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RELIABILITY AND RATIONING COST: SOME ANALYTICAL IMPLICATIONS FOR A POWER SYSTEM

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ABSTRACT

The present paper attempts to analyze the implications of the relationship between reliability and rationing cost involved in a power supply system in the framework of the standard inventory analysis, instead of the conventional marginalist approach of welfare economics. The study is substantiated by fitting a normal distribution to the daily internal maximum demand of the Kerala power system during the four-year period from 2010-11 to 2013-14, and also by estimating, based on the techno-economic parameters of a 660 MW thermal power plant, the rationing costs implied in different reliability target criteria.

Keywords: Power supply, Maximum demand, Installed capacity, Reliability, Rationing cost, Normal distribution, Exponential distribution.

JEL Classification: C44, L94, Q41.

Contribution/ Originality

This study suggests a new approach to the relationship between reliability and rationing cost involved in a power supply system; it also contributes in the existing literature to the standard inventory analysis by linking it to reliability analysis.

1. INTRODUCTION

Reliability of a component (or system) in general refers to the probability that the component (or system) does not fail during any given interval (or, equivalently, it implies the probability that the component (or system) is still functioning at any given time). For instance, if the reliability of a system is given as 0.90, this means that the system, under normal conditions, will be functioning for 90 per cent of the time in any given interval. In a power supply system, reliability means the ability of the system to meet the demand for power at any given time. Most of the reliability criteria are measured in terms of the probability of failing to meet the expected demand, thus implying shortage of power.

Power shortage is experienced whenever demand (or load)for it exceeds its available supply, just like any other good or service. Besides random deviations of demand from expectations, many other factors (on the supply side) cause shortages, such as inadequate water level in the reservoirs, forced outages of generators, their scheduled preventive maintenance, and generator deratings. A power shortage or 'loss of load' occurs whenever load (L) exceeds available generating capacity (K). The probability of such shortage, or the expected fraction of time the available capacity fails to meet the demand, $P(L>K)$, is known as loss of load probability (LOLP). Another measure, known as loss of energy probability (LOEP), refers to the expected quantum of un-served energy, (i.e., energy shortage), as a ratio of mean demand.

It should be noted that LOLP implies the expected accumulated quantum of time in any given period during which available capacity falls below load. Thus for a one year period, LOLP expressed in terms of days per year is LOLP = $365 \times P(L>K)$. When expressed as a fraction of time, LOLP gives the probability that there will be a shortage of power (or loss of load) of any magnitude in a given period. Hence the name. It has been a common practice in some of the advanced countries to adopt in electricity supply an arbitrary reliability target, such as a one-day-in-ten-years LOLP [1]. Note that this does not suggest a full day of shortages once every 10 years; it rather gives the total accumulated time of shortages, expected not to exceed one day in 10 years, or, equivalently, 0.03 per cent of a day.

This traditional approach, however, considers only the peak hour of the days that have significant LOLP. Therefore a new metric, loss of load hours (LOLH), is now used that considers all hours during which there may be a risk of insufficient supply. However, the present study is based on the traditional measure of LOLP with its peak load pressing against the demand. It should be noted here that Telson [2] had criticized this reliability criterion of one-day-in-ten-years as too high from an economic standpoint, and suggested a five-day-in-ten-years LOLP (about 0.14 per cent of a day) as more appropriate. For some Asian and African countries (such as Korea, Hong Kong, Thailand, and Zimbabwe), LOLP criteria are found to vary from 12 to 24 hours per year, equivalent to five to ten days in ten years [3].

In the case of India, a reliability target planning criterion of 5 per cent (i.e., 18.25 days a year) was adopted in the First National Power Plan (NPP) in 1983 and in the Second NPP in 1987.This high level of LOLP target was justified considering the financial inability to undertake larger quantum of new capacity additions essential for a desirably low target. Subsequently, for the Third NPP in 1991, an LOLP target of 2 per cent (i.e., 7.3 days a year) was adopted in view of the general improvement in the technical performance parameters of generating plants, as well as the advent of new vintage larger size plants with higher efficiency. The same level was proposed to be retained during the 9th Five Year Plan (FYP) period in the Fourth NPP in 1997also, in view of the unexpected wide gap between the target and the achievement in capacity addition. Along with this, active participation of independent power producers (IPP) in terms of substantial capacity addition in the near future had also been expected. This hence facilitated to propose an improvement in the reliability target planning criterion further to 1 per cent (i.e., 3.65 days a year) LOLP by the end of the 10th FYP. The target for LOEP, however, was accepted for less than 0.15 per cent.

Stochastic nature of demand implies situations of capacity shortage or excess demand. The power shortage in turn necessitates rationing of the un-served electricity among the consumers. There are different rationing schemes, from simple rotating blackouts to sophisticated load shedding, and these have different effects on consumers' surplus. Besides such surplus loss, rationing has some administrative costs also. Summing these two yields the short run social rationing cost (or outage cost); the particular rationing scheme in force determines its magnitude.

The perfect load shedding scheme, proposed originally by Brown and Johnson [4] assumes costless rationing according to willingness to pay. Taking this as a base case, [5, 6] defined (incremental) rationing cost as any surplus losses and administrative costs, that are incurred over and above those under the Brown and Johnson scheme, and that depend only on the level of excess demand. Random rationing and rationing in order of lowest willingness to pay are other forms of rationing examined by Visscher [7] and Carlton [8]. The former case serves the consumers in random order until capacity is fully met, with no additional penalty costs involved. Our experiences of load shedding in India correspond with this scheme. With the latter, it is assumed that the consumers with the lowest willingness to pay may be willing to stand the longest in the queue for the service, and hence the rationing will be in order of lowest willingness to pay until capacity is exhausted, again with no social costs. The simplest rationing scheme (with the most convenient, linear, functional form) assumes that each unit demanded but not supplied involves a constant marginal outage cost [9]. This can be viewed as a special case of random rationing [10].

Despite the significance of rationing cost in determining both price and capacity at optimal level, its actual numerical estimates are very rare. One way of estimating it is from LOLP itself, as a certain level of rationing cost is implied in a given LOLP target [10-12]. The estimate of the rationing cost is obtained by maximising expected net social welfare (gross welfare less capital and operating costs less rationing cost); it varies inversely with the LOLP target chosen, and depends on the capital and operating costs of a given technology.

This paper seeks first to unravel the link between LOLP and LOEP, and then on the basis of this relationship, attempts to derive an estimator of rationing cost in general, and also in terms of the actual power demand distribution. We deviate from the conventional comparative static analysis of welfare economics, involving long and tedious derivations, and, instead, make use of the simple results on reliability from the standard inventory analysis – a novel approach, that surprisingly and most assuredly *yields the same results on rationing cost as the former method.* An attempt also is made to analyse the estimated relationship for its implications for the peak load pricing rule under uncertainty.

The next section of the paper discusses the most indices of LOLP and LOEP in the framework of the standard results of inventory analysis, in order to set up the relationship between them; and section 3 then applies these rules to power system reliability. Section 4 goes on to establish a relationship between outage cost on one hand and these two reliability criteria on the other. A numerical example is given to illustrate the implications of the relationship based on the techno-economic parameters of a representative thermal power plant. The final section briefly summarizes.

2. RELIABILITY INDICES

The excess demand situation in the case of any good or service is a major problem. If demand were uniform, supply could adequately be geared to the expected demand. However, random fluctuations in demand (L) about its mean value [$E(L) \equiv \mu$, where E is the expectation operator] involve some chances of shortage and necessitate a further safety margin or buffer or reserve (R). The standard inventory analysis therefore gives the supply or stock (S) as

$$S = \mu + R \qquad \text{.....(1)}$$

The reliability of service depends on the level of the reserves. With a small R come about excess shortages; and a large R requires excessive holding costs. Hence the significance of obtaining an optimum level of reserves.

The simplest and the most frequently used approximation of reserve levels is based only on the mean (μ) and the standard deviation (σ) of the demand distribution. Here R is equated to the demand deviation from its mean, ($L - \mu$), which in turn is set equal to some value Z_s times the standard deviation, i.e., $R = L - \mu = Z_s\sigma$; the value of Z_s is chosen such as to fix at some predetermined level, the probability that demand exceeds supply. Thus for the normal distribution, one σ demand deviation, ($Z_s = 1$) results in the expected demand exceeding supply for 15.9 per cent of the time; and 2σ deviation ($Z_s = 2$), for 2.3 per cent of the time only. In other words, if demand deviates from its mean by one σ point, then for 84.1 per cent of the time, we are confident, supply will meet demand. For other distributions, however, the risks are different. For instance, when $Z_s = 1$, the risk of the expected demand exceeding supply is 13.5 per cent in the case of exponential distribution, and 21.1 per cent with uniform distribution; when $Z_s = 2$, the risks are respectively 5 per cent and 6.7 per cent.

Thus the reserve margin required is defined to be equal to $Z_s\sigma$, and (1) becomes

$$S = \mu + R = \mu + Z_s\sigma. \qquad \text{..... (2)}$$

The probability of shortage (or LOLP) and hence the reliability of service varies with demand distribution. Suppose the demand for a good has a probability distribution function $f(L)$, the probability for the demand to lie between L and ($L + dL$). The probability of shortage, i.e., the probability that demand (L) will exceed available supply (S), is then given by

$$\text{LOLP} = \text{P}(L>S) = \int_S^\infty f(L)dL, \qquad \text{.....(3)}$$

and the expected amount of shortage (Q) is

$$Q = \text{E}(L-S) = \int_S^\infty (L-S)f(L)dL \text{ ...... (4)}$$

The reliability of service, (ρ), can be defined as the ratio of mean value of supply to mean value of demand, and is given by

$$\rho = (\mu - Q)/\mu = 1-(Q/\mu) = 1 - \text{LOEP}, \qquad \text{..... (5)}$$

where LOEP is the loss of energy probability (also called shortage factor) in power system reliability analysis, defined as LOEP = $\text{E}(L-S)/\text{E}(L) \equiv Q/\mu$.

This roughly corresponds to the unit loss function in inventory analysis. Usually it is given as Q/σ, rather than as Q/μ, that refers to our LOEP. Unit loss function, Q/σ, appears more as a convenient formulation, than as a definitional one, as, for example, the unit loss function under the normal distribution can thus be made equal to the term within the parentheses of equation (16) below, that is, independent of v, the demand variability, as σ is cancelled out in the formulation.

From (4) and (5), we have an inverse relationship at the margin between shortage/LOEP and available supply expressed through LOLP:

$$\frac{\partial Q}{\partial S} = \mu\frac{\partial}{\partial S}(LOEP) = -\int_S^\infty f(L)dL = -\text{LOLP}. \qquad \text{.....(6)}$$

Because $\frac{\partial Q}{\partial S}$ is negative,

$$\left|\frac{\partial Q}{\partial S}\right| = \left|\mu\frac{\partial}{\partial S}(LOEP)\right| = \text{LOLP}. \qquad \text{..... (7)}$$

We can also express the above (7),making use of (2), as

$$\left|\frac{\partial}{\partial Z_S}(LOEP)\right| = \text{LOLP}\,\sigma/\mu = \text{LOLP}\,v, \qquad \text{.....(8)}$$

where $v = \sigma/\mu$ is the coefficient of variation (CV) of demand.

Thus, LOLP may be defined as a marginal rise in shortage for a one unit fall in supply. Two (equivalent) implications follow from this.

i) It implies from (7) that LOLP may also be expressed as a fraction of the expected demand, the fraction being determined by the marginal change in LOEP for a unit change in the available supply; it also shows, for example, for a 10 per cent LOLP, that the marginal rise in LOEP associated with a one unit fall in supply is equivalent to 10 per cent of the inverse of the expected demand.

ii) LOLP may also be interpreted [from (8)] as a fraction of demand variability, the fraction being determined by the marginal change in LOEP with respect to the standardized supply; it also follows that for a 10 per cent LOLP, the marginal rise in LOEP for a unit fall in standardized supply corresponds to 10 per cent of the coefficient of variation of demand. The second implication is significant, as it establishes a direct relationship between LOLP and marginal change in shortage factor (LOEP) in terms of demand variability.

To evaluate the reliability criteria, we need to consider the actual demand distribution, since the risks are different for different distributions, as we have already seen. For illustration, we take up the distribution of the average daily maximum (peak) demand for electricity on the power system in the Kerala state of India during the four-year period from 2010-11 to 2013-14, presented in Table 1. When the demand series is distributed in suitable class intervals, the tail (lower and higher) values are found to be fewer in frequency, and hence we have tried to explore whether the data fit a normal distribution well. We have found that this closely approximates a normal distribution with mean = 2809 MW and standard deviation = 291.5 MW, giving a coefficient of variation (CV) of 10.38 per cent.

Note that for a long time, Kerala has been reeling under severe power shortage, and power cut/load shedding has become the rule of the day. The very low variability in the maximum demand distribution obtained here might be a reflection of the ironed-out pattern of the supply-constrained demand. Thus the very reliability of the data is in question, but we use the same just for illustration.

Hence below we consider the relevant properties of the reliability criteria in the context of the normal distribution.

2.1. Normal (Gaussian) Distribution

The normal distribution is given by

$$f(L) = \left(\sigma\sqrt{2\pi}\right)^{-1} exp\left[\left(-1/2\sigma^2\right).\left(L-\mu\right)^2\right]. \qquad \text{….. (9)}$$

Table-1. Distribution of the average daily maximum demand for electricity on the Kerala power system during the four-year period from 2010-11 to 2013-14

Maximum Demand (MW)	Actual Frequency	Theoretical Frequency
2000 – 2150	4	2
2150 – 2300	9	10
2300 – 2450	32	25
2450 – 2600	44	47
2600 – 2750	60	63
2750 – 2900	78	80
2900 – 3050	61	63
3050 - 3200	42	42
3200 – 3350	28	21
3350 – 3500	7	8
3500 - 3650	1	3

Mean = 2809.43 MW

Standard Deviation = 291.48 MW

Chi Square value = 7.80

Chi Square Critical value for 10 degrees of freedom at 5 per cent level = 18.31

Hence we cannot reject the null hypothesis that the data fit the theoretical distribution well.

Defining $Z = (L-\mu)/\sigma$, ….. (10)

wecan estimate the probability of shortage (LOLP) as

$$\text{LOLP} = P(L>S) = \left(\sqrt{2\pi}\right)^{-1}\int_{Z_S}^{\infty} exp\left(-Z^2/2\right)dZ = \phi(Z_s), \qquad \text{….. (11)}$$

which is a well-tabulated function, giving the area under the standard normal distribution.

Now let us consider the expected shortage. As explained above, the random fluctuations in demand (L) about its mean value μ involve some chances of shortage and necessitate a safety margin or reserve (R). Hence the stochastic demand can be expressed in terms of its deterministic equivalent as

$$L = \mu + R = \mu + Z_s\sigma. \qquad \text{….. (12)}$$

The expected shortage (L > S) is then

$$Q = \int_L^{\infty}(S-L)f(S)dS\ ,$$

where $f(S)$ is the probability density function (pdf) of normal distribution. Now substituting for L from above (12) and defining the standard normal variate $z = (S-\mu)/\sigma$, we get

$$Q = \sigma\int_{Z_S}^{\infty}(z-Z_S)\varphi(z)dz, \qquad \text{….. (13)}$$

where φ(z) is the probability distribution function of the standard normal distribution.

Now evaluating the first part of the above equation (13),

$$\int_{Z_S}^{\infty} z\varphi(z)dz = \frac{-1}{\sqrt{2\pi}} e^{-z^2/2}\Big|_{Z_S}^{\infty} = \frac{1}{\sqrt{2\pi}} \exp(-Z_S^2/2).$$

Thus we have the expected shortage

$$Q = \sigma\left(\sqrt{2\pi}\right)^{-1} exp\left(-Z_S{}^2/2\right) - \sigma Z_S\left(\sqrt{2\pi}\right)^{-1}\int_{Z_S}^{\infty} exp\left(-Z_S{}^2/2\right)dZ$$

$$= \sigma[\Psi(Z_s) - Z_s(\text{LOLP})], \qquad \text{…..} (14)$$

where $\Psi(Z_s) = \left(\sqrt{2\pi}\right)^{-1} exp\left[-Z_s^2/2\right],$ ….. (15)

is the height (ordinate) of standard normal distribution N(0, 1), again a well-tabulated function; and LOLP = $\phi(Z_s)$.

We can now express the shortage factor (LOEP) in terms of LOLP as

$$\text{LOEP} = \nu[\Psi(Z_s) - Z_s(\text{LOLP})], \qquad \text{…..} (16)$$

where $\nu = \sigma/\mu$ is the coefficient of variation (CV) of demand distribution (also see (8)). Note that the expected shortage Q is now given by μLOEP; also note LOEP < LOLP, and falls with demand variability. Thus, given the normally distributed maximum demand, we can estimate LOLP and LOEP for the corresponding standard variate, Z_s. But how shall we determine Z_s in the context of a power system? The Section 3 below discusses our method of estimation of Z_s.

Below we present, for comparative illustration, the results on the relationship between LOLP and LOEP in the case of exponential and uniform distributions.

2.2. Exponential Distribution

The exponential distribution is given by

$f(L)dL = \lambda\exp(-\lambda L)dL$,

where $\lambda = 1/\mu$, and standard deviation, σ, = μ (= $1/\lambda$).

The shortage probability (LOLP) in this case is

LOLP = P(L>S) = exp($-\lambda S$).

Now from (2) and (17), we get

LOLP = P(L>S) = exp($-\lambda S$) = exp[$-(1 + Z_s)$], ….. (18)

Along with the expected shortage,

$Q = \sigma\exp[-(1 + Z_s)]$.

From the above, we get the LOEP as

LOEP = $\nu\exp[-(1 + Z_s)]$ = ν LOLP = LOLP,

as $\nu = 1$; this implies that the reliability of service is not influenced by CV and LOEP = LOLP, in contrast to normal distribution case.

For exponential distribution, we have from (8) and (18):

$$\frac{\partial}{\partial Z_k}\ \text{LOEP} = -\,\text{LOLP} \ = -\exp[-(1 + Z_k)];$$

2.3. Uniform (Rectangular) Distribution

The uniform distribution is given by

$f(L)dL = 1/(L_1 - L_0)$, $L_0<L<L_1$

$= 0$ otherwise.

We have

$\mu = (L_1 + L_0)/2$, and

$\sigma = (L_1 - L_0)/2\sqrt{3}$

The limits on the distribution are

$$L_0 = \left(\mu - \sigma\sqrt{3}\right)$$

and

$$L_1 = \left(\mu + \sigma\sqrt{3}\right)$$

Considering the definition of Z_s implied in (2), we get $\text{LOLP} = \text{P}(L > S) = \frac{\sqrt{3}-Z_S}{2\sqrt{3}}$

Now directly from (2) we get

$$\text{LOLP} = \text{P}(L > S) = \frac{1}{2\sigma\sqrt{3}}(L_1 - S) \ldots\ldots(19)$$

Note that LOLP is *not* defined for values of Z_s greater than the square root of 3.

The expected quantum of shortage is

$$Q = \frac{1}{4\sigma\sqrt{3}}(L_1 - S)^2 = \frac{\sigma}{4\sqrt{3}}\left(\sqrt{3} - Z_s\right)^2$$

Then the LOEP is

$$\text{LOEP} = \frac{v}{4\sqrt{3}}(\sqrt{3} - Z_S)^2 = \frac{v}{2}\left(\sqrt{3} - Z_S\right)\text{LOLP}$$

Like the normal distribution, LOEP < LOLP, and falls with demand variability.

3. AN APPLICATION TO POWER SYSTEM RELIABILITY

This section seeks to apply the above results, derived using the standard inventory analysis, to an assess the reliability of a power system. In general, it is considered that the major safeguards against power supply shortages come in terms of sufficient redundancy and excess capacity in the system designed to meet any contingent exigency. While excess capacity in generation is taken care of in terms of buffer or reserve margins, possible fluctuations in load from its expectation (μ) are duly taken into account in determining new capacity to be installed to meet an expected peak load. If demand were uniform, installed capacity could correspond just to the expected maximum demand. However, the random deviations of demand as well as the day-to-day variations in the available capacity necessitate some reserve margin to account for them. This then suggests that in determining installed capacity required in a power system to ensure reliability in meeting the contingent demand deviations, due considerations be given to the expected maximum demand along with a certain reserve margin. Thus with 10 per cent reserve margin (PRM), installed capacity (K) equals 1.1 times the expected maximum demand (μ). That is,

$$K = \mu + R = \mu(1 + \text{PRM}). \qquad \text{..... (20)}$$

Comparing (20) with (2), we find that

$$\mu\,\text{PRM} = Z_k\sigma, \quad \text{or}$$

$$\text{Zk} = \text{PRM}\,\mu/\sigma = \text{PRM}/\nu, \qquad \text{...... (21)}$$

where $\nu = \sigma/\mu$ is the CV of maximum demand.

Thus the standardized variate, Z_k, that determines LOLP and reliability, as shown above, is in turn determined, in the reliability analysis of a power system, by both demand and supply factors, i.e., demand variability and PRM. We note three possible cases here:

i) PRM is just enough to contain demand variability, i.e., PRM = ν, then Z_k= 1, and for the normal distribution, installed capacity falls short of the expected maximum demand only for 15.87 per cent of the time; i.e., LOLP = 0.1587 [from (11)], LOEP = 0.083 times CV [from (16)], and the reliability of service is $\rho = 1 - 0.0833\nu$. For example, for the Kerala power system with ν = 10.38 per cent as during 2010-14, assuming PRM = ν, we have LOEP = 0.0086 and ρ = 0.9914, so that the expected peak power shortage (with μ = 2809 MW) is 24.29 MW only [from (14)and (16)]. It can also be seen that for every one unit drop in available supply, LOEP increases by 0.0086 per cent [from (7)].

ii) With PRM $> \nu$, we have $Z_k > 1$, that reduces the chances of shortage and raises the service reliability. For example, the installed capacity of the Kerala power system in 2013-14 was 2892 MW and the state's power quota from the central pool was 1536 MW; this, provided the Kerala system has undisturbed access to its share of central allocation, implies a PRM of more than 50 per cent of the mean maximum demand (2809 MW), and Z_k = 5.55, and hence LOLP and LOEP are just nil – evidently, the largesse of a very low demand variability.

iii) On the other hand, if the reserves are inadequate in relation to demand variability, i.e., PRM $< \nu$, then $Z_k < 1$, and the probability of demand exceeding available supply increases and reliability decreases. In this case, disregarding the Central share, Kerala's own installed capacity in 2013-14 (2892 MW), in relation to the mean maximum demand, gives a PRM of only 2.94 per cent, whereas the annual maximum demand (3268 MW, maximum of the daily peak loads), stood about 16 per cent above the mean maximum demand (and about 13 per cent above the own installed capacity). This necessitates that the Kerala system in 2013-14 must have had a PRM of at least 20 per cent to meet the maximum demand with an *available* capacity of nearly 4000 MW, implying Z_k = 1.93 and very high reliability (LOLP = 0.0269 and LOEP = 0.00107). If, on the other hand, the PRM were only 13 per cent, just enough to cover the peak load (of 3268 MW), then the implied LOLP would be 1.05 per cent (i.e., 0.01051, for Z_k = 1.25) and LOEP = 0.0052. However, in the actual situation, in relation to the annual maximum demand, we have Z_k= −1.109, and hence LOLP = 0.866 and LOEP = 0.122. Though the LOLP is very high, the reliability of service turns out to be higher (0.878), thanks to the very low demand variability. (If the CV were 25 per cent, LOEP would be about 30 per cent.) The significance of our formulation of the relationship (21) is very much evident from this discussion.

4. RELIABILITY AND RATIONING COST

Now that we have determined LOLP and LOEP, as also Z_k, in terms of PRM and ν, required in their estimation in the context of a normally distributed maximum demand in a power supply system, we now derive a relationship among rationing cost, and reliability (LOLP, LOEP) in electricity supply in the event of excess (maximum) demand. Instead of the conventional marginalist approach of analyzing the expected net social welfare, we seek to minimize the total cost made up of costs of capacity, output (generation) and shortage that yields exactly the same result as does the former. We assume the simplest rationing scheme, with a constant marginal penalty cost of excess demand, that is a special case of random rationing, that we usually experience in our country. Thus the rationing costs we estimate later at the end of this section approximate the reality. For sake of simplicity, again, we dispense with subscripts and symbols for diverse technology and periods.

From the preceding sections, we have the fraction of expected energy shortage, LOEP, as LOEP = Q/μ, giving the availability factor as (1 − LOEP). During the length (θ) of a certain period (i.e., peak period), the energy shortage is $\theta Q = \theta\mu$ LOEP units, involving a penalty price of 'r' per unit short. The energy available for supply, then, is $\theta\mu$ (1 − LOEP) units with an operating cost of 'b' per unit. The capacity cost is 'β' per kW of capacity, K. Then the total cost is:

$$\text{TC} = \beta K + b\,\theta\mu\,(1 - \text{LOEP}) + r\,\theta\mu\,\text{LOEP}.$$

It should be noted that the analysis can be extended to multiple period case also; for example, shortage and non-shortage periods as well as peak and off-peak periods. Only during the shortage duration (in peak period) is the third term in the above equation, $r\theta\mu$LOEP, active; and this is the only case we consider here.

We minimize the total cost to get

$$\frac{\partial}{\partial K}\,\text{TC} = 0 = \beta + (r - b)\,\theta\mu\frac{\partial}{\partial K}\,\text{LOEP}. \qquad \text{….. (22)}$$

Using the relation in (10) above, we get

$$Z = (L - \mu)/\sigma \rightarrow (K - \mu)/\sigma = Z_k,$$

Now we can rewrite (22) as

$$\beta + (r - b)\ \frac{\theta}{\upsilon}\frac{\partial}{\partial Z_k}\ \text{LOEP} = 0, \qquad \ldots..(23)$$

where $\nu = \sigma/\mu$.

$\frac{\partial}{\partial Z_k}$ LOEP, the rate of change in LOEP with respect to standardized capacity, depends on the particular demand distribution. Using (8) and the definition of LOLP in (11), we obtain for normally distributed demand:

$$\frac{\partial}{\partial Z_k}\ \text{LOEP} = -\ \nu\text{LOLP} = -\ \nu\phi\,(Z_k), \qquad \ldots..(24)$$

Now, from (23) and (24), we get LOLP as

$$\text{LOLP} = \phi(Z_k) = \beta/\theta(r - b). \qquad \ldots..(25)$$

Equation (25) presents a number of significant implications:

The denominator in (25) represents the net expected social cost of shortage. Each unit of power cut imposes a penalty of '*r*', but saves a marginal operating cost of '*b*'. As the net social cost of rationing increases, LOLP falls. The dynamics behind this is clear. As (non-price) rationing (in the event of excess demand) becomes more inefficient, rationing cost increases; this necessitates to place more reliance on price rationing (price rise) to reduce excess demand, which in turn leads to increased profits and capacity expansion. This in turn raises reliability also. Equation (25) shows that for any given LOLP target criterion, a certain level of rationing cost is implied in it. This then provides an estimate of rationing cost:

in general,

$$r = b + \beta/\theta\text{LOLP}, \qquad \ldots..(26)$$

Or, in particular, for normally distributed demand:

$$r = b + \beta/\theta\phi(Z_k), \qquad \ldots..(27)$$

where $Z_k = \text{PRM}/\ \nu$.

In the case of the exponential distribution, the rationing cost is:

$$r = b + \beta/\theta\exp[-(1 + Z_k)].$$

and for uniform distribution, it is:

$$r = b + \frac{\beta}{\theta}\frac{2\sqrt{3}}{\left(\sqrt{3} - Z_k\right)}$$

Note that r also is *not* defined for values of Z_s greater than the square root of 3.

The general expression, (26), for '*r*' that explicitly states the relationship between the rationing cost and LOLP, is a significant result in that it brings out an economic justification in setting an outage cost vis-à-vis an optimal reliability target planning criterion (LOLP); it also marks the effects on this reliability criterion of the assumptions of capital costs, generation costs and rationing costs; it is significant to note that the rationing cost derived here is the same as the one obtained by Chao [10] and [11, 12] under conditions of stochastic demand and conventional costs and ordering of n-technology in an expected social welfare maximizing model.

Equation (25) is significant again in that it sets the optimal marginal capacity cost as a fraction (equal to LOLP) of the net social cost of rationing implied in that level of LOLP. That is, for a 10 per cent LOLP, the marginal capacity cost corresponds to 10 per cent of the net rationing cost. Thus we obtain an optimal investment rule in shortage period. The denominator in (25) may also be interpreted as the net expected benefits from a unit increase in capacity. Each unit increase in capacity implies that with each unit of power cut avoided, a rationing cost of 'r' is escaped, but a marginal operating cost of 'b' is incurred. Thus the investment rule in (25) states that the marginal capacity cost (β/θ) equals the net expected benefits from a unit increase in capacity in shortage period times the probability of that period, i.e., $(r - b)$LOLP (Turvey and Anderson 1977: Chapter 14).

Another significant implication of (25) is its potential in yielding the stochastic equivalent of the deterministic peak load pricing rule, whereby peak period price equals $b + \beta/\theta$. Now substituting here for β/θ from (25), we have the stochastic pricing rule:

$$P = b(1 - \text{LOLP}) + r\text{LOLP}, \qquad \ldots.. (28)$$

a probability-weighted average of the marginal operating cost and the marginal rationing cost [9]. Since (1 – LOLP) is the probability of meeting demand and LOLP, that of power cut, the average price is applicable to both the situations. This is in contrast to the deterministic pricing rule that charges the off-peak customers only the marginal operating cost (b).

Compared with the peak-period price, rationing cost is much higher through the effect of LOLP on the unit capacity cost. If, for example, PRM is just sufficient to meet demand variability (v) such that $Z_k = 1$, then as we have already seen, LOLP for normal distribution is 0.1587; for exponential distribution, it is 0.1353 and for uniform distribution, 0.2113. Then from (26), we find that the capacity charge component of the rationing cost is about (1/0.1587 =) 6.3 times higher than that of peak-load tariff rate if demand follows normal distribution; it is about 7.4 times higher, if demand is exponentially distributed, and about 4.7 times higher for uniformly distributed demand. If PRM $> v$, then LOLP falls, with a much higher implied rationing cost; and if PRM $< v$, reliability falls, with a lower implied rationing cost.

A numerical example will illustrate the implications of the relationship between LOLP and rationing cost. Let us consider a 660 MW thermal power plant, and estimate the outage costs for different LOLP targets, as well as peak- and off-peak-period prices (representing generation cost only, and assuming a peak period of 4.5 hours a day, i.e., from 6 to 10.30 in the evening). The basic data are from the Tariff Regulations of the Central Electricity Regulatory Commission (CERC) of India for the five-year period 2009-14 [13].

(a) Capital cost : Rs. 3960 crores

Annuitized capital cost (at 12 per cent discount rate and for 25 years of plant life)

: Rs. 7650/kW/year

Marginal capital cost at peak demand (with 10 per cent transmission and distribution loss and 20 per cent reserve margin): Rs. 10200/kW/year.

(b) Fuel costs

Coal consumption norm : 6.38 kg/kWh

Price of coal : Rs. 2000/tonne

Oil consumption norm : 1 ml/kWh

Price of oil : Rs. 35000/kl

Total fuel cost : Rs. 1.315/kWh

(c) Operation and maintenance cost

(14.62 per cent of capital cost) : Rs. 1.001/kWh

(d) Total operating cost (off-peak price) : Rs. 2.32/kWh

(e) Peak period price : Rs. 8.53/kWh

(f) Average (accounting) price : Rs. 5.42/kWh.

(g) Rationing cost (Rs./kWh) with

10 per cent LOLP : 64.42

5 per cent LOLP : 126.52

2 per cent LOLP : 312.82

Five-day-in-ten-years LOLP : 4535.65

One-day-in-ten-years LOLP : 22668.98

5. CONCLUSION

The present paper employs a novel approach to power system reliability study by utilizing the results on reliability in the standard inventory analysis, making use of particular (normal) demand distribution for the average daily internal maximum (peak) demand of the Kerala power system during 2010-14. Thus the concepts of buffer stock, shortage probability and unit loss function are extended to power system reliability in terms of percentage reserve margin, LOLP and LOEP respectively. We find that the inverse relationship at the margin between LOEP and available supply corresponds to LOLP weighted by the inverse of the expected demand. It is found that in the case of normally distributed demand, LOEP < LOLP, and falls with demand variability.

Rationing cost involved in power shortage includes loss of consumers' surplus and cost of administering a certain rationing scheme. Minimizing the total cost incurred in power supply in a shortage period yields a significant inverse relationship between LOLP and rationing cost along with other (capacity and operating) cost components. Various implications of this relationship are examined, for optimal investment rule, stochastic version of peak load pricing, etc.

Rationing costs implied in different LOLP target criteria are also estimated, based on the techno-economic parameters of 660 MW thermal power plant. The assumption in our model of a random rationing scheme brings these estimates up as representing the actual penalty costs of the excess demand for power that we exert on the system in India. It is found that the rationing cost implied for a 10 percent LOLP is about 8 times higher than the peak-period price (representing generation cost only), and higher levels of reliability involve prohibitive tariff rates.

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