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A MODEL OF OPTIMAL TIME-OF-DAY PRICING OF ELECTRICITY UNDER DIVERSE WELFARE ASSUMPTIONS

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ABSTRACT

An important feature of an electric power system is that its customer's load varies greatly at random according to time of day, and day of season. Time-of-day pricing of electricity is an indirect load management against such variable demand according to which electricity is charged in relation to the time-differential cost of supply. The present study is an attempt at modeling seasonal time-differential pricing of electricity. The static, deterministic model that we present incorporates diverse technology, as well as soft deterministic equivalents of chance constraints representing stochastic demand and inflows. The model is solved for two types of power systems – pure hydro and hydro-thermal – under the umbrellas of four structural assumptions – first best, second-best, monopoly and constrained monopoly.

Keywords: Time-of-day pricing, Marginal cost pricing, Monopoly pricing, Ramsey pricing, Constrained monopoly pricing.

Contribution/ Originality

The present study models seasonal time-differential pricing of electricity under diverse welfare assumptions of first best, second best, monopoly and regulated monopoly pricing, in the case of two typical power systems – hydro and hydro-thermal.

1. INTRODUCTION

What is unique for an electric utility is that its product must be generated and supplied at the very instant of demand. Any anticipated demand can be met by the utility if it has an excess generating capacity, which in turn remains expensively idle during normal periods. Moreover, no other product has greater concern for quality and reliability; in practice, electric power systems are expected and thus designed to have enough reserves to check black-outs and brown-outs. Another significant component of a power system is its customer's load with an inherent nature of time-variability. This is represented by a load curve, showing changes in energy demand against time. Derived from the load curve is the load duration curve (LDC) that shows the amount of time during which a given load equals or exceeds a given capacity – an important tool in power system planning. There has been a consistently rising trend in general for electricity costs across the globe over a long period now. The electric utilities have successfully resorted to a number of methods of load management of electricity usage; the methods include direct mechanical control of end-use equipments and indirect time-of-day pricing.

Load management meets the dual objectives i) of reducing growth in peak load, thus nipping the need for capacity expansion, and ii) of shifting a portion of the load from the peak to the base-load plants, thereby securing some savings in peaking fuels. This can lead to some cut in operating and capacity costs for the electric utilities and hence lower tariff for the consumers, besides providing a partial solution to the country's energy problem.

Time-of-day pricing of electricity is an indirect load management against the customers' variable demand according to which electricity is charged in relation to the time-differential cost of supply. In contrast to the block rate tariffs, the seasonal time-differential (STD) pricing has a potential for improving the utility's load factor in that its price signals reflecting the cost structure motivate customers to modify their consumption; this in turn helps the system to achieve the above goals. However, implementing STD pricing requires new meters, capable of reading time-differential load, and thus involves substantial capital expenditure. In fact, this pricing structure has been in force in a few advanced countries for a long time; initially it has been applied only to the very large industrial consumers for whom the metering costs come out to be trivial compared with their total power bill. The spot markets for electricity that have come up in the wake of the reforms in the power sector have rendered the wholesale energy price to vary hourly, which in turn has required the distribution companies to apply real-time pricing to the power consumers. The progress in solid state technology has now introduced *smart meters* with many advantages over simple automatic meter reading, such as real-time or near real-time readings, power outage notification, and power quality monitoring. These smart meters have opened up avenues for time-of-day pricing in these countries to cover almost all consumers.

The present study attempts to model seasonal time-differential pricing of electricity under the umbrellas of diverse welfare assumptions. In deriving such pricing rules, we consider the case of two typical power systems – hydro and hydro-thermal – each under four welfare assumptions – marginal cost or first best pricing and monopoly pricing and their constrained cases, second best or Ramsey pricing and regulated monopoly pricing. Though in its basic form our static model follows Turvey and Anderson [1] and Munasinghe and Warford Jeremy [2], the model is an entirely modified one incorporating diverse technology as well as soft deterministic equivalents of chance constraints representing stochastic demand and inflows and their implications under the diverse set of welfare assumptions. This study falls into four sections. The next section briefly discusses the salient features of the generally accepted four welfare models in the context of pricing. The third section illustrates the basic peak load pricing theory and the fourth one goes into the modeling of STD pricing of electricity for two types of power systems – pure hydro and hydro-thermal – under the above four structural assumptions. The study concludes with the last section.

2. A REVIEW OF THE WELFARE ASSUMPTIONS

2.1. First Best or Marginal Cost Pricing

Historically the use of gross surplus as a measure of welfare[1] was apparently first proposed by Dupuit [10] while evaluating public works projects. It was Marshall [11] who developed the concept, which was later fulfilled in Hotelling [12], Hotelling [13] public utility pricing theory.[2]

We proceed with the traditional measure of welfare used in evaluating public utility policies in terms of net social welfare (W), defined as the sum of total revenue (R) and consumers' surplus (S) less total costs (C):

$$W = R + S - C. \quad ....(2.1)$$

The total surplus (R + S) is equivalent to the area below the uncompensated demand curve. The welfare is then given by,

$$W = \int_0^x p(y)\mathrm{d}y - C(x)), \quad ....(2.2)$$

where $p(x)$ is the inverse demand function and $C(x)$ is the total cost function.

Maximizing (2.2)gives the first best pricing rule: $p(x) = \mathrm{d}C/\mathrm{d}x$, i.e., price equals marginal cost [16]. Note that the net welfare function given in (2.2) suffers from the assumption of independent demand [17]. That is welfare in respect of each good or service can be found out separately and their sum equals the total net welfare:

$$\mathrm{W} = \sum_{i=1}^{n} w_i = \sum_{i=1}^{n}\left(\int_0^{x_i} p_i(y_i)\mathrm{d}y_i - C(x_i)\right) \quad .... (2.3)$$

However, in reality, we have to relax this assumption and consider the interdependent demands. And this renders the definition of gross surplus more complicated [12, 17]. Suppose $\boldsymbol{x} = (x_1, ..., x_n)$ represent a typical commodity bundle. Also let $\boldsymbol{x}(\boldsymbol{p}) = (x_1(\boldsymbol{p}), ..., x_n(\boldsymbol{p}))$ be the n demand function for x and $\boldsymbol{p}(\boldsymbol{x}) = (p_1(\boldsymbol{x}),....,P_n(\boldsymbol{x}))$ be their inverse demand function. In this multi-product case the net social welfare at the vector of outputs $\boldsymbol{x} = (x_1, ...,x_n)$ would be (2.3). But in general, because of the substitute/complement property of products, $\boldsymbol{p}_i$ may be expected to depend on the entire output vector $\boldsymbol{x}$, rather than just on x_i, as in (2.3). For the multi-dimensional welfare function with dependent demands, Hotelling [12] has suggested a line integral of the form:

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[1]Although there have been detractors [3] Samuelson , [4] Little, [5] Silberberg, [6] Bös., the use of surplus is widespread in applied welfare economics [7] Mishan, [8] Mishan, [9] Willig. has given further justification for its use by demonstrating, under conditions quite reasonable for the utility sector, that consumer surplus closely approximates the consumer benefit in money terms.

[2]Traditional interest in the efficiency issues sprang up from pricing aspects only. Later on the realm of efficiency concerns has broadened to involve such considerations as *X*- efficiency [14] Leibenstein, [14] ibid. and transaction costs [15] Williamson, [15] ibid..

$$W = \oint_0^x [\sum_{i=1}^n p_i(y)\mathrm{d}y_i] - C(x) \quad .....(2.4)$$

where σ is some designated path connecting the origin (of *n*-space) and the output vector.

This formulation presents two major difficulties. First, differentiability of *W*, and second, *W*, as it is now defined, depends on the particular path σ chosen and is thus not unique [17]. Thus an indeterminacy arises with variation of the value of the integral when the path of integration between the same end points is varied. The condition that all these paths of integration shall give the same value, i.e., *W* in (2.4) will depend on *x* and not on the path only under what is called the 'integrability conditions', invoked from the Independence of Path Theorem for line integrals, given by

$$\frac{\partial p_j}{\partial x_k} = \frac{\partial p_k}{\partial x_j}, \ \forall j, k. \quad .... (2.5)$$

Hotelling [13], Hotelling [12] has shown that there is a good reason to expect these integrability conditions to be met, at least to a close approximation, in an extensive class of cases [17, 18].

Thus, with the integrability conditions, the line integrals of the form (2.4) become differentiable and their value, *W*, independent of the path σ; so that the first-order conditions for maximizing *W* in (2.4) again lead to marginal cost pricing.

2.2. If No First Best, Then Second-Best

Though marginal cost pricing has got strong argument appeal, it is not without significant problems. First, departures from marginal cost pricing in some sectors of the economy owing to the immutable violation of any of the competitive equilibrium conditions in those sectors pose serious questions against thieving Pareto optimality in the other sectors of the economy.

Such violations in the first-best atmosphere accumulate as what are termed 'second-best' problems. Some of the early contributors on second-best, Lipsey and Lancaster [19], for example, argue that "To apply to only a small part of an economy welfare rules which would lead to a Paretian optimum if they were applied everywhere, may move the economy away from, not toward, a second-best optimum position" [19]. Some positive developments also followed. Farrel [20], for example, argues that the second-best optimum is likely to be close to the first-best optimum, implying that price should be set at least equal to MC, and in the case of substitutes, above MC. It has also been pointed out that first-best rules may be optimal even with the particular Lipsey-Lancaster formulation of the second-best problem [21-23]. Davis and Whinston [24] indicate that in the face of separability or little or no interdependence between sectors, first-best conditions are optimal in the competitive sectors even when they turn out to be unattainable in the other sectors [25]. Lancaster [26] has later on summarized the whole second-best arguement in the context of the electric utility industry. The small size of individual regulated industries in relation to the whole economy entails a very large manipulation of these sectors in order to counter-balance the distortions of the economy. Since all the regulated industries could not be under a common control, the alternative appears to be to optimize in individual sectors.[3]"Unless a simultaneous second-best solution is determined for the complete regulated sector, therefore, it would seem that the next best thing (the 'third best'?) is to ignore second-best elements in pricing policy at the decentralized level."[26].

But still another critical problem remains there-the problem of decreasing costs even if costless regulation could enforce marginal cost pricing policy. The traditional approach, as explained above, defines a natural monopoly in terms of everywhere decreasing average cost curve. Let AC(x) denote average costs, $C(x)/x$, and MC(x), marginal costs, d$C(x)$/dx. Then it can be shown that dAC(x)/dx = [MC(x) − AC(x)]/x, so that for any positive output level x, if dAC(x)/dx < 0, then MC(x) < AC(x). Also if MC(x) is everywhere decreasing (concave costs), then assuming $C(x) \geq 0$, we have MC(x) < AC(x).

Thus either decreasing average or decreasing marginal costs lead to marginal costs being less than the average. This results in incurring deficits under marginal cost pricing posing many a problem.[4] Attempts to have recourse to taxation for covering deficits will only lead to significant allocative distortions. Discussions upon the issue of decreasing costs have converged on two alternatives, fair rate of return regulation and welfare optimal break-even pricing.

2.3. The Other Extreme: Monopoly Pricing

First consider the case of a profit maximizing monopolist who would set price and output such as to

$$\text{Max}_{x\geq 0}\, \pi(x) = \sum_{i=1}^{n} x_i p_i(x) - C(x) \quad \text{.... (2.6)}$$

This leads to the familiar result that MR = MC, i.e., $\partial R(x)/\partial x_i = \partial C(x)/\partial x_i$, where $R(x) = \sum x_i p_i(x)$, or from (2.6),

$$p_i(x) + \sum_{j\in N} x_j \frac{\partial p_j(x)}{\partial x_i} - \frac{\partial c(x)}{\partial x_i} = 0, \quad i\in N = (1, \ldots, n) \quad \text{.... (2.7)}$$

$$\Rightarrow p_i = \frac{\text{MC}_i}{1+\sum_{j\in N}\frac{R_j}{R_i}\eta_{ji}}, \quad i\in N; \quad \text{.... (2.8)}$$

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[3]This, in effect, seems to take us back to the case-by-case approach of applied welfare economics used by Meade and others in the beginning of the 1950s and represented in later and technically more elaborate studies by, e.g., [27] Boiteux, [28] Rees. and [29] Guesneries.

[4]The very existence of MC pricing equilibria is challenged [30] Beato, [31] Cornet. Moreover, the optimality of MC pricing also is challenged [29] Guesneries.; [32] Brown and Heal, [33] Brown and Heal, [34] Brown and Heal,.; [35] Tillmann. If the production possibilities are non-convex, MC equilibria may fail to be Pareto optima. Though many an attempt has been made to find conditions under which at least one equilibrium is Pareto efficient, there exist examples showing that even in very simple cases such conditions cannot be found [32] Brown and Heal.

where $\eta_{ji} = \frac{\partial p_j}{\partial x_i}\frac{x_i}{p_j}$ is the 'flexibility' of p_j w.r.t. x_i [36] and $R_i = p_i x_i$ is the revenue from product i. When cross price elasticities of demand are zero, we get the inverse elasticity rule [37], pregnant with price discrimination potential.

Depending on the sign of $\partial P_j/\partial x_i$ in (2.8), various possibilities arise; but the usual presumption favours own effects, $\partial P_i / \partial x_i < 0$, to dominate cross effects, $\partial p_j/\partial x_i$, such that the second term there would be negative, resulting in higher prices $p_i(x)$ and lower output x than under MC pricing.

2.4. Regulated Monopoly Pricing

The welfare losses due to monopoly pricing may be limited by regulating[5] the level of profits to some 'fair' level, say, high enough to pay at competitive rates the various factors used, including capital. Assuming a fair rate of returns, v, larger than the market cost of factors k, the rate of return regulation may, in general, be captured in the constraint,

$$\sum x_i P_i(x) - \alpha C(x) \leq 0, \qquad ....(2.9)$$

with $\alpha = v/k > 1$. Inclusion of this constraint in the above monopoly pricing model yields the optimal prices,

$$p_i = \mathrm{MC}_i \left[1 - \frac{\lambda}{1-\lambda}(\alpha - 1)\right] / \left[1 + \sum_{j \in N} \frac{R_j}{R_i} \eta_{ji}\right], \quad i \in N; \qquad ....(2.10)$$

where λ is the shadow price of a rupee of profit regulated. In contrast to the unconstrained monopolist who equates MR and MC, the monopolist under rate of return regulation sets MR equal to something less than MC, the deduction being determined by λ and α. The limiting cases refer to zero profits ($\lambda = \alpha = 1$) and to monopoly profits ($\lambda = 0$).[6]

2.5. The Break-Even Ramsey Pricing

The second approach, originated with Ramsey [42] and developed mainly by Boiteux [27] and Baumol and Bradford [43], deals directly with the deficit problem by allowing optimal departures from MC pricing such as to break even. This optimal departure is obtained by maximizing the welfare function (2.4) subject to an explicit break-even constraint:

$$\pi(x) \geq \pi_0(x) \qquad ....(2.11)$$

where $\pi(x)$ is as defined in (2.6) and π_0 is the required profit level.

Assuming the integrability conditions to hold, the optimal, second-best prices derived are:

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[5] [38] Bailey, [39] Sheshinski. have examined the welfare implications of increased regulation.

[6] Though regulation may be able to reduce the abuse of monopoly power, it is fraught with a lot of knots in the context of privately owned public utilities, e.g., Averch-Johnson effect [40] Averch and Johnson. and the tar baby effect. [16] Mckie, [16] ibid.. Also see [41] Crew and Kleindorfer. for a discussion on the tar baby effect in electricity regulation in private enterprise economies.

$$p_i = \mathrm{MC}_i(1+\gamma)/\left[1+\gamma(1+\sum_{j\in N}\frac{R_j}{R_i}\eta_{ji})\right], \quad i \in N; \qquad \text{... (2.12)}$$

where γ is the shadow price of a rupee of revenue raised. Rewriting it,

$$\left[\frac{p_i - \mathrm{MC}_i}{p_i}\right] S_i = -\rho, \quad i \in N; \qquad \text{... (2.13)}$$

where $\rho = \frac{\gamma}{1+\gamma} \geq 0$ is the 'Ramsey number' and $S_i = \frac{1}{\sum_{j\in N}\frac{R_j}{R_i}\eta_{ji}}$ is the 'super elasticity' of x_i [36].[7] ρ is positive except at the welfare optimum, where $\rho = 0$, and the conditions for the profit-maximizing solution are identical to the above with $\rho = 1$.

Hence a regulated monopoly under Ramsey pricing regime behaves as if it were an unconstrained profit maximizing monopolist faced with a demand curve whose elasticity is inflated by the factor $1/\rho = (1+\gamma)/\gamma$. It must be noted that if we neglect all cross-price elasticities of demand, the Ramsey price structure reduces to the 'inverse elasticity rule':

$$(p_j - \mathrm{MC}_j) / p_j = -\rho/e_{jj}, \; j \in N; \qquad \text{.... (2.14)}$$

where e_{jj} denotes own price elasticity. The price-cost margin of a product is larger, the smaller the absolute value of its price elasticity. The normal own-price elasticity of demand being negative, the Ramsey pricing in general results in positive price cost margins. Under 'low pricing procedures', $\rho<0$, and we have the case of negative price-cost margins. The positive price-cost margins lead to higher prices of price-inelastic goods and to lower prices of price-elastic goods.

The reverse holds in the case of negative price-cost margins. Thus, in general, the poor who are comparatively price inelastic are burdened in the case of positive price-cost margins and favoured in the negative ones.[8]

3. THE PEAK-LOAD PRICING THEORY: A REVIEW

Apparently, the first pace of exploration into the basic ideas of peak-load pricing started with Brown and Heal [32] of *Electricite de France*.[9] In the USA it was independently originated by Steiner [52] and developed by Hirshleifer [53] and Williamson [54]. While Boiteux and Steiner assumed two equal periods, Williamson showed how to work out with periods of any length. Steiner interpreted his peak-load pricing results in terms of price discrimination. Hirshleifer, taking issue with this, suggested that they could be more usefully interpreted in MC pricing terms.

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[7] It should be noted that if we defined the net social benefit function over the 'budget space', the optimal solution would be in terms of the usual cross-price elasticity of demand, ε_{ij}, which can be easily interpreted. Note that $\eta_{ji} \neq 1/\varepsilon_{ij}$, and the interpretation of S_i and therefore (2.13) becomes complicated. In fact, ε_{ij} and η_{ji} need not even have the same sign; see [44] Nguyen and MacGregor-Reid.

[8] See, for equity aspects of pricing, [45] Feldstein, [46] Feldstein, [47] Feldstein, and [48] Wilson Leonard.

[9] However, according to [49] Ault Richard, Robert and Ekelund., the theory of peak load pricing goes back at least to the work of [50] Bye., [51] Bye., who first developed the peak load model.

The additional contributions made include Buchanan [55], [56-60], Pressman [17], Mohring [61], Littlechild [62],Littlechild [63],Crew and Kleindorfer [64], [65-67], and Bailey [68]. The major result common to all these works is that peak-load price should equal marginal peak running costs plus marginal capacity costs, while off-peak price equals only marginal off-peak running costs, since the peak consumers, not the off-peak ones, are solely responsible for raising the 'capacity lid'. The first major extension to the basic model was provided by Pressman [17] who synthesized the earlier works by the MC pricing school (for example, Hotelling [12]), Dreze [69] and Nelson [70] in constructing a peak-load pricing model with time-interdependent demands and a more general specification of technology. Crew and Kleindorfer [71] presented a further theoretical generalization by looking for the implications of a diverse technology for both pricing and capacity decisions. Dansby [72], based on the same technology specifications as Crew and Kleindorfer [73], Crew and Kleindorfer [74], allowed demand to vary continuously with time within each of the finite number of pricing periods.

Bailey and White [75] set up a scenario of reversals in peak and off-peak prices as enacted by a monopoly, a welfare maximizing firm with increasing returns to scale, a monopoly under rate of return (RoR) regulation and a firm with a two-part tariff. Their results implied, inter alia, that for customer changes of almost the same size, regulatory authorities with tighter RoR regulations might encourage lower usage prices to peak business users of electricity leaving the prices to off-peak residential users substantially unchanged.

Panzar [76] presented a reformulation of the peak-load problem in which technology was specified through a neo-classical production function. The best-known result that optimal peak- load pricing requires only those consumers who utilize plant to capacity to bear the marginal capacity costs was shown to result from the fixed proportions technological assumptions of the traditional literature and not from the fundamental nature of the peak-load problem. When a neo-classical technology was specified, it was found that optimal pricing required consumers in *all* periods to contribute towards the capacity cost.

3.1. A Basic Peak-Load Model

Steiner [52] has adopted the conventional welfare maximizing approach. He assumes a typical 'day' divided into two equal-length periods, each with its own independent demand curve.

Costs are assumed to be linear: b is operating cost per unit per period and β the unit capacity cost per day. Neo-classical substitutability between variable and capital costs is ignored. This and the single technology are the critical assumptions that yield 'Steiner's results' for the finite period case. The objective is to maximize welfare as given by:

$$W = \sum_i \int_0^{q_i} p_i(y_i)\mathrm{d}y_i - \beta q_p - b \sum_i q_i, \qquad i = p, o; \qquad \ldots (3.1)$$

where q_p and (q_o are demands in the peak (q_p) and off-peak (q_o) periods respectively, with peak period demand equaling capacity, and $p_p(q_p)$ and $p_o(q_o)$ are prices in the peak (p_p) and off-peak (p_o) periods respectively. The optimal prices corresponding to these periods are:

$$p_p = b + \beta, \text{ and } p_o = b, \qquad ....(3.2)$$

which indicate that peak price covers both the marginal capacity and operating costs, whereas off-peak price just covers marginal operating costs. Moreover, it is clear that if there are constant costs, welfare maximization automatically requires the peak price to be higher than the off-peak one.

3.2. Peak-Load Pricing Under Uncertainty

All the above models assume that demand is deterministic. But in general, many public utilities face demands that are not only strongly periodic as in the peak-load model but also stochastic. After the contributions of the French economists discussed by Dreze [69], Brown and Johnson [77] sparked off a new controversy as to the effects of stochastic demand on public utility pricing. Brown and Johnson used the familiar cost assumptions of the Boiteux-Steiner-Williamson peak-load model, but with a one-period stochastic demand. Their expected welfare maximization yielded the optimal solution as $p = b$, in stark contrast to the corresponding one period deterministic solution of $p = b + \beta$.

Moreover, there lurked at their optimal solution a possibility of excess demand to occur frequently. Turvey [78] criticized[10] this low level of reliability at optimum as implausible, which spurred Meyer [80] to reformulate the Brown-Johnson model by adding reliability constraints to it; this, in turn, raised a new issue as to determining the optimum levels of such constraints. Carlton [81] and Crew and Kleindorfer [82] tried on this issue, still leaving much to be resolved.

Rationing in the event of demand exceeding capacity was another vulnerable point in Brown-Johnson model Visscher [83].They assumed a zero-cost rationing process in accordance with the willingness to pay of the consumers, which appeared highly implausible. Crew and Kleindorfer [84] subsequently examined the simultaneous effects of a diverse technology, stochastic demand and rationing costs on the peak-load pricing policy of an expected-welfare maximizing public utility. Both uncertain demand and uncertain capacity were considered simultaneously in a simple model by Chao [85]. He examined demand uncertainty in a more general framework within which the hitherto specifications of demand uncertainty, in either additive or multiplicative form,

[10] [79] Salkever. also joined issue with Brown and Johnson in *American Economic Review*.

were seen as special cases. The work took explicit account of the random availability of installed capacity, a major source of uncertainty contributing to electricity supply shortages.

The theoretical refinements have not attracted much attention of late, possibly because the classical framework and the inevitable result have been taken for granted, and the research interest has shifted from theory to empirics. However, Pillai [86] has taken up the basic peak load model to question the classical framework and its result and shown that if the off-peak period output is explicitly expressed in terms of capacity utilization of that period, the result will be an off-peak price including a fraction of the capacity cost in proportion to its significance relative to total utilization.

Analyzing the implications of the relationship between reliability and rationing cost involved in a power supply system in the framework of the standard inventory analysis, instead of the conventional marginalist approach of welfare economics, he has also formulated indirectly a peak period price in terms of rationing cost [87]. The present paper is one in this continuum.

3.3. Empirical Studies on Peak-Load Pricing

As already mentioned, theoretical interest on peak load pricing has waned over time and given way to empirical analysis of residential electricity demand by time of use. Most of the published studies have sought to estimate electricity demand by time-of-day, using data at the household level obtained from 'rate experiments'. During the last three decades, in countries such as the US (see, for example, Faruqui and Malko [88] and Faruqui and George [89], the UK (see [90, 91] and France (see Aubin [92], several demonstration projects on residential electricity consumption by time-of-use were promoted in an attempt to better understand the effects of time-of-day pricing on residential electricity consumption. Generally, in a rates experiment, residential consumers of an electric utility are selected randomly and placed on various time-of-use rates for a time horizon ranging from two to six months. The electric utilities collect monthly data on the electricity consumption of each of the selected customers during various daily time periods, which on aggregation provide a data set on residential time-of-use electricity consumption. Among the studies making use of such data set we have on the one hand those undertaken by Hill [93] and Filippini [94] that analyze the electricity demand by time-of-use using a system of log-linear demand equations in an 'ad hoc' way; that is, the models do not reflect completely the restrictions imposed by the neoclassical theory of consumer behaviour. On the other hand are studies by Caves and Christensen [95], Filippini [96], Aubin [92], Baladi, et al. [97] that bring out the implications of apportioning the electricity expenditure to peak and off-peak consumption based on conditional demand. For an overview of these studies see Hawdon [98] and, recently, Lijesen [99] and Faruqui and Sergici [100]; for a review on price and substitution elasticities under time-of-use rates, see Acton and Park [91] and King and Chatterjee [101].

Empirical evidences on the response of larger commercial and industrial customers to real time pricing (RTP) are reported in Herriges, et al. [102], Patrick and Wolak [103], Taylor, et al.

[104], and Boisvert, et al. [105]. Barbose, et al. [106] gives a detailed review of real time pricing programs of the US utilities. On the other hand, in spite of significant hourly variation in the wholesale market price, most of the US residential customers are charged a near-constant retail price for electricity. The first significant effort to introduce real time pricing, that is, hourly market-based electricity pricing to residential customers (called Energy Smart Pricing Plan) was developed by Chicago Community Energy Cooperative in association with Commonwealth Edison (ComEd) as a voluntary programme with 1500 households in Chicago in 2003. The four-year pilot Plan demonstrated the potential benefits of real-time electricity pricing on a limited basis. Its success paved the way for expanding real-time pricing to all household across the state of Illinois, starting in 2007. Allcott [107] evaluates this first program to expose residential consumers to hourly real-time pricing and finds that the enrolled households were statistically significantly price elastic and that consumers responded by conserving energy during peak hours, but remarkably did not increase average consumption during off-peak times.

4. MODELING OPTIMAL TIME-OF-DAY PRICING OF ELECTRICITY

Programming and simulation models are regularly used to compare the techno-economic performance of different combinations of power plants and to evaluate the optimal schedule. However, they generally tend to be impotent in revealing the underlying principles of the optimal plant mix. To analyze this problem, the marginalist approach has been widely employed by electric utilities that rely on thermal sources of power.[11]But systems depending primarily on hydroelectric power have not received that much extent of analysis.[12] The marginalist approach, however, is constricted in its scope of comprehension in that it usually reduces the operation of a multi-reservoir multi-plant system to that of an 'equivalent' single composite reservoir.

Equivalent composite representation of multi-reservoir systems is often used by engineers in evaluating optimal operation of hydro-electric systems.[13] In the absence of a well-knit sophisticated planning model and of accessibility to solution techniques, and in view of intricate complications involved in dynamic analysis, such simple, static model comes in handy with the essential features to be analyzed for structuring long-run marginal cost (LRMC). Again it is an immediate alternative for taking into account the stochastic inflows, and it enables the use of stochastic dynamic programming.[14]

In what follows we present our simple, static, deterministic model.

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[11]See, for example, the seminal work of [108] Turvey.

[12]This may be because, except Canada, most of the industrialized countries make little use of hydro-power. [109] Bernard. presents a marginalist analysis of the specific characteristics of limited hydro-power in a Ricardian framework in the context of Canada.

[13]See, for instance, [110] Arvanitidis and Rosing, [111] Arvanitidis and Rosing.

[14]See [112] Neto, Araripe, Pereira and Kelman.

4.1. A Hydro-Power System with Seasonal Costs

The power generation of a hydro-system is subject to two constraints, viz., the available hydraulic energy (i.e., kinetic energy of falling water) that drives the turbines and the available installed capacity that sets a ceiling on the pace of conversion of hydraulic energy into electric energy. Given the capacity, hydraulic energy is determined jointly by nature (rainfall) and by engineering works (dam, river diversion and dredging). The seasonality of water inflows entails storages for impounding water in the wet season to help meet the dry season requirements. Storage begins and rises with the wet season and once the reservoirs are full, spilling and/or sluicing occurs and continues as long as effective inflow exceeds energy demand. Discharge begins as the latter outgrows the former and consequently reservoir level falls. If the spilling and sluicing period spans quite long with a likelihood of this pattern recurring for many years, then the marginal costs of energy in the wet season will be essentially zero; because, with the energy inflows exceeding energy demands plus storage, extra energy in the wet season can be generated just by running through the turbines more water that might otherwise be spilled or sluiced away, provided there is enough plant capacity. The operating and maintenance (O&M) costs may increase a little to make up marginal costs. In contrast, during the dry season, when energy inflows skimp in relation to outflows, extra reservoir capacity is required to meet extra energy demands and the corresponding costs of providing storage capacity represent marginal energy costs during the dry season. In certain instances allocating a fraction of the dam costs to the capacity costs may be justifiable, which, however, may depend on the nature of the specific case: for example, whether or not more storage is required to firm up the additional capacity. Given this picture of supply cost characteristics, if we now superimpose on it demand for power with its random features bouncing between peak and off-peak points, we get an optimal schedule of generating costs.

Now the above model with the system assumptions can be more compactly and precisely be couched in terms of a marginalist approach. First we turn to the assumptions designing the load duration curve (LDC), pivotal to our analysis. Our models consider only independent demands during a period divided into two seasons, wet and dry, $s = w, d$. The time-varying demand for power during each season is represented by a LDC (Fig. 1) which describes the width of the time-interval, θ, that demand equals or exceeds a given capacity level q:

$$Q = F(\theta), \quad 0 \leq \theta \leq T; \qquad \ldots. (4.1)$$

where T refers to the duration (hours) of the season. Because of its monotonicity and continuity, the function $F(\theta)$ can be inverted to obtain the width of the time-interval when capacity level q is in use:

$$\theta = F^{-1}(q) \equiv \Phi(q), \quad 0 \leq q \leq \overline{q} = F(0) = \text{peak load}. \qquad \ldots. (4.2)$$

The LDC is broken down into two discrete blocks, t, of power demand – peak and off-peak, $t = p, o$.

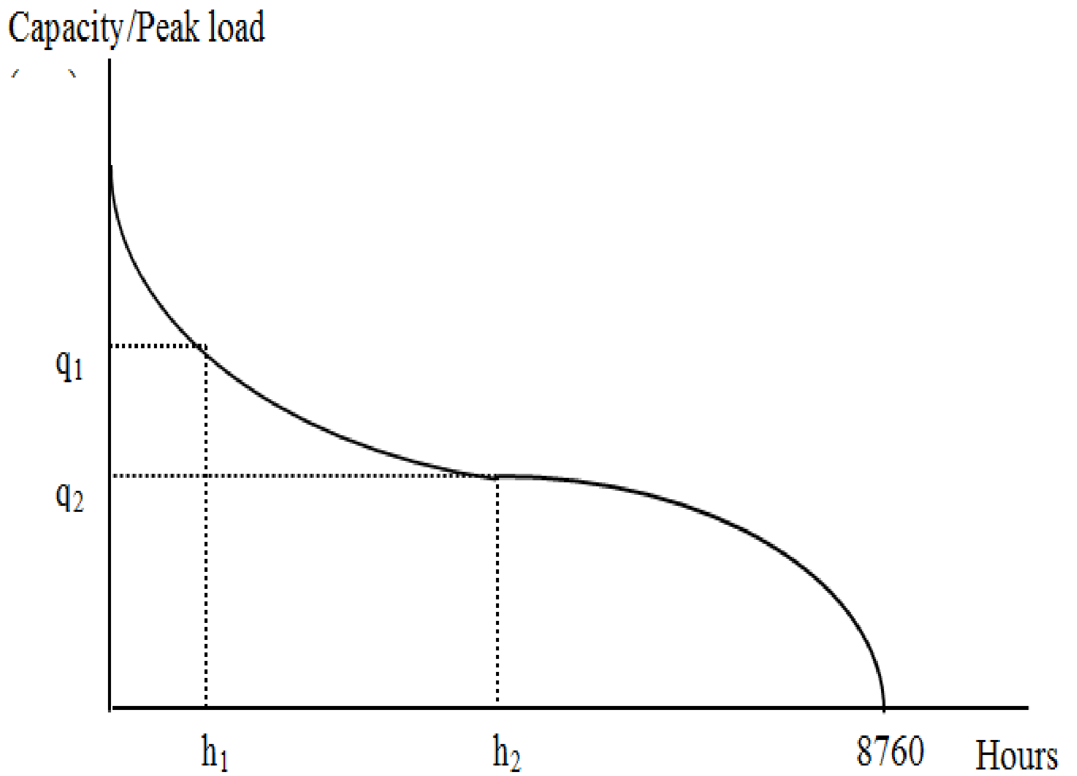

Fig-1. Load duration curve

4.2. The Marginal Cost or First-Best Prices

4.2.1 A Pure Hydro System

The first model considers the ramifications of the state-owned utility's welfare-commitments for its pricing policy. The mathematical formulation of the model portrays the maximization of the sum of consumers' and producers' surplus, given by the integrals of inverse demand curves less the costs:

$$W = \sum_s \sum_t \int_0^{Q_{st}} P_{st}(\mathrm{Y_{st}}) \mathrm{d}Y_{st} - \left\{\sum_i \beta^i q^i + \sum_i \rho^i R^i + \sum_i \sum_s \sum_t b^i q^i_{st} \theta_{st}\right\} \ldots (4.3)$$

where

Q_{st} : energy demand in season s, period t;

q^i: power capacity of the i^{th} hydro-plant (kw);

β^i : the corresponding constant annuitized marginal (turbine) capacity cost;

R^i : peak period capacity of reservoir (or hydraulic energy) of the i^{th} plant (kwh)

ρ^i: the corresponding constant annuitized marginal capital cost;

q^i_{st}: power output of the plant i in season s, period t (kw);

b^i: the corresponding (output inelastic) constant marginal operation and maintenance costs;

and

θ_{st} : the length of the period t in season s;

Among the constraints on this maximization, first let us consider what the French writers call the 'guarantee conditions', to ensure supply, to an acceptable probability limit, in the face of contingencies-water shortages in dry seasons, peak-load above mean expectations, or plant outages. These conditions are incorporated into the model in two forms: one for peak power supplies and the other for energy supplies in critical periods. Thus the first one gives the chance constraint that the capacity will be enough to meet the peak-load at least 100α per cent of the time:

$$\Pr\{\sum_i q^i \geq Q^*\} \geq \alpha,$$

where Q^* refers to the stochastic peak load; and $0 \leq \alpha \leq 1$.

This guarantee condition is often simplified in practice in terms of a 'margin of available capacity' over and above that required to meet the mean expected peak demand, as found by Cash and Scott [113] while reviewing the practices in European countries in planning system reliability. Hence we may write

$$\sum_i q^i \geq \mathrm{E}(Q^*)(1 + \mathrm{PRM}),$$

where PRM is percent reserve margin. This constraint may better be added implicitly to the model, since its effect is tantamount to interpreting Σq^i as actual capacity less an allowance for the risk of peak-load outgrowing its mean expected value; that is, Σq^i is 1/(1 + PRM) of actual capacity which in turn implies that βs are now (1 + PRM) times the cost of a kw of new capacity. Hence, hereafter βs represent these adjusted costs and qs, the available capacities.

The second guarantee condition, relating the energy availability especially in dry seasons, takes on the chance constraint that the total power output may be insufficient to meet the instantaneous demand at most $100(1-\alpha_{st})$ per cent of the time:

$$\Pr\{\sum_i q_{st}^i \geq Q_{st}\} \geq \alpha_{st}, \quad 0 \leq \alpha_{st} \leq 1 \qquad \ldots (4.4)$$

The inclusion of a penalty cost term in the objective function is in fact a direct effect of this chance constraint likely to be violated, i.e., the social cost of the failure to meet requirements. Hence suffice it to replace this constraint by the following relation in an equality:

$$\sum_i q_{st}^i - Q_{st} = 0, \quad \forall s, t; \ \text{(dual variables } \mu_{st}) \qquad \ldots (4.4')$$

Next are the capacity constraints that plant output can never exceed the corresponding available capacity:

$$q_{st}^i - q^i \leq 0, \ \forall i, s, t; \ \text{(dual variables } C_{st}^i) \qquad \ldots (4.5)$$

The stochastic water flows and storage are captured in a chance constraint that the energy release during a season plus water in storage at the end of the period cannot exceed, at least $100\alpha_s$ per cent of the time, the inflow during the period (corrected for evaporation and seepage) plus the water in store at the beginning:

$$Pr\{\sum_i q_{st}^i \theta_{st} + S_s^i - S_{s-1}^i \leq I_s^i\} \geq \alpha_s, \qquad \ldots (4.6)$$

where S_s^i is the water in i^{th} storage at the end of s, I_s^i is water inflow into it corrected for losses during s, and $0 \leq \alpha_s \leq 1$; all variables are expressed in kwh. Conversion of this chance constraint into its equivalent deterministic form requires information on the probability distribution of the stochastic inflow I_s^i. Assuming the probability distribution is known and its fractiles are completely determined by its mean, $\mathrm{E}(I_s^i)$, and standard deviation, σI_s^i, and defining $k\alpha_s$ by the relationship $F(k\alpha_s) = \alpha_s$, $0 \le \alpha_s \le 1$; where F(.) is the cumulative distribution function of $\{I^i_s - E(I^i_s)\}/\sigma I^i_s$, the chance constraint may be written in its deterministic equivalent as

$$\sum_i q^i_{st}\theta_{st} + S^i_s - S^i_{s-1} \le E(I^i_s) + k_{\alpha s}\sigma I^i_s. \quad .... (4.6')$$

For a marginalist analysis, however, this specification lends little help; and hence for practical purposes, we qualify the energy release, $q^i_{st}\theta_{st}$, in order to atone for the stochastic impacts of inflow, with a water availability factor, ω^i_s, which in effect, if lower, imposes a penalty in terms of higher storage costs. Thus the water balance constraint we consider is

$$\sum_t \frac{q^i_{st}}{\omega^i_s}\theta_{st} + S^i_s - S^i_{s-1} \le E(I^i_s), \quad \forall i, s \text{ (dual variables } H^i_s). \quad .... (4.6'')$$

The last, upper storage constraint, requires that the quanta of water stored, S^i_s, can never exceed capacity, R^i:

$$S^i_s - R^i \le 0, \ \forall i,s \quad \text{(dual variables } X^i_s). \quad .... (4.7)$$

The Kuhn-Tucker conditions for maximization subject to these constraints are:

$$Q_{st} > 0;\ P_{st} - \mu_{st} = 0; \quad ....(4.8)$$

$$q_i > 0;\quad -\beta^i - \sum_s \sum_t C^i_{st} = 0; \quad .... (4.9)$$

$$S^i_s \ge 0;\ H^i_s - H^i_{s+1} + X^i_s \le 0; \quad .... (4.10)$$

$$R^i > 0;\ -\rho^i - \sum_s X^i_s = 0; \quad ..... (4.11)$$

$$q^i_{st} \ge 0;\ -b^i\theta_{st} + C^i_{st} + \frac{H^i_s}{\omega^i_s}\theta_{st} + \mu_{st} \le 0; \quad .... (4.12)$$

The last equation when q^i_{st} is positive yields seasonal time-of-use long-run marginal cost per kwh, μ_{st}/θ_{st}, and together with the first one gives the usual first-best solution, P = MC. Assuming there is only one hydro-plant in the system, an equivalent composite reservoir case, and S_s and q_{st} are positive, we get the following results. The constraint on water (4.6'') is not binding during spilling periods, $s = w$, and hence H_w is zero, which is its lower value. From (4.12) we have, then, during the wet season

$$\frac{\mu_{wt}}{\theta_{wt}} = b - \frac{C_{wt}}{\theta_{wt}}. \quad .... (4.13)$$

The constraint on capacity (4.5) is not binding during the off-peak period, $t = o$, so that C_{wo} is zero. Thus marginal cost of hydro-generation during wet off-peak periods is just equal to b, the O &M costs per kwh involved. When the capacity constraint is binding so that C_{wt} is positive in periods $t = p$, (4.9) gives $-C_{wp} = \beta$, and hence marginal cost per kwh during wet peak periods is

$$\frac{\mu_{wp}}{\theta_{wp}} = b + \frac{\beta}{\theta_{wp}}. \quad ..... (4.14)$$

The upper storage constraint (4.7) may be binding for several successive periods of spilling; but X_w will be positive only for the last of these spilling periods because extra reservoir capacity is useful only if it provides more water for discharge. Hence, if $d + 1$ is the first discharge period, then from (4.10) we get $H_d = X_{d-1}$, (as $H_{d-1} = 0$). Since X_s is positive only in $d-1$, (4.11) gives $p = -X_{d-1}$, so that $-H_d = \rho$. Hence in the dry off-peak periods, marginal cost per kwh is

$$\frac{\mu_{do}}{\theta_{do}} = b + \frac{\rho}{\omega_d}. \qquad .... (4.15)$$

i.e., the unit O &M cost plus the annuitized cost per kwh of storage capacity weighted by the water availability factor. In contrast, in the dry peak period, $C_{dp} = \beta$ and hence

$$\frac{\mu_{dp}}{\theta_{dp}} = b + \frac{\rho}{\omega_d} + \frac{\beta}{\theta_{dp}}. \qquad .... (4.16)$$

4.2.2. A Hydro-Thermal Power System

Now we will find out the rules for optimal plant mix and the corresponding prices when there are two plants in the system. This will be such as to be in keeping with the direction of our empirical exercise (in the next chapter), so that we assume that a thermal plant is added to our system with a single representative reservoir. Thermal plant will be used in the dry season continuously on base-load operation with hydro meeting the peak; and vice-versa in the wet season. Such a specification entails new definitions for some of the elements in our earlier model. Let us denote the sets of hydro and thermal plants by h and f respectively; then our generalized model (4.3) becomes

$$W = \sum_s \sum_t \int_0^{Q_{st}} P_{st}(Y_{st})dY_{st} - \{\sum_{i\in h,f} \beta^i q^i + \sum_{i\in h} \rho^i R^i + \sum_{i\in h,f} \sum_s \sum_t b^i q^i_{st} \theta_{st}\} \ldots (4.3')$$

where b^i, $i \in h, f$, are now O& M costs for hydro plants and fuel costs plus O& M costs for thermal plants. It needs no mention that the water balance constraints apply only to the hydro-plants. Hence the last of the Kuhn-Tucker conditions may be rewritten more specifically as

$$q^i_{st} \geq 0;\ -b^i\theta_{st} + C^i_{st} + \frac{H^i_s}{\omega^i_s}\theta_{st} + \mu_{st} \leq 0; \quad i\in h; \qquad .... (4.13')$$

and

$$q^i_{st} \geq 0;\ -b^i\theta_{st} + C^i_{st} + \mu_{st} \leq 0; \quad i\in f; \qquad .... (4.13'')$$

Now let us consider the system with two plants, one hydro (h) and one thermal (f), in the dry season, assuming $q^i_{dt}>0$, $i = h, f$. Then, eliminating μ_{dt} and substituting for C^i_{dt}, $i = h, f$, and for H^i_d, $i = h$, in the above equations, we get the familiar rules for optimal load scheduling.

$$\frac{\beta^f - \beta^h}{\theta_{dt}} + b^f = b^h + \frac{\rho}{\omega_d}, \qquad .... (4.17)$$

i.e., the marginal generating cost should be equal at the optimum for both the plants. More precisely, it requires that the marginal capacity cost per kwh saved if hydro-plant were used instead of thermal, should be equal to the savings in marginal running cost per kwh if thermal were operated instead of hydro. It also implies that if the hydro-plant has cheaper marginal running cost, then it should be more expensive to construct. Note that the right side term in (4.17) is the optimal price (= MC) per kwh in the dry off-peak period for a single hydro-plant system. Hence on the strength of the economic rationale that extra thermal capacity means commensurately less hydro-capacity in need and therefore a saving in its cost, the L.H.S. in (4.17) may be taken as the marginal cost per kwh in the dry off-peak period for the hydro-thermal system.[15] And in the peak period, as we know, the MC per kwh will be higher by β^h/θ_{dp}, i.e.,

$$\frac{\mu_{dp}}{\theta_{dp}} = \frac{\beta^f - \beta^h}{\theta_{do}} + b^f + \frac{\beta^h}{\theta_{dp}}. \qquad \text{.... (4.18)}$$

Rewriting (4.18),

$$\frac{\mu_{dp}}{\theta_{dp}} = \beta^h\left(\frac{1}{\theta_{dp}} - \frac{1}{\theta_{do}}\right) + \beta^f\left(\frac{1}{\theta_{do}}\right) + b^f, \qquad \text{.... (4.19)}$$

$$\text{or, } \mu_{dp} = \beta^h\left(1 - \frac{\theta_{dp}}{\theta_{do}}\right) + \beta^f\left(\frac{\theta_{dp}}{\theta_{do}}\right) + b^f\theta_{dp}. \qquad \text{.... (4.19')}$$

In other words, peak-load operation of the hydro requires a capacity $1/\theta_{do}$ less than its peak capacity, but no additional hydraulic power, the decrease being compensated for by the thermal with extra fuel provisions. This means that, as (4.19') indicates,[16] it is possible for adding one kw of hydro-capacity to be used during θ_{dp} hours without extra hydraulic energy. Since hydraulic energy remains the same, this leaves θ_{dp}/θ_{do} of a hydro-plant without hydraulic energy during θ_{do} hours, so that the net capacity increase is only $1 - \theta_{dp}/\theta_{do}$ with no change in energy. To counter this deficiency, however, both capacity, (θ_{dp}/θ_{do}kw) and energy, (θ_{dp} kwh) provisions are required for the thermal. Now it is straightforward to find out the marginal costs in the wet season, when hydro will be continuously on base-load operation and thermal on the peak. The same logic as above yields an off-peak price in terms of i) cost savings if thermal were used instead of hydro, plus ii) O &M costs of hydro, (the sum to be equal to thermal fuel costs). The peak price is obtained by adding to it, the marginal annuitized thermal capacity costs per unit. Below we tabulate the first-best seasonal time-of-day (STD) prices per kwh of electricity for an all-hydro (single representative reservoir) system and a hydro-thermal (one hydro-one thermal: both representative) system:

Table-1. STD prices per kwh of electricity for an all-hydro and a hydro-thermal system under the first best assumption:

Seasonal Time-of-day	Pure Hydro	Hydro-thermal
Wet off-peak	b^h	$\frac{\beta^h - \beta^f}{\theta_{wo}} + b^h$
Wet peak	$b^h + \frac{\beta^h}{\theta_{wp}}$	$\beta^f\left(\frac{1}{\theta_{wp}} - \frac{1}{\theta_{wo}}\right) + \beta^h\left(\frac{1}{\theta_{wo}}\right) + b^h$
Dry off-peak	$b^h + \frac{\rho}{\omega_d}$	$\frac{\beta^f - \beta^h}{\theta_{do}} + b^f$
Dry peak	$b^h + \frac{\rho}{\omega_d} + \frac{\beta^h}{\theta_{dp}}$	$\beta^h\left(\frac{1}{\theta_{dp}} - \frac{1}{\theta_{do}}\right) + \beta^f\left(\frac{1}{\theta_{do}}\right) + b^f$

15 See [1] Turvey and Anderson.

16 For a similar result for two hydro-power 'sites', see [109] Bernard.

4.3. The Monopoly Pricing

Our second model is set to look for the pricing implications of the utility's objective ingrained in its monopoly status to maximize profit rather than welfare. The objective function here is

$$\pi = \sum_s \sum_t P_{st}Q_{st} - \text{COST}, \quad \text{... (4.20)}$$

where π denotes profit and COST refers to the cost terms in parentheses in (4.3) for a pure hydro system and in (4.3') for a hydro-thermal one. The maximization subject to the relevant production constraints we have considered earlier – (4.4) through (4.7) – yields the monopoly prices which we tabulate below for our two systems:

Table-2. STD prices per kwh of electricity for an all-hydro and a hydro-thermal system under the monopoly assumption:

Seasonal Time-of-day	Pure Hydro	Hydro-thermal
Wet off-peak	$\frac{b^h}{1-\frac{1}{e_{wo}}}$	$\frac{(\beta^h - \beta^f)/\theta_{wo} + b^h}{1-\frac{1}{e_{wo}}}$
Wet peak	$\frac{b^h + (\beta^h/\theta_{wp})}{1-\frac{1}{e_{wp}}}$	$\frac{\beta^f\left(\frac{1}{\theta_{wp}} - \frac{1}{\theta_{wo}}\right) + \beta^h/\theta_{wo} + b^h}{1-\frac{1}{e_{wp}}}$
Dry off-peak	$\frac{b^h + \rho/\omega_d}{1-\frac{1}{e_{do}}}$	$\frac{(\beta^f - \beta^h)/\theta_{do} + b^f}{1-\frac{1}{e_{do}}}$
Dry peak	$\frac{b^h + \rho/\omega_d + \beta^h/\theta_{dp}}{1-\frac{1}{e_{dp}}}$	$\frac{\beta^h\left(\frac{1}{\theta_{dp}} - \frac{1}{\theta_{do}}\right) + \beta^f/\theta_{do} + b^f}{1-\frac{1}{e_{dp}}}$

where e_{st}, $s = w, d$; $t = o, p$; is the price elasticity of demand in season s, period t.

As usual, monopoly price attaches an elasticity term to the welfare price and is hence pregnant with price discrimination potential. Depending upon the degree of the period elasticity and marginal capacity cost per kwh, there is a possibility of pricing reversals, as found by Bailey and White [75].

4.4. The Ramsey Pricing

Our constant cost model ensures under the marginal cost pricing rule that the utility just exactly breaks even. The guidelines laid down by the Venkataraman Committee characterize the Electricity Boards in effect as commercial-cum-service organizations and require them not merely to break-even, but also to generate a surplus after meeting all expenses properly chargeable to revenues, including O&M expenses, taxes, depreciation and interest [114]. Hence we add to the welfare function model an additional constraint of the following form:

$$\pi \geq \pi_0 \text{(dual variables } \gamma), \quad \text{.... (4.21)}$$

where π is as in (4.20) and π_0 is some desired profit level. The maximization of the welfare function [(4.3) or (4.3')] subject to the relevant production constraints, (4.4) through (4.7), and the profit level constraint (4.21) gives the following second-best prices for our two simple systems. Here the prices equal marginal costs inflated with weights imposed by the profit level constraint as well as the price-elasticity of period demand. These Ramsey prices warrant that the price-cost margin for each period is proportional to the marginal deficit (MR less MC) incurred in that period.[17] Note that we have the Bailey-White pricing reversal possibility here also.

Table-3. STD prices per kwh of electricity for an all-hydro and a hydro-thermal system under the second best assumption:

Seasonal Time-of-day	Pure Hydro	Hydro-thermal
Wet off-peak	$\frac{b^h(1+\gamma)}{1+\gamma(1-\frac{1}{e_{wo}})}$	$\frac{\{(\beta^h-\beta^f)/\theta_{wo}+b^h\}(1+\gamma)}{1+(1-\frac{1}{e_{wo}})}$
Wet peak	$\frac{\{b^h+(\beta^h/\theta_{wp})\}(1+\gamma)}{1+\gamma(1-\frac{1}{e_{wp}})}$	$\frac{\{\beta^f\left(\frac{1}{\theta_{wp}}-\frac{1}{\theta_{wo}}\right)+(\beta^h/\theta_{wo})+b^h\}(1+\gamma)}{1+\gamma(1-\frac{1}{e_{wp}})}$
Dry off-peak	$\frac{(b^h+\frac{\rho}{\omega_d})(1+\gamma)}{1+\gamma(1-\frac{1}{e_{do}})}$	$\frac{\{(\beta^f-\beta^h)/\theta_{do}+b^f\}(1+\gamma)}{1+\gamma(1-\frac{1}{e_{do}})}$
Dry peak	$\frac{\{b^h+\frac{\rho}{\omega_d}+\frac{\beta^h}{\theta_{dp}}\}(1+\gamma)}{1+\gamma(1-\frac{1}{e_{dp}})}$	$\frac{\{\beta^h\left(\frac{1}{\theta_{dp}}-\frac{1}{\theta_{do}}\right)+(\beta^f/\theta_{do})+b^f\}(1+\gamma)}{1+\gamma(1-\frac{1}{e_{dp}})}$

4.5. Constrained Monopoly Pricing

It needs no note that care should be taken to reduce the abuse of monopoly motive to push up the prices beyond certain levels and thus to safeguard the socio-economic development. At the same time the utility should strive to reap a reasonable return on its capital. Hence on the assumption of a fair return, *v*, larger than the market cost of capital, *k*, the monopoly behaviour (4.20) may be constrained under a rate of return regulation of the form:

$$\sum_s \sum_t P_{st} Q_{st} - \{\sum_i \alpha\beta^i q^i + \sum_i \rho^i R^i + \sum_i \sum_s \sum_t b^i q^i_{st} \theta_{st}\} \leq 0,$$

(dual variables λ); …. (4.22)

where $\alpha = v/k > 1$, and the superscript *i* should be defined in accordance with whether the system is pure hydro or hydro-thermal one [40].

Maximizing profit subject to the original set of constraints, (4.4) through (4.7), and (4.22), we get the following time-varying prices for our two systems under consideration:

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[17] Cf. [43] Baumol and Bradford. and [27] Boiteux, [115] Boiteux. Our profit-ensuring pricing rules are reminiscent of those in the general model of optimal departures from marginal cost pricing to deal with the deficit dilemma in the context of increasing returns in capacity provision.

Table-4(a).STD prices per kwh of electricity for an all-hydro system under the constrained monopoly assumption:

Seasonal Time-of-day	Pure Hydro
Wet off-peak	$\dfrac{b^h}{1-\frac{1}{e_{wo}}}$
Wet peak	$\dfrac{b^h + (\beta^h/\theta_{wp})\{1-\frac{\lambda}{1-\lambda}(\alpha-1)\}}{1-\frac{1}{e_{wp}}}$
Dry off-peak	$\dfrac{b^h + \rho/\omega_d}{1-\frac{1}{e_{do}}}$
Dry peak	$\dfrac{b^h + \rho/\omega_d + (\beta^h/\theta_{dp})\{1-\frac{\lambda}{1-\lambda}(\alpha-1)\}}{1-\frac{1}{e_{dp}}}$

First let us consider the hydro system; a surprise springs up in that the rate of return regulation appears not to affect the off-peak pricing policy of the utility, if cross-elasticity effects are zero, as the off-peak prices under rate of return regulation in both the seasons are identical to those obtained for a profit-maximizing monopoly. All the onus of regulation falls on the peak prices.

In the case of hydro-thermal system, see that the off-peak prices also include the capacity charges. Except for pure hydro off-peak periods, regulation sets MR equal to something less than MC; and thus the period prices, except the hydro off-peak ones, under rate-of-return regulation are lower than those of an unconstrained profit maximizer.

Table-4(b). STD prices per kwh of electricity for a hydro-thermal system under the constrained monopoly assumption:

Seasonal Time-of-day	Hydro-Thermal
Wet off-peak	$\dfrac{\frac{(\beta^h-\beta^f)}{\theta_{wo}}\{1-\frac{\lambda}{1-\lambda}(\alpha-1)\} + b^h}{1-\frac{1}{e_{wo}}}$
Wet peak	$\dfrac{\{\beta^f\left(\frac{1}{\theta_{wp}}-\frac{1}{\theta_{wo}}\right) + (\beta^h/\theta_{wo})\}\{1-\frac{\lambda}{1-\lambda}(\alpha-1)\} + b^h}{1-\frac{1}{e_{wp}}}$
Dry off-peak	$\dfrac{\frac{(\beta^f-\beta^h)}{\theta_{do}}\{1-\frac{\lambda}{1-\lambda}(\alpha-1)\} + b^f}{1-\frac{1}{e_{do}}}$
Dry peak	$\dfrac{\{\beta^h\left(\frac{1}{\theta_{dp}}-\frac{1}{\theta_{do}}\right) + (\beta^f/\theta_{do})\}\{1-\frac{\lambda}{1-\lambda}(\alpha-1)\} + b^f}{1-\frac{1}{e_{dp}}}$

Comparing the prices under these four models, it is clear that, as expected, the monopoly prices constitute the upper bound of the price domain and the first best prices form the floor except when a higher value of $\lambda$is imposed upon the regulated monopoly. Between these lie other model prices, given enough flexibility for the concerned constraint to exert itself upon the respective model. Thus a very high value of λ (low γ) tends to constrict the constraint, driving prices to the minimum.

5. CONCLUSION

It has long been advocated that the sale of electricity and other services, in which periodic variations in demand are jointly met by a common plant of fixed capacity, should be at time-differential tariffs. Despite a very rich tradition of modeling in peak load pricing, there has not been of late any major work. The present study has sought to model seasonal time-of-day pricing rules for electricity for two types of power systems – pure hydro and hydro-thermal under the various umbrellas of assumptions in the first-best, second-best, monopoly and constrained monopoly domains. The results are summarized in the following:

First let us consider the first best pricing regime.

In the case of the pure hydro system, the marginal cost of hydro-generation during wet off-peak period is just equal to the O &M costs per kwh involved, whereas the marginal cost per kwh during wet peak period includes, besides the former, the constant annuitized marginal (turbine) capacity cost also. In the dry off-peak period, marginal cost per kwh equals i) the unit O & M cost *plus* ii) the annuitized cost per kwh of storage capacity, weighted by the water availability factor. In contrast, the dry peak period price equals these two components (of the dry off-peak period price) *plus* the constant annuitized marginal (turbine) capacity cost.

In the case of a hydro-thermal system, the marginal generating cost should be equal at the optimum for both the plants. The marginal costs in the wet season (when hydro is continuously on base-load operation and thermal on the peak), yields an off-peak price in terms of i) the constant annuitized marginal (turbine) capacity cost of hydro *less* that of thermal plant (representing cost savings if thermal were used instead of hydro), *plus* ii) the O & M costs of hydro, (the sum being equal to thermal fuel costs). The peak price is obtained by adding to it, the marginal annuitized thermal capacity cost per unit.

The marginal cost per kwh in the dry off-peak period for this hybrid system equals i) the constant annuitized marginal (turbine) capacity cost of thermal *less* that of hydro plant (representing cost savings if hydro were used instead of thermal), *plus* ii) the thermal fuel cost. And in the peak period, the marginal cost per kwh will be higher by the marginal annuitized hydro capacity cost per unit.

The monopoly STD prices attach corresponding elasticity terms to the welfare prices. Thus these prices offer immense scope for price discrimination.

The Second best [42] prices equal the welfare marginal costs inflated with weights imposed by the profit level constraint as well as the price-elasticity of period demand.

Finally let us consider the constrained monopoly STD prices. For a hydro system, the off-peak prices (in both wet and dry seasons) are identical to those obtained for a profit-maximizing monopoly. The regulation affects only the peak prices. In the case of hydro-thermal system, the off-peak prices also include the capacity charges. Except for pure hydro off-peak periods, regulation sets marginal revenue less than marginal cost; and thus the period prices, except the hydro off-peak ones, under rate-of-return regulation are lower than those of an unconstrained profit maximizer.

These simple, static and deterministic rules appear to be well-adapted for less developed power systems, and in the face of inaccessibility of computerized dynamic models, capable of being applied to actual tariff estimation.

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