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PANEL DATA ESTIMATORS IN THE PRESENCE OF SERIAL AND SPATIAL CORRELATION WITH PANEL HETEROSCEDASTICITY: A SIMULATION STUDY

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ABSTRACT

Panel data analysis is often faced with the issue of errors having arbitrary correlation across time for a particular individual "I" (serial correlation) and/or errors having arbitrary correlation across individuals at a moment in time (spatial correlation) with error disturbances having non constant variance. This study examined some panel data estimators in the presence of serial and spatial autocorrelation with panel heteroscedasticity. The study was done using two different sets of data simulated separately with $\rho=0.95$ & 0.50. For each set of simulations short and long panels were considered for different sample sizes. The analysis considered two settings were rho is considered to be panel-specific (ρ_i) and where rho is considered to be common for all panels (ρ). The estimators were examined based on bias, overconfidence and relative efficiency. The results produced evidence that the size of the autocorrelation coefficient ρ affects the general performance of an estimator. Comparison of the estimators showed that Panel Corrected Standard Error Estimator (PCSE) produced better results than the other estimators considered in this work. But it was seen not to do very well in small samples and short panels. In terms of relative efficiency Park-Kmenta estimator was found to be more efficient that PCSE and PWLS (Panel Weighted Least Square Estimator). This paper has been able to show that the size of rho at the long run has an impact on the performance of the estimators, it showed that a small size of rho tends to increase overconfidence. The paper also revealed that Park-Kmenta estimator even with its flaws is still the most efficient estimator compared to the PCSE and PWLS. and it also substantiated the fact that PCSE performs badly in samples especially when $N>T$.

Contribution/Originality: This study is one of the few studies which have investigated the effect of the size of autocorrelation coefficient on the estimators based on the flaws of Parks-Kmenta estimator, which has mislead most researchers into becoming more inclined to use PCSE without considering others credits of Parks-kmenta estimator.

1. INTRODUCTION

Statistical methods can be characterized according to the type of data to which they are applied. In survey statistics the cross-sectional data describing each of many different individuals or units at a single point in time is usually used while in economic data analysis time series data describing a single entity over several period of time is used. The econometrics literature reveals another type of data called "panel data", which is the pooling of observations on a cross-section of households, countries, firms etc. over several time periods. Panel data simply refers to a cross section of observations (individuals, groups, countries, regions) repeated over several time periods. The Panel data compared with purely cross-sectional data or time series are more attractive in many ways. Since they often contain far more information than single cross-sections it allows for an increased precision in estimation and because the effects are also studied over several time periods, it creates the possibility of learning more about the dynamics of individual behavior. When the number of cross-sectional observations is constant across time periods the panel is said to be balanced. Economists and policy makers are frequently faced with the problem of drawing inferences from panel data. In such situations, it has become standard practice to base inferences on pooled regression. For such methods to be valid, the assumption that the error terms are not correlated across different cross-sectional units, either contemporaneously or at leads and lags must be met. This assumption is directly analogous to the usual requirement that the residuals from different observations in a single cross-sectional regression be independent of each other. If this condition is not met, estimates of standard errors will be inconsistent, and the inference will not be valid.

To ensure valid statistical inference when some of the underlying regression model's assumptions are violated, relying on robust standard errors is common. Cameron and Trivedi (2005) stated that correlated observations have less information than independent observations. Therefore, erroneously ignoring possible correlation of regression disturbances over time and between subjects can lead to biased statistical inference. Most recent studies that include a regression on panel data therefore adjust the standard errors of the coefficient estimates for possible dependence in the residuals in order to ensure validity of the statistical results. Furthermore, the standard error components model has been extended to take into account serial correlation (see (Bera, Sosa-Escudero, & Yoon, 2001; Galbraith & Zinde-Walsh, 1995; Hong & Kao, 2004)). This model has also been generalized to take into account heteroscedasticity by Baltagi and Griffin (1988); Lejeune (2006); Holly and Gardiol (2000); Roy (2002) and Baltagi.., Bresson, and Pirotte (2006). In an early attempt to account for heteroscedasticity as well as for temporal and spatial dependence in the residuals of panel models, Parks (1967) proposed a feasible generalized least-squares (FGLS)–based algorithm that later became popular through (Kmenta, 1986). However, the Parks–Kmenta method was found to be infeasible if the time period is smaller than its cross-sections because obtaining a nonsingular estimate of the $N \times N$ matrix of cross-sectional covariance when $T < N$ is impossible and secondly, Beck and Katz (1995) showed that the Parks–Kmenta method tends to produce unacceptably small standard error estimates. In other to mitigate this problem of the Parks–Kmenta method; Beck and Katz (1995) suggest relying on OLS coefficient estimates with panel-corrected standard errors (PCSEs). Beck and Katz (1995) convincingly demonstrate that their large-T asymptotic– based standard errors, which correct for contemporaneous correlation between the subjects, perform well in small panels. Nevertheless PCSE estimator are rather poor when the panel's cross-sectional dimension N is large compared to the time dimension (Moundigbaye, Rea, & Reed, 2018) in their work stated that when it comes to choosing an estimator for efficiency, it uses the size of the panel dataset (N and T) to guide the researcher to the best estimator. They also showed that when it comes to choosing an estimator for hypothesis testing, it identifies one estimator as superior across different data. In cross-sectional setting, Romano and Wolf (2017) obtained asymptotically valid inference of the FGLS estimator, combined with heteroscedasticity consistent standard errors without knowledge of the conditional heteroscedasticity functional form. Moreover, Miller and Startz (2018) adapted machine learning methods (i.e., support vector regression) to take into account the mis-specified form of heteroscedasticity. However, these strands of literature are almost separate in the panel data error components literature. An unusual finding in most literature of panel regression showed that when one deals with heteroscedasticity, serial correlation is ignored, and when one deals with serial correlation, heteroscedasticity is ignored (Baltagi., 2005). Again, most researchers often neglect the role of the size of the autocorrelation coefficient during simulations, hence neglecting a right guide to the best estimator to use; in fact little has been done to investigate the effect of the size of autocorrelation coefficient on the estimators, from literature we observed that there could be a level of effect that the size of the rho could have on the estimate, and we also noticed that because of the work of Beck and Katz (1995) on the flaws of Parks-Kmenta estimator, some researchers seem to become more inclined to use PCSE without considering others credits of Parks-kmenta estimator, which in recent studies is very relevant. The review of literature has it that PCSE yields poor results when N>T and these issues stated above led to our quest to investigate the size of autocorrelation coefficient on estimators using a simulation study. In this paper therefore we investigated the effect of the size of autocorrelation coefficient rho on the estimators and made a comparison of the estimators, namely the Panel Corrected Standard Error estimator, Parks-Kmenta estimator and Panel Weighted Least Square estimator based on their bias and overconfidence in the presence serial and spatial autocorrelation with panel heteroscedasticity. The study also looked at the relative efficiency of these estimators. We investigated the notion that PCSE does very badly when the number of cross sections is larger than the time index. This we did using two different ratios of N:T (number cross sections: time period) for both large and small sample sizes.

The rest of the paper is structured as follows: section 2 introduces the model overview and explanation of comparison factors, section 3 presents the method of analysis, section 4 contains the process of simulation, section 5 presents the results and discussion and section 6 concludes the paper.

2. MODEL OVERVIEW

The general model for panel data permits the intercept and slope coefficient to vary over both individual and time, with the model given as;

$$y_{it} = \alpha_{it} + X_{it}'\beta_{it} + e_{it}, \; i = 1, \dots N \; and \; t = 1, \dots, T. \qquad (1)$$

Our methodological approach is to pool cross-sectional time series. This technique incorporates both the cross-sectional effect of the independent variables as well as the time-series effects. The critical assumption of pooled cross-sectional times series models is that of "pooling". That is, all units are characterized by the same regression equation at all points in time, hence we consider a restricted model given by;

$$y_{it} = X_{it}\beta_{it} + e_{it} \qquad i = 1,\dots,N; \; t = 1,\dots,T \qquad (2)$$

where y_{it} and x_{it} are observations for the ith unit at time t and β is a vector of coefficients and $e_{it}$is the residual with the usual properties (mean 0, uncorrelated with itself, uncorrelated with x, and homoscedastic).

$e_{it} = u_i + v_{it}$, we assume, $u_i = 0$

for common rho we have that the error is modelled as first-order auto regression.

$$e_{it} = \rho e_{it-1} + v_{it} \qquad (3)$$

where $v_{it} \sim iidN(0, \sigma_{vi}^2) and \; e_{i0} \sim N(0, \sigma_{vi}^2 / 1 - \rho^2)$, while ρ is the autocorrelation coefficient/parameter.

And for varying rho we have, $e_{it} = \rho_i e_{it-1} + v_{it}$. (4)

2.1. Paris-Winsten Corrected Robust Standard Errors

The generalized least square can be used to estimate Equation 2 irrespective of any complexities of the error process so far as the error covariance matrix (Ω) is known. The generalized least square is fully efficient and yields consistent estimates of the standard error. The GLS procedure involves transforming of the model with the general error covariance matrix to another linear equation where error covariance matrix is suitable for ordinary least square estimation.

The GLS estimate for the regression coefficient is then given by;

$$\beta = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y \quad (5)$$

And it requires the error covariance matrix, but unfortunately it is never known in practice so it is usually estimated, the process that uses the estimated error covariance matrix is known as feasible generalized least square.

Prais –Winsten FGLS follows this process and estimates its rho ρ using the linear regression model in Equation 3 which is implemented using the sample residuals e_i from OLS regression, Paris then uses the estimated rho to transform the data. The Prais-Winsten estimator transformation proceeds thus;

$$y_{it}^* = \begin{bmatrix} \sqrt{1-\hat{\rho}^2}y_{i1} \\ y_{i1} - \hat{\rho}y_{i2} \\ \vdots \\ y_{iT} - \hat{\rho}y_{iT} \end{bmatrix} \text{and } x_{it}^* = \begin{bmatrix} \sqrt{1-\hat{\rho}^2}x_{i1} \\ x_{i1} - \hat{\rho}x_{i2} \\ \vdots \\ x_{iT} - \hat{\rho}x_{iT} \end{bmatrix}$$

This transformation takes care of the serial autocorrelation.

2.2. Panel Weighted Least Square

The method of weighted least squares can be used when the ordinary least squares assumption of constant variance in the errors is violated. Panel weighted least square differs by incorporating a weight with each data point. The weighted least estimate of the regression coefficient is given by;

$$\beta_{wls} = (X'WX)^{-1}X'WY$$

Where $w_i = \frac{1}{\sigma_i^2}$ this process addresses the heterogeneity problem.

2.3. Parks-Kmenta Estimator

The Parks method is a FGLS estimator for panel models where their errors have panel heteroscedasticity and exhibits spatial and serial correlation. The method is in two steps; firstly the serial correlation is eliminated after which the spatial correlation is eliminated. The process obtains residual from the OLS estimates; this residual is then used to estimate the panel specific correlation coefficient which in turn is used to transform the model into one with serial independent errors. Secondly the residuals from these estimates are then used to estimate the spatial correlation coefficient which again is transformed to allow for OLS estimation with independent errors that have constant variance. In this situation with panel heteroscedasticity, panel specific AR(1) autocorrelation, and time-invariant cross-sectional correlation, the classic Parks-Kmentan estimator has a total of $\left(\frac{N^2+3N}{2}\right)$ unique parameters in the error variance-covariance matrix (EVCM), given that N is the number of cross-sectional units. The Park-Kmenta Feasible Generalized Least Square estimator cannot be estimated when the number of time periods, T, is less than N, because the associated error variance-covariance matrix (EVCM) cannot be inverted, that is, the problem of singularity. Also in this situation where $T \geq N$, there may be relatively few observations per error variance-covariance matrix (EVCM) parameter, causing the associated elements of the error variance-covariance matrix (EVCM) to be estimated with great imprecision, Moundigbaye et al. (2018).

2.4. Panel Specific Corrected Errors Estimator (PCSE)

Beck and Katz (1995) addressed the problem Park-kmenta estimator, by proposing a modification of the full Park-Kmenta Feasible generalized Least square estimator called Panel-Corrected Standard Errors (PCSE). The Panel-Corrected Standard Errors (PCSE) preserves the Prais-Winsten weighting of observations for autocorrelation, but uses a sandwich estimator to incorporate panel dependence when calculating standard errors. The Panel-Corrected Standard Errors (PCSE) estimator has proven very popular, as evidenced by many citations in Web of Science. All of this has opened up a myriad of choices for applied researchers when it comes to choosing a panel data estimator. The PCSEs thus corrects for panel heteroscedasticity and spatial correlation. When panel heteroscedasticity and spatial correlation are present, the Ordinary Least Square estimates are inefficient and its standard errors are inaccurate. Accurate estimation of the variability of the Ordinary Least Square estimates can only be achieved if the standard error is corrected. This correction takes into account spatial correlation and panel heteroscedasticity of the errors, but any form of serial correlation has to be taken care of before the panel corrected specific errors are calculated.

The variance of the OLS estimate is given by;

$$Var(\hat{\beta}) = (X'X)^{-1}\{X'\Omega X\}(X'X)^{-1} \tag{6}$$

In the presence of panel heteroscedasticity and spatial correlation Equation 6 above yields inaccurate standard errors. The model in Equation 2 has an NT x NT error covariance block diagonal matrix with an N x N matrix of spatial covariance $'\Sigma'$.

Let E denote the matrix of the OLS residuals, Σ is estimated thus;

$$\hat{\Sigma} = \frac{(E'E)}{T} = \frac{1}{T}\begin{bmatrix} \sigma_{e,11} & \sigma_{e,12} & \cdots & \sigma_{e,1N} \\ \sigma_{e,21} & \sigma_{e,22} & \cdots & \sigma_{e,2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{e,N1} & \sigma_{e,N2} & & \sigma_{e,NN} \end{bmatrix} \tag{7}$$

The estimate $\hat{\Sigma}$ is used for transformation and hence Ω is estimated thus;

$$\hat{\Omega} = \frac{(E'E)}{T} \otimes I_T = \frac{1}{T}\begin{bmatrix} \sigma_{e,11} & \sigma_{e,12} & \cdots & \sigma_{e,1N} \\ \sigma_{e,21} & \sigma_{e,22} & \cdots & \sigma_{e,2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{e,N1} & \sigma_{e,N2} & & \sigma_{e,NN} \end{bmatrix} \otimes \begin{bmatrix} 1 & \rho & \cdots & \rho^{T-1} \\ \rho & 1 & \cdots & \rho^{T-2} \\ \vdots & \rho & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \cdots & 1 \end{bmatrix} \tag{8}$$

Where $\otimes$ is the kronecker product. $\hat{\Omega}$ incorporates panel heteroscedasticity, time-invariant cross-sectional dependence, and first-order, common autocorrelation.

Hence, $$Var(\hat{\beta}) = (X'X)^{-1}X'\left(\frac{(E'E)}{T} \otimes I_T\right)X(X'X)^{-1} \tag{9}$$

PCSEs are thus computed by taking the square root of the diagonal elements of Equation 9.

3. METHODOLOGY & ANALYSIS

This article compares the performance of three different estimators using the panel pooled method (restricted model) under two settings. The first setting is where autocorrelation coefficients are panel-specific and the second setting is where the autocorrelation coefficient is common among the panels. The three different estimators considered are; panel-corrected standard errors (PCSEs), Parks-Kmenta Feasible Generalized Least Square estimator and Panel weighted least squares estimator. First data are generated with serial correlation and heteroscedastic disturbances. Then serial autocorrelation was addressed via Prais-Winsten feasible generalized least squares (FGLS) procedure and panel heteroscedasticity, was addressed using the Panel weighted least squares estimator and generalized least square method by Parks. Secondly the simulated data with panels that both heteroskedastic and contemporaneously correlated were also analyzed using the panel-corrected standard errors (PCSEs) after serial correlation has been eliminated. The performances of these estimators were investigated by looking at their bias and their accuracy in predicting variability (overconfidence). We considered situations where the time period is larger than the number of cross-sections (T>N) and situations where the cross-sections is larger than the time period (N>T). Unfortunately for N larger than T, generalized least square method by Park-Kmenta is not feasible because it renders the estimate of the variance covariance matrix singular. So for that, we considered only the panel weighted least square estimator and the Panel Corrected Standard Error estimator in that instance. We also investigated the relative efficiency of the three estimators.

3.1. BIAS

Bias can be defined as the tendency of a statistic to overestimate or underestimate a parameter. Bias can arise for a number of reasons including failure to respect either comparability or consistency, in data collection, statistical procedure followed and the calculation and aggregation formula employed. Holton (2014) defined bias of an estimator H, as an expected value of an estimator less than the theta being estimated. And he also defined standard error of an estimator as its standard deviation. Most times two standard performance measures for assessing accuracy of an estimator are bias and mean square error (MSE). Bias and MSE are denoted as;

$$bias(\hat{x}) = E\{\hat{x}\} - x \qquad (10)$$

$$MSE(\hat{x}) = E\{(\hat{x} - x)^2\} \qquad (11)$$

It is desired that $bias(\hat{x}) = 0 \; or \; E\{\hat{x}\} = x,$ indicating that the estimator is unbiased and MSE is as small as possible. For an unbiased estimator, MSE is equal to variance. Bias can be measured based on the mean square error of an estimate. Mean square error combines the notion of bias and standard error and it follows thus;

$$VAR(\hat{x}) = E\{(\hat{x} - E\{\hat{x}\})^2\}$$

$$= E\{(\hat{x} - x)^2\}$$

$$= MSE(\hat{x}) \qquad (12)$$

In general the mean squared error of an estimator decomposes as a sum of the bias squared and the variance

$$MSE(\hat{x}) = VAR(\hat{x}) +^2 (bias(\hat{x}))^2 = (standard\ error)^2 + (bias)^2 \qquad (13)$$

$$\therefore \sqrt{MSE} = RMSE = stndard\ error + bias \qquad (14)$$

We can deduce that;

$$Bias = RMSE - standard\ error \qquad (15)$$

3.2. Overconfidence

The quality of the estimate of variability of the regression coefficient can be accessed by comparing the root mean square of the standard errors with the corresponding standard deviation of the estimates. The quality of this estimate of variability is determined by calculating the overconfidence. Overconfidence can also be defined as the level by which variability is understated. Formally, we say that overestimation is exhibited if E[Xi|yi] > xi and underestimation is exhibited if E[Xi|yi] < xi. The measure of accuracy of the estimators that is the ability not to overestimate or underestimate is considered in this work. This measure of accuracy is determined using bias and overconfidence. Overconfidence which is the percentage by which an estimator understates variability is given by;

$$\text{Overconfidence} = 100 * \frac{RMSE}{Standard\ Deviation} \tag{16}$$

while RMSE is the root mean square root of an estimate.

Relative Efficiency: In general the relative efficiency is a function of θ, and so some estimators will have lower MSE than others for some values of θ but not for other values of θ. In some cases, however, the relative efficiency does not depend on θ but points at a clear advantage of one estimator over another in terms of the MSE. Relative Efficiency can be calculated thus;

$$RE = \frac{var(\theta_1)}{var(\theta_2)} \tag{17}$$

$if\ we\ have\ \frac{var(\theta_1)}{var(\theta_2)} = \frac{n+2}{3}$, if n > 1, then θ_2 is has a lower variance thus more efficient than θ_1, $where\ var(\theta_1) is\ the\ variance\ of\ \theta_1\ and\ var(\theta_2)\ is\ the\ variance\ of\ \theta_2$, ((Nwakuya & Nwabueze, 2016).

4. SIMULATIONS

All simulated data were generated to mimic panel data properties, using the model in Equation 2. we use sample sizes T = 80, 30, 9 and N = 20, 10, 2, for (N<T) and T = 20, 3, and N = 200, 6, for case of (N>T).The setup were similar for a given N and T to reflect long or short panel. The values of the independent variable X_{it} were generated as independent normally distributed random variables with constant mean and variance. The values were allowed to differ for each cross-sectional unit and it was held fixed for all simulations. The errors were generated as AR(1) process given in Equation 4 from a normal distribution with zero mean and variance. The simulation was done using rho=0.95 & 0.5 and we considered two settings; where rho is panel specific and where rho is constant.

5. RESULTS & DISCUSSIONS

Tables 1-6 present the simulation results at different level of rho, ρ = (0.95, 0.5) where rho is high and constant. We also considered both short and long panels at different sample sizes. In each table, the root mean square error (RMSE), standard deviation (St. D), bias and overconfidence (OC) of the estimators are reported. PCSE refers to Panel Specific Corrected Errors Estimator, PARK is Parks-Kmenta Estimator while PWLS refers to Panel Weighted Least Square. We considered each simulation for Panel-Specific Autocorrelation of Order 1 (PSAR1) and Common Autocorrelation of Order 1 (AR 1).

Table-1. Simulated data Results with rho = 0.95 for T > N.

T	N	Panel-specific Autocorrelation of order 1 (PSAR 1)				Common Autocorrelation of order 1 (AR1)			
			PCSE	PARK	PWLS		PCSE	PARK	PWLS
		RMSE	1.1361	1.1362	1.1361	RMSE	1.1360	1.1360	1.1360
80	20	St. D	1.2422	1.2461	1.2435	St. D	1.2471	1.2495	1.2478
		Bias	-0.1061	-0.1099	-0.1074	Bias	-0.1111	-0.1135	-0.1118
		OC	91.4587	91.1804	91.3630	OC	91.09	90.91	91.04
30	10	RMSE	1.0416	1.0418	1.0410	RMSE	1.0413	1.0414	1.0414
		St. D	0.9806	0.9790	0.9770	St. D	0.9771	0.9711	0.9751
		Bias	0.0609	0.06280	0.0639	Bias	0.0642	0.0703	0.0659
		OC	106.221	106.414	106.55	OC	106.57	107.24	106.76
9	2	RMSE	3.563	3.858	3.573	RMSE	3.544	3.818	3.549
		St. D	4.5679	3.9532	4.514	St. D	4.8621	4.420	4.9832
		Bias	-1.0049	-0.0952	-0.941	Bias	-1.3181	-0.602	-1.4342
		OC	78.00	97.59	79.15	OC	72.89	86.38	71.22

Table 1 above shows the simulated results with rho = 0.95 for T > N (we considered rho to be high). The results show that the bias of all estimators are lower when the rho is panel-specific than when rho is common. All the estimators has less than 100% overconfidence and there is presence of underestimation due to negative bias when T=80 and N=20 and also for T=9 and N=2, but for T=30 and N=10 there was overestimation (positive bias) with more than 100% overconfidence for all estimators. Comparison of accuracy between the three estimators shows that Panel Corrected Standard Error has the least bias except for the small sample where T=9 and N=2. We can conclude that the estimators perform generally better in terms of bias when the rho is panel specific and the only over predicted variability in the case where there is overestimation (T=30 and N=10). We also conclude that Panel Corrected Standard Error produces not to good result in small samples but performs better than the other estimators in large samples.

Table-2. Simulated data Results with rho = 0.95 for N > T (The number of cross section is higher the time period).

T	N	Panel-specific Autocorrelation of order (PSAR1)			Common Autocorrelation of order 1 (AR1)		
			PCSE	PWLS		PCSE	PWLS
		RMSE	1.1633	1.163	RMSE	1.162	1.162
20	200	St. D	0.8449	0.8232	St. D	0.9039	0.8841
		Bias	0.3184	0.3398	Bias	0.2581	0.2779
		OC	137.67	141.28	OC	128.55	131.43
3	6	RMSE	1.156	1.1523	RMSE	1.1584	1.156
		St. D	0.7312	0.7960	St. D	0.5148	0.8795
		Bias	0.4248	0.3563	Bias	0.6436	0.2763
		OC	158.09	144.76	OC	225.02	131.442

Table 2 above investigated only the Panel Corrected Standard Error and Panel Weighted Least Square because PARK-Kmenta estimator is not feasible when the N>T due to singularity of matrix. The table shows that PCSE performed better in large sample with respect to bias and overconfidence for both panel-specific rho and common. While for small sample the PWLS performed better. We noticed that PCSE showed more than 200% of overconfidence for common rho and over 150% of overconfidence for panel-specific rho in small sample. We can conclude the PCSE does not do well in small samples just like we saw in Table 1.

Table 3 above shows the simulated results with rho = 0.5 for T > N. The results shows that the bias for all the estimators is higher when the rho is panel specific than when the rho is common. But in small sample the estimators produce interesting result, here the bias of all the estimators are lower for panel-specific rho than common rho.

Table-3. Simulated data Results with rho = 0.50 for T > N.

T	N	Panel-specific Autocorrelation of order 1 (PSAR 1)				Common Autocorrelation of order 1 (AR1)			
			PCSE	PARK	PWLS		PCSE	PARK	PWLS
		RMSE	1.3042	1.3312	1.3087	RMSE	1.2775	1.3177	1.2881
80	20	St. D	0.2302	0.0862	0.1950	St. D	0.4575	0.1590	0.3503
		Bias	1.074	1.2451	1.1137	Bias	0.8200	1.1587	0.9378
		OC	566.69	1546.11	671.03	OC	279.24	828.69	367.71
30	10	RMSE	1.3022	1.3123	1.3127	RMSE	1.2816	1.3022	1.2974
		St. D	0.2274	0.1411	0.1904	St. D	0.3509	0.1407	0.3174
		Bias	1.0748	1.1712	1.1223	Bias	0.9307	1.1616	0.9800
		OC	572.72	930.03	689.59	OC	365.23	925.80	408.81
9	2	RMSE	1.0388	0.9950	1.0018	RMSE	0.9683	0.9861	0.9980
		St. D	0.4996	0.4119	0.4299	St. D	0.2628	0.1108	0.09577
		Bias	0.5392	0.5831	0.5719	Bias	0.7055	0.8753	0.9022
		OC	207.92	241.52	232.99	OC	368.44	890.09	1042.21

All estimators showed an alarming overconfidence with PWLS having overconfidence over 1000% in small sample for common rho and PARKS having over 1500% of overconfidence in large sample for panel-specific rho. On comparison the PCSE performed better than other estimators showing the lowest bias and overconfidence. We can conclude that the value of rho has effect on the performance of the estimators.

Table-4. Simulated data Results with rho = 0.50 for N > T.

T	N	Panel-specific Autocorrelation of order 1 (PSAR 1)			Common Autocorrelation of order 1 (AR1)		
			PCSE	PWLS		PCSE	PWLS
		RMSE	1.3366	1.3363	RMSE	1.3183	1.3269
20	200	St. D	0.3013	0.3062	St. D	0.4474	0.3772
		Bias	1.0353	1.0301	Bias	0.8709	0.9497
		OC	443.58	436.36	OC	294.66	351.82
3	6	RMSE	1.3018	1.1045	RMSE	1.1249	1.1727
		St. D	0.3956	0.2934	St. D	0.2679	0.4544
		Bias	0.9062	0.8111	Bias	0.8570	0.7183
		OC	329.09	376.48	OC	419.81	258.06

Table 4 shows results for N>T for rho=0.50. This result shows that bias of PCSE and PWLS are higher for panel-specific rho than that of common rho in the two samples. Comparison on level of accuracy we can conclude that PCSE did not perform very well. This result agrees with literature that PCSE produces bad results when N>T.

Table-5. Relative Efficiency for Simulated data with rho=0.95.

		Relative Efficiency for panel-specific rho			Relative Efficiency for common rho		
T	N	PCSE Vs PWLS	PWLS Vs PARK	PCSE Vs PARK	PCSE Vs PWLS	PWLS Vs PARK	PCSE Vs PARK
80	20	1.0021	1.0041	1.0063	1.0011	1.0027	1.0038
30	10	0.9927	1.0041	0.9967	0.9959	0.9918	0.9877
9	2	0.9791	0.7669	0.7489	1.0504	0.7867	0.8264
20	200	0.9492	**************	**************	0.9566	**************	**************
3	6	1.1851	**************	**************	2.9187	*************	**************

Table 5 indicates that PARK was more efficient compared to the other estimators when T>N, except when T=80 and N=20 where PCSE showed more efficiency compared to the others.

Table-6. Relative Efficiency for Simulated data with rho=0.50.

		Relative Efficiency for panel-specific rho			Relative Efficiency for common rho		
T	N	PWLS Vs PCSE	PWLS Vs PARK	PCSE Vs PARK	PWLS Vs PCSE	PWLS Vs PARK	PCSE Vs PARK
80	20	0.7175	0.1954	0.7175	0.5865	0.2060	0.1207
30	10	0.7010	0.5491	0.3850	0.8181	0.1965	0.1607
9	2	0.7404	0.1328	0.9180	1.3385	0.6797	0.1777
20	200	1.4435	*************	*************	0.7180	*************	*************
3	6	0.5500	*************	*************	2.8769	************	*************

Table 6 still shows that PARK is more efficient compared to PCSE and PWLS. For the case of N>T where N=200 and T=20, PCSE was more efficient than PWLS for panel-specific rho while PWLS was more efficient than PCSE for common rho, but for N=6 and T=3, the reverse is the case. The relative efficiency results indicate that PARK is more efficient relative to PCSE and PWLS.

6. CONCLUSIONS

In conclusion, it was deduced that overestimation leads to over prediction of variability while underestimation leads to under prediction of variability. The results pointed out that the size of rho has effect in the performance of the estimators. The data simulated using rho=0.95, was observed in Table 1 to have lower bias for panel-specific rho than that of common rho but when data was simulated using rho=0.50, the bias of the panel-specific rho became bigger than that of common rho except in small sample. Also the overconfidence with rho=0.95 was much lower than the one with rho=0.50, based on this we can say that the lower the rho the more variability is over predicted, giving rise to more overconfidence. This observation makes us conclude that the size of rho has a major effect on the performance of the estimators and researchers should take note of that. The comparison of the three estimators based on bias and overconfidence showed that Panel Corrected Standard Error performed better than the rest except in small samples and situations where N>T, this led us to conclude that Panel Corrected Standard Error (PCSE) does not do well in small samples and when the number of cross sections is larger than the time index. The relative efficiency results showed that Park-Kmenta estimator is more efficient than Panel Corrected Standard Error and Panel Weighted Least Square. Our contribution to knowledge at this point is that, we have shown that in simulation of auto correlated with panel heteroscedastic data, the size of rho is of great importance, also we have shown that Park-Kmenta FGLS estimator is efficient relative Panel Corrected Standard Error estimator and Panel Weighted Least Square estimator, but in terms of general performance Panel Corrected Standard Error estimator was seen to better than others.

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