/Acknowledgement: Both authors contributed equally to the conception and design of the study.

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