



THE EFFECTS OF DIFFERENT SCORING SYSTEMS ON UPSET PERCENTAGE AND MATCH LENGTH IN TENNIS: A SIMULATION STUDY

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ABSTRACT

Tennis has used many different scoring systems throughout its history. Researchers have been examining these scoring systems for over forty years. While several intuitive themes have emerged, no systematic examination on how different scoring systems affect the stronger player's probability of winning over a range of serving strengths has been done. This research study used simulation to examine the three most common scoring systems used in the United States and their effect on match win probability, total points played, and expected length of the match. Moving from ad to no ad or 10-point scoring can result in a more than 3.5% and 5% respective decrease in the probability of the stronger player winning a match. When applied to NCAA Division I team tennis, moving from ad to no ad can decrease the stronger team's probability of winning by over 4%. Changes between scoring systems are more pronounced at relatively weaker serving strengths.

Keywords: Discrete event simulation, Tennis scoring, Tennis match length, Upset probability, NCAA tennis, Skill difference.

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Contribution/ Originality

This study formally quantifies, through a systematic design of experiment, how different scoring systems affect upset percentage, match length, and match duration in tennis, thus extending upon earlier work which touched upon these aspects in a less comprehensive and quantitative manner.

1. INTRODUCTION

Based on year-end 2014 numbers, approximately 17.9 million people play tennis in the United States, with youth and teenage participation growing to 2.14 million and 2.23 million, respectively (Tennis Industry Association, 2015). When considering certain metrics, tennis is the 4th most popular sport in the world, behind soccer, basketball, and cricket (Total Sporttek, n.d). Scoring systems have evolved throughout the sport's history.

For a complete summary of tennis scoring, refer to the International Tennis Federation's (ITF) "Rules Tennis 2016" (International Tennis Federation, 2016). A tennis match consists of nested points, games, and sets. Games consist of points, sets consist of games, and a match consists of sets. In the traditional advantage or "ad" game scoring, a game is won when a player wins four points and the margin of difference of points between the players is at least two (4-0, 4-1, or 4-2). If both players reach three points, then the game continues until one player has won two more total points than the other (5-3, 6-4, etc.). In "no ad" scoring, if the game score reaches three points apiece, a sudden death 7th point occurs where the receiver chooses which side the server will serve from and the winner of that point wins the game. Players rotate serving between the deuce and ad courts within games and also rotate who serves each game.

The non-tiebreak (also referred to as "classical" or advantage) set is won when a player wins six total games and the margin of difference between games is at least two (6-0, 6-1, 6-2, 6-3, or 6-4). If the game score reaches 5-5,

then the set continues until the margin of difference is two games (7-5, 8-6, etc.). To combat the sometimes excessively long sets that occur in this format, the tiebreaker gradually came into play during the 1970s. In tiebreak sets, a 12-point tiebreaker occurs when the game score reaches 6-6. Players alternate serving, where the first player will serve one point, then the second player serves two points, then the players alternate serving two points apiece for the rest of the tiebreaker. The first player to win seven total points with a margin of difference of at least two points wins the tiebreak and consequently the set. If the tiebreaker score is 6-6, then the tiebreaker continues until the margin of difference is two points. In best of three set matches, which is used in virtually every tournament except men's professional grand slam events, the first player to win two sets wins the match.

In the early 2000s, the 10-point match tiebreaker was introduced as an alternative to playing a full third set. The match tiebreaker occurs when players have each won one set. It follows the same format as the 12-point tiebreaker, except a player must win 10 total points instead of 7. The winner of the match tiebreaker wins the third set and, consequently, the match. The United States Tennis Association (USTA) now uses the 10-point match tiebreaker for the consolation draws at most junior tournaments (United States Tennis Association, 2014).

The use of different scoring systems presents some interesting questions to researchers, such as how these systems affect the probability of an upset occurring and how much do they differ in expected match duration. While a number of research papers have touched on certain aspects of these questions, this study will formally quantify the answers through a structured design of experiment using discrete event simulation.

2. RELATED WORK

Researchers have analyzed tennis with mathematical and probability models since the 1970s. Schutz (1970) developed a mathematical model to evaluate and compare various tennis scoring systems that were prevalent at the time. In this model, the author assumed a single constant probability for each player winning a point regardless of who was serving. Schutz then used a Markov chain and random walks to calculate expected win probabilities and match durations, which was measured by total points played. Hsi and Burych (1971) expanded upon Schutz (1970) by incorporating the more realistic assumption of two distinct probabilities of who wins a point depending on which player is serving. The authors then calculated the probability of a player winning a single set based upon the two service point win probabilities. George (1973) employed probability models to compare the efficacy of different serving strategies.

With the invention of the tiebreaker, Carter Jr and Crews (1974) analyzed how win probabilities change under different scoring systems such as tiebreak sets and a "win a set by three games" system. The authors averaged out the two distinct probabilities from Hsi and Burych (1971) and calculated their results using a single probability model similar to Schutz (1970). Carter Jr and Crews (1974) found that the odds of a stronger player winning a tiebreak set are slightly less than a non-tiebreak set and that a similar dynamic occurs in a hypothetical "no ad game and win a set by one game" scenario. Conversely, the authors found that the odds of a stronger player winning a set increase under the "win a set by three games" system. In a very important paper, Pollard (1980) tested whether the assumptions of constant service point win probabilities and point independence in tennis modeling are adequate and, using data from 35 matches at a professional tennis tournament, showed that they are indeed sufficient.

Pollard then wrote a number of articles examining how different scoring systems affect match win probability and duration. Pollard (1983) derived the match duration distribution and probability of a player winning a two out of three tiebreak set match and two out of three non-tiebreak set match. The author concludes that the probability of the better player winning a non-tiebreak set is greater than winning a tiebreak set. Motivated by a number of delays in scheduled tournament match times, Pollard (1987) developed a new scoring system that has a lower match length variation. In this system, players play best of five "sets", where a set is won if a player wins 4 games and the opponent 2 or less games and a set is "halved" if the game score is 3-3. The author simulated 10,000 matches to show how this system reduces match length variance, especially in matches between strong servers, which are the

exact instances where match length variation is greatest. Pollard and Noble (2002) examined additional ITF scoring proposals, including three out of five short sets formats with different starting game scores. Pollard and Noble (2004) analyzed the 50-40 scoring format, where the server must reach 50 (4 points) to win the game or the returner reach 40 (3 points) to win the game, and concluded this format is more efficient with a lower match length variance for two players with strong serves.

Pollard and Barnett (2006) considered how psychological factors might influence the probabilities of winning a tennis match. The authors show that when equal players possess a psychological advantage when leading a set, the player to start serving that set wins more than half the time. The authors also consider when the opposite dynamic occurs in which the player behind in the set has the psychological advantage (referred to as the “back to the wall” advantage). Pollard and Pollard (2009) examine the fairness of scoring systems when considering different probabilities of winning a service point between the deuce and ad courts. The authors consider identical, symmetrically identical, and equal players in the ad, no ad, and 50-40 scoring formats as well as a separate analysis for the traditional 12-point tiebreaker format. Brown *et al.* (2008) develop recursive formulas to calculate higher moments of the total number of points played in a match given service point win probabilities. Barnett (2012) gives results for 13 different scoring systems with two different serving percentage scenarios and recommends that different events use different scoring formats.

In addition to singles matches, researchers have also examined doubles play. Pollard (2005) shows that the traditional set is “unfair” in doubles regardless of serving order when considering all four possible serving orders between identical teams with a stronger and weaker player. Pollard *et al.* (2007) and Brown *et al.* (2008) examine multiple scoring systems in men’s doubles, in which the service point win probability is generally higher than in singles. The authors used recursive methods and the NP approximation from Pesonen (1975) to calculate win probabilities and match length distributions.

Researchers have also looked at the efficiency of tennis scoring systems. Miles (1984) used symmetric sequential analysis to show that the traditional scoring systems in tennis are inefficient. But, as Pollard (1987) and Brown *et al.* (2008) note, although the proposed systems in Schutz (1970) and Miles (1984) are more efficient, they are not practical because of their high variance in match length. Pollard and Pollard (2015) extended upon Miles (1984) for more general scoring structures. Betting on tennis is a lucrative industry, and researchers have also developed models to calculate match win probabilities between two specific players both before and during the match (Barnett and Clarke, 2005; Spanias and Knottenbelt, 2012).

A few intuitive themes emerge from this body of research. First, matches with ad games are longer, or have more expected points per match, than matches with no ad games. Similarly, matches without tiebreakers are longer than matches with tiebreakers. Second, the longer the scoring system, the higher the probability of the better player winning that game, set, or match. Third, matches between very strong servers have a higher match length variance. This last consideration is important given the increasing power new racket technology provides players on their serves (Haake *et al.*, 2007). This variance is amplified if the scoring system does not contain tiebreakers.

The objective of this paper is to expand upon these findings by formally quantifying how much match win probabilities and match length change as a result of different scoring systems and whether these changes are statistically significant, using a systematic design of experiment. While researchers have proposed a plethora of different scoring systems, this paper will focus on the three most common systems utilized in the United States: best of three tiebreak sets with ad games (used in the main draw at virtually all USTA tournaments), best of three tiebreak sets with ad games with a 10-point match tiebreaker in lieu of a third set (some USTA leagues and consolation draw at most USTA junior tournaments), and best of three tiebreak sets with no ad games, which was recently implemented in NCAA Division I tennis (Johnson, 2015). One of the main attractions of the latter two scoring systems is the reduction in match length and the variance of that length, which makes it easier for tournament organizers (Pollard, 1987) and TV broadcasts (Johnson, 2015). However, it does not appear that a

systematic analysis of just how much these two scoring systems reduce the better player's or team's probability of winning has been considered prior to their implementation. Furthermore, expected match length has always been expressed in total points played; this paper lays the groundwork to express match length not only in expected point played, but also in actual time elapsed.

3. METHODOLOGY

For the purposes of this paper, each simulated tennis match occurs between two players, Player A and Player B. As in previous papers such as Pollard (1983) and Brown *et al.* (2008) there are two separate probabilities in the model: p_a , which represents the probability that Player A wins a point when serving, and p_b , which represents the probability that Player B wins a point when serving. Like the majority of research in this field, the key assumptions in the simulation model are that p_a and p_b remain constant and that individual points are independent of one another; Pollard (1980) showed these assumptions are adequate. In the simulation, Player A will always be the better player, that is, $p_a > p_b$. The term "skill difference" denotes how much better Player A is than Player B in terms of service point win probabilities. In other words, skill difference equals $p_a - p_b$.

The simulation logic follows the nested nature of tennis scoring. At the lowest level, a point is simulated based on which player is serving and the respective service point win probability. After each point, the logic updates the score, number of points played, current match duration, and checks if the game has been completed. If the game has not been completed, then additional points are simulated until the game is complete. Once the game is completed, the set score is updated and the logic checks to see if the set is completed. If not, then another game is played with the other player serving. This logic follows for sets, matches, and the number of matches simulated at each p_a and skill difference. Figure 1 shows the conceptual logic for the simulation. Note that figure 1 presents the high-level simulation logic and does not contain details such as changeovers, tiebreakers, who starts serving each game, among others.

For the simulation design of experiment, five different service point win percentages were considered for Player A, ranging from a relatively weaker serve for the better player ($p_a = 0.55$) to a stronger serve ($p_a = 0.75$), incrementing every 0.05. Barnett (2012) gives service point win percentages data from professional grand slam tournaments and this data was used as a basis for selecting the range of p_a . Although 0.7 and 0.75 are considered rather high win probabilities that generally only occur in professional men's tennis between stronger servers on fast surfaces, they will still be included in the experiment. At each p_a , the skill difference ranges from 0.01 to 0.2, incrementing every 0.01. The simulation metrics of interest are the percentage of matches won by Player A, total points played, and match duration in time. The standard deviation of each metric of interest is also determined to calculate confidence intervals for statistical comparison. To calculate match duration in time, the times for points, changeovers, and other aspects were inserted into the simulation logic. These times were estimated based off the rules of the game (e.g. 90 seconds for changeovers) and estimates, however, there was no comprehensive data collection conducted to determine actual values; the intent was to build these times into the simulation logic, which would allow match duration in time to be further explored in future research once more comprehensive data collection is done. Furthermore, logic for a first serve percentage, p_{1srv} , probability of winning a point given a made first serve, $p_{w|1srv}$, and probability for winning a point given a second serve, $p_{w|2srv}$, were incorporated into the model instead of a simple p_a and p_b . This level of detail is not necessarily vital, as they reduce to a p_a and p_b that give the same results. However, it allows the realistic inclusion of a small time delay that occurs when a player misses a first serve.

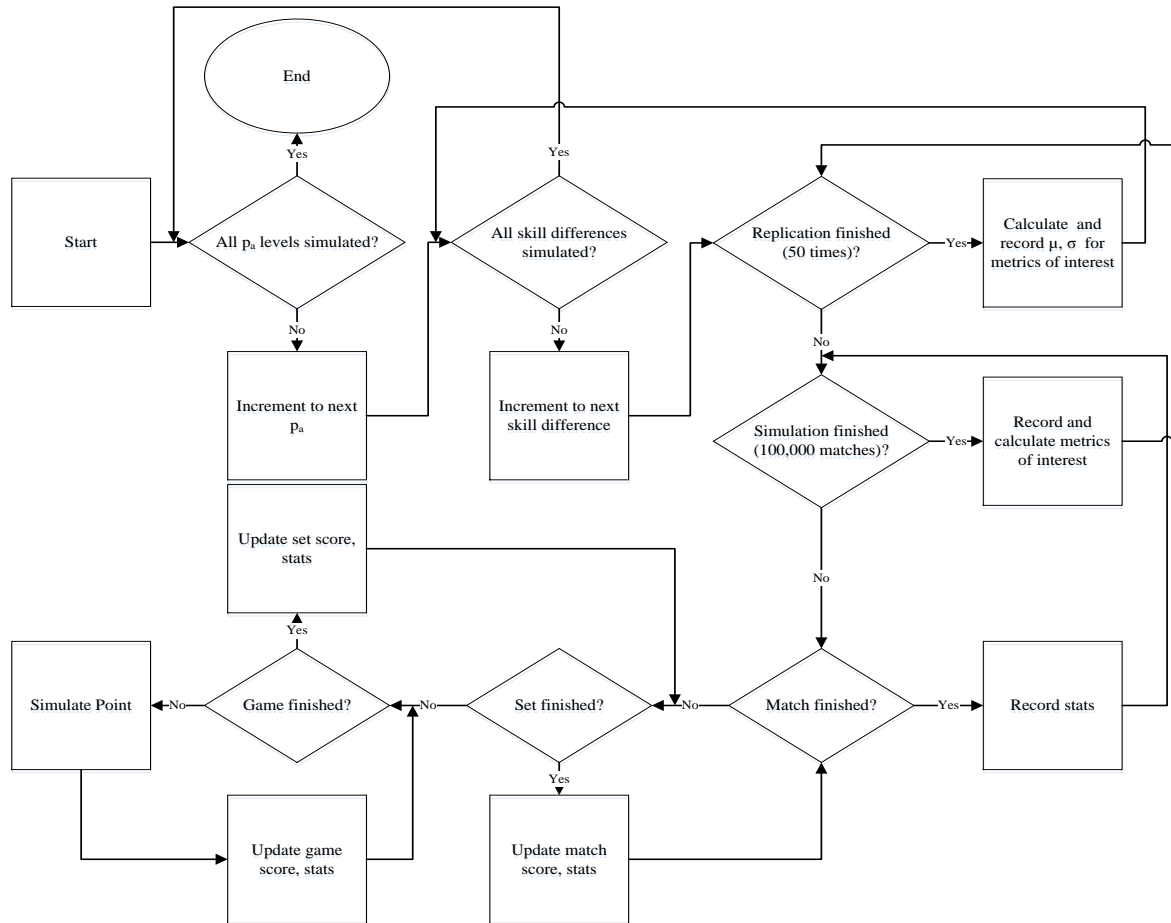


Figure-1. Simulation Conceptual Model

Like Pollard and Noble (2002) 100,000 total matches are simulated at each p_a and skill difference level, however, this will be replicated 50 times to help ensure normal distributions of the metrics of interest. The two players alternate the first to serve each match. The simulation was run using Java version 1.8.0_101 on Windows 8. Normality tests were randomly conducted on four separate occasions for the three metrics of interest using the Kolmogorov-Smirnov Test, alpha value of 0.05. In all 12 cases, normality assumptions were adequate. To validate the simulation, results were compared to previously calculated results from Pollard (1983) and Brown *et al.* (2008). Table 1 displays the results. For purposes of brevity, in column 4, “ad” refers to best 2 out of 3 tiebreak sets with ad games, “no ad” to best 2 out of 3 tiebreak sets with no ad games, and “10 point” to best 2 out of 3 tiebreak sets with ad games and a 10-point match tiebreaker in lieu of a third set. These terms will be used to represent the three scoring systems for the rest of the paper.

Table-1. Simulation Validation

Paper	Method	p_a, p_b	Scoring System	Paper Results (Player A win %, total points)	Simulated Results, 95% Confidence Interval (Player A win %, total points)
Pollard (1983)	Probabilistic Models	0.6, 0.5	Ad	0.9078, 142.1	(0.906, 0.909), (141.89, 142.38)
		0.7, 0.5	Ad	0.9947, 112.3	(0.994, 0.995), (112.1, 112.4)
		0.7, 0.6,	Ad	0.8910, 145.1	(0.889, 0.893), (144.8, 145.3)
Brown <i>et al.</i> (2008)	Recursive Methods	0.77, 0.73	Ad	0.669, 166.3	(0.667, 0.672), (166.0, 166.5)
		0.77, 0.73	No Ad	0.672, 155.2	(0.669, 0.675), (151.9, 152.4)*
		0.77, 0.73	10 Point	0.656, 142.8	(0.654, 0.659), (142.7, 143.0)
		0.62, 0.58	Ad	0.697, 160.0	(0.695, 0.700), (159.8, 160.2)
		0.62, 0.58	No Ad	0.681, 140.7	(0.677, 0.683), (140.5, 140.9)
		0.62, 0.58	10 Point	0.670, 137.8	(0.667, 0.673), (137.7, 138.0)

As table 1 shows, the 95% confidence interval of the simulated results fell within the calculated results from previous papers in 17 of 18 cases. The only occurrence where it did not is marked by an * in the table. It occurs with the total points played for no ad with $p_a = 0.77$ and $p_b = 0.73$, where the simulated result disagreed with the calculated result by about 2.6%. Given these results, the simulation model can be deemed adequate.

4. RESULTS

4.1. Scoring System and Win Probability

As figure 2 illustrates, the probability of the stronger player winning decreases as the scoring system changes from ad to no ad to 10 point. This decrease is greater when there are relatively weaker servers, or a lower p_a . As skill difference increases, win percentage increases in a slightly convex manner.

Figure 3 shows the relative win percentage decrease from one scoring system to another. 95% confidence intervals were calculated for each win percentage to determine whether the decrease was statistically significant or not. If the intervals overlapped, they were not statistically significant and are denoted as 0% on figure 2. When p_a increases, the difference between ad and no ad scoring systems decreases to the point where when $p_a = 0.75$, there is no statistical difference between ad and no ad from a skill difference of 0.01 to 0.09, after which the decrease is less than 1%.

Peak difference in win percentage occurs somewhere in the first third of the skill difference range and increases slightly as p_a increases. The highest win percentage decrease from ad to 10-point is 5.05% ($p_a = 0.55$, skill difference = 0.07) and the highest win percentage decrease from ad to no ad is 3.57% ($p_a = 0.55$, skill difference = 0.06). The decrease in win percentage at relatively weaker serves is particularly interesting because in general, juniors have weaker serves than professional players and are more likely to fall in this range. Therefore, the areas where a 5% decrease in the better player winning from ad to 10-point occur exactly in the primary population that play in USTA junior tournaments, where the 10-point system is used in the consolation rounds.

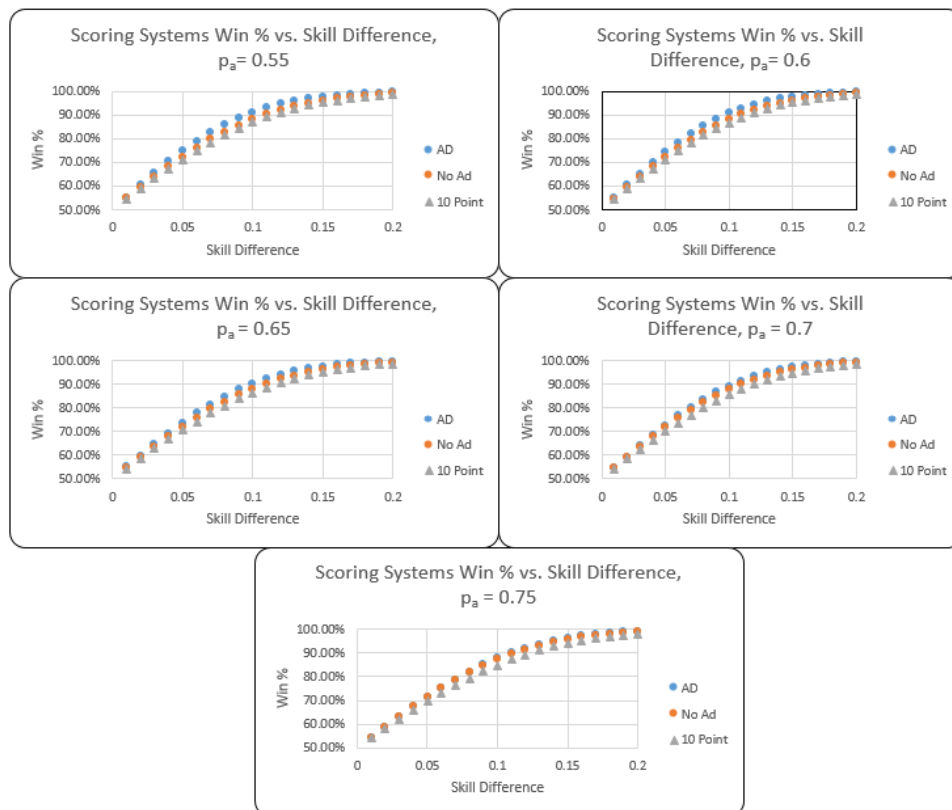


Figure-2. Player A Win % vs. Skill Difference

Figure 4 illustrates the three scoring systems win percentage vs. skill difference. These results suggest that no ad could be considered the most robust scoring system in that the win percentage hardly changes from weak servers to strong servers, while ad is the least robust.

4.2. Scoring System and Points Played

Figure 5 shows total points played vs. skill difference. Not surprisingly, total points decrease as the skill difference increases and ad scoring takes more points than no ad or 10-point. What is interesting however, is the dynamic between points played and no ad versus 10-point scoring. At lower skill differences, no ad requires fewer points than 10-point. At a skill difference ranging from 0.08 ($p_a=0.55$) to 0.12 ($p_a=0.75$), this changes and 10-point requires fewer total points than no ad. Figure 6 displays the relative percentage decrease in total points from different scoring systems

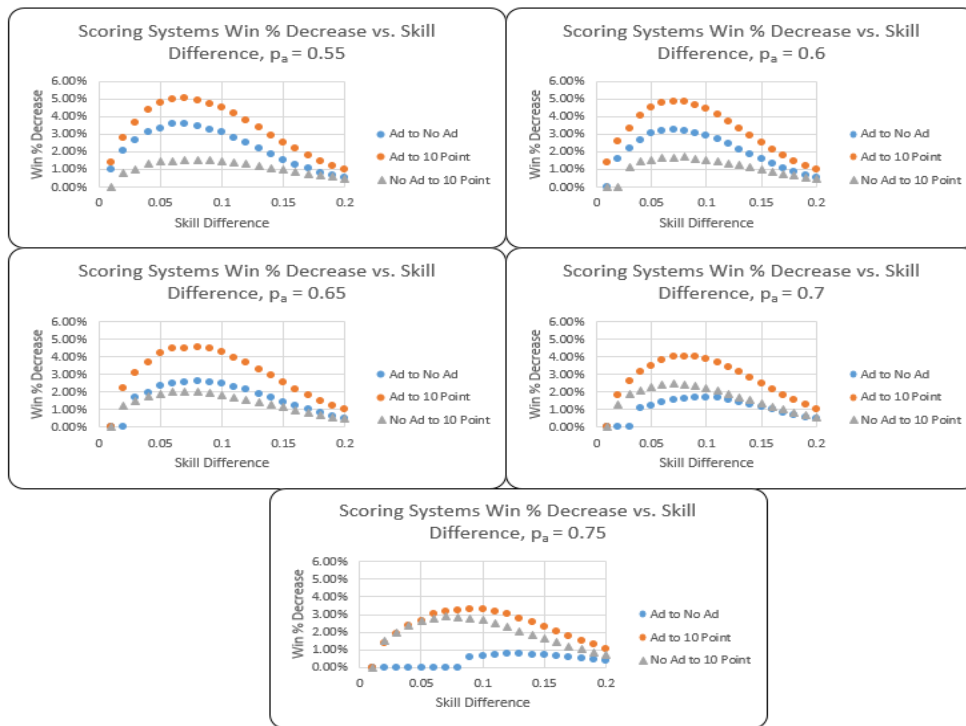


Figure-3. Player A Relative Win % Decrease

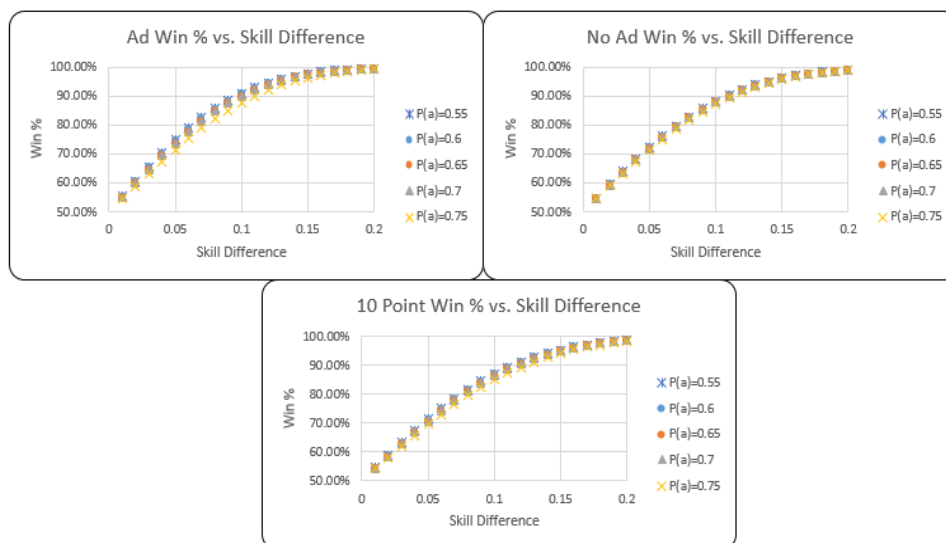


Figure-4. Player A Win % vs. Skill Difference by Scoring System

Again, results that are not statistically significant are denoted as 0 on figure 6. The highest points played percentage decrease from ad to 10-point is 14.74% ($p_a = 0.55$, skill difference = 0.01) and the highest points played percentage decrease from ad to no ad is 13.32% ($p_a = 0.55$, skill difference = 0.01).

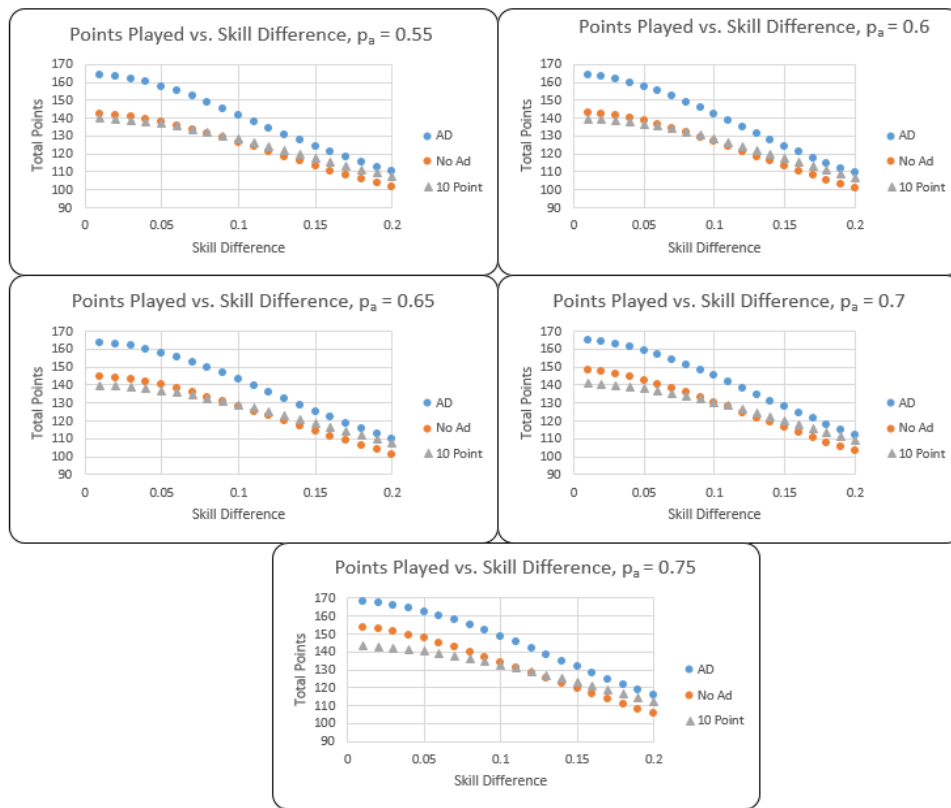


Figure-5. Points Played vs. Skill Difference

This dynamic between no ad and 10-point can seem counterintuitive. At low skill differences, players will often “split sets” and must play a third and final set. The 10-point scoring system would seem to reduce the total points played in situations where players would be likely to split sets more often and thus not play out a third and final set. One explanation for this is that no ad makes up for playing out a third set by having shorter games throughout the whole match. In other words, although 10-point scoring system requires a shorter third set, the long ad games that occur throughout the match end up taking more total points than no ad scoring, where players play out a full third set but never the long games because of the sudden death point when the game score is 3-3.

4.3. Scoring System and Match Length

The relationship between scoring system and match length in minutes follows a similar pattern to total points played as figures 7 and 8 illustrate. While the results fall within the realm of common match time lengths, meaningful insights cannot necessarily be gleaned from these results for two reasons. First, there are no results from previous research that calculate the match length in time, either mathematically or through simulation, to compare. Second, as was previously mentioned, there was no comprehensive data collection done, nor is there any existing data set, that contains the actual time and probability distributions of points played at different serving strengths and skill differences. In addition, the actual time elapsed and probability distributions of other factors such as time between points and changeover times was estimated based off rules, not observation or existing data. Consider changeovers: players change sides of the court whenever the total number of games played in the set is odd.

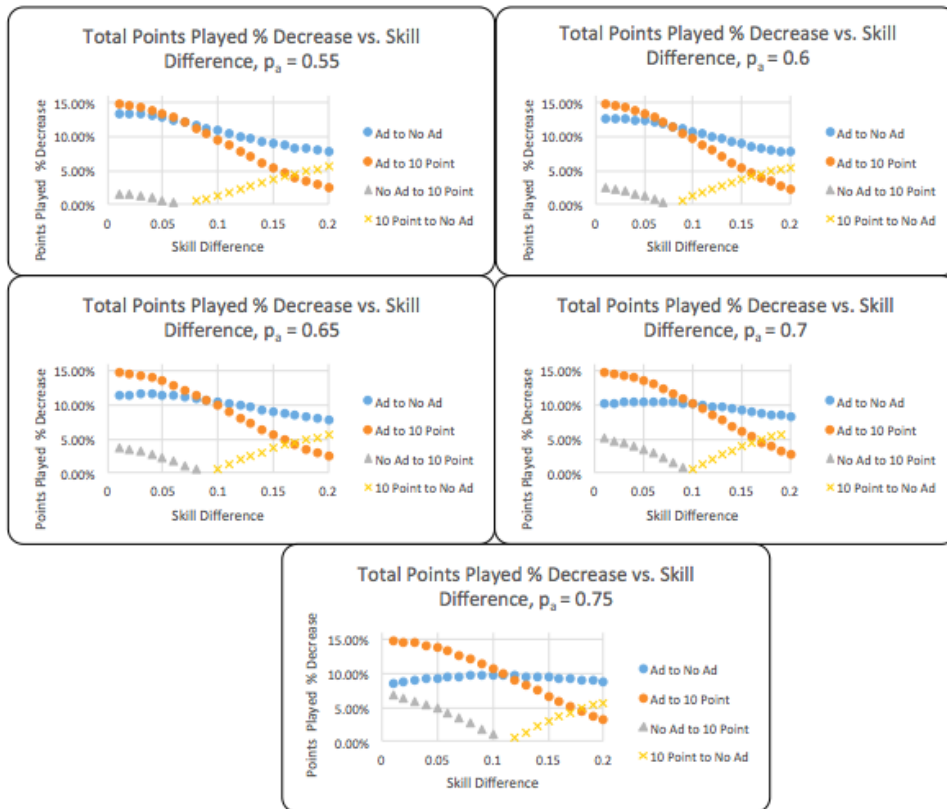


Figure-6. Points Played Relative % Decrease

Players are allotted 90 seconds of rest during changeovers with the exception being after the first game of the set. It is not known whether players actually take the full 90 seconds and what the probability distribution of those changeovers looks like. Therefore, future research lies in examining this dynamic and whether the match length in time actually does mirror the total points played.

4.4. Impact of Ad to No Ad in NCAA Division I Tennis

The results from the simulation can be analyzed with respect to the recent change from ad to no ad in NCAA Division I team tennis. A college team match consists of a best of seven point format with six no ad singles matches each counting for one point and one doubles point that consists of 3 separate “matches” of a single no ad set. Whatever team wins best two out of three doubles matches wins the doubles point. Doubles is played prior to singles. Therefore, to win the match, a team must win three singles matches and the doubles point or at least 4 singles matches. By using the somewhat unrealistic assumption that the serving strength and skill difference between the players in the six matches are identical, the binomial distribution can be used to get an idea of how the team win percentage changes from ad to no ad scoring. Essentially, team A consists of six identical Player A's and team B consists of six identical Player B's. The doubles point will not be considered. Instead, the two different scenarios for Team A to win are considered: it wins the doubles point and only has to win at least 3 singles matches or it loses the doubles point and has to win at least 4 singles matches. Figure 9 shows the win percentage vs. skill difference for this college team match. A similar trend emerges where the difference between win percentage in ad and no ad is more pronounced for relatively weaker servers.

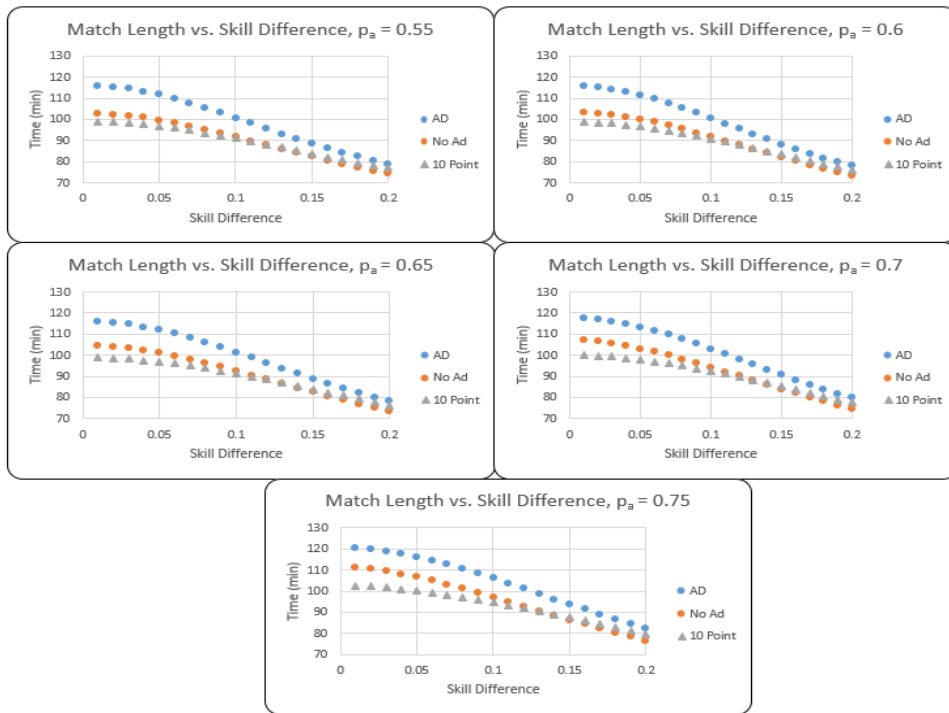


Figure-7. Match Length vs. Skill Difference

Figure 10 shows the relative team win percentage decrease that occurs from ad to no ad scoring for both scenarios. Player A relative win percentage decrease for a single match from ad to no ad from figure 3 is also shown for context. Figure 10 shows that in certain scenarios, the relative win percentage decrease for a single match amplifies under the college team format and gives the weaker team up to a 4.18% better chance of upsetting the stronger team.

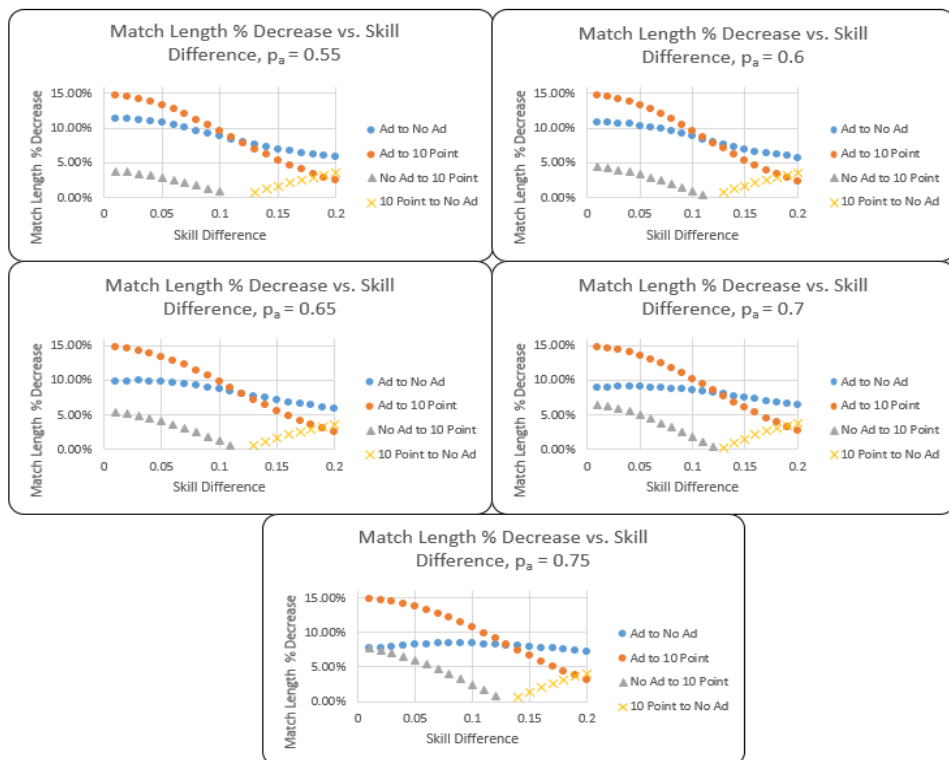


Figure-8. Match Length Relative % Decrease

4.5. Discussion

The simulation results agree with the intuitive themes from previous research, however, they also provide insight into the actual amount that scoring systems affect upset percentage and match length. At USTA junior tournaments, for example, the change from ad to 10-point scoring from the main draw to the consolation draw can decrease the better player’s chances of winning by over 5%. The drop can be over 4% for the stronger NCAA Division I team from ad to no ad. This is not to say that one scoring system is necessarily better than the other. While scoring systems such as no ad and 10-point give weaker players or teams a better chance of winning, they also have the benefit of shortening the match, which can be easier for TV or tournament scheduling. Additionally, tennis governing bodies may want more parity in their competitions for one reason or another. Players may also have a smaller chance of overuse injuries in a shorter match format. Conversely, shorter scoring systems may not give players who are more physically fit the advantage they used to have. Governing bodies must consider and weigh a number of factors when selecting what scoring format to use.

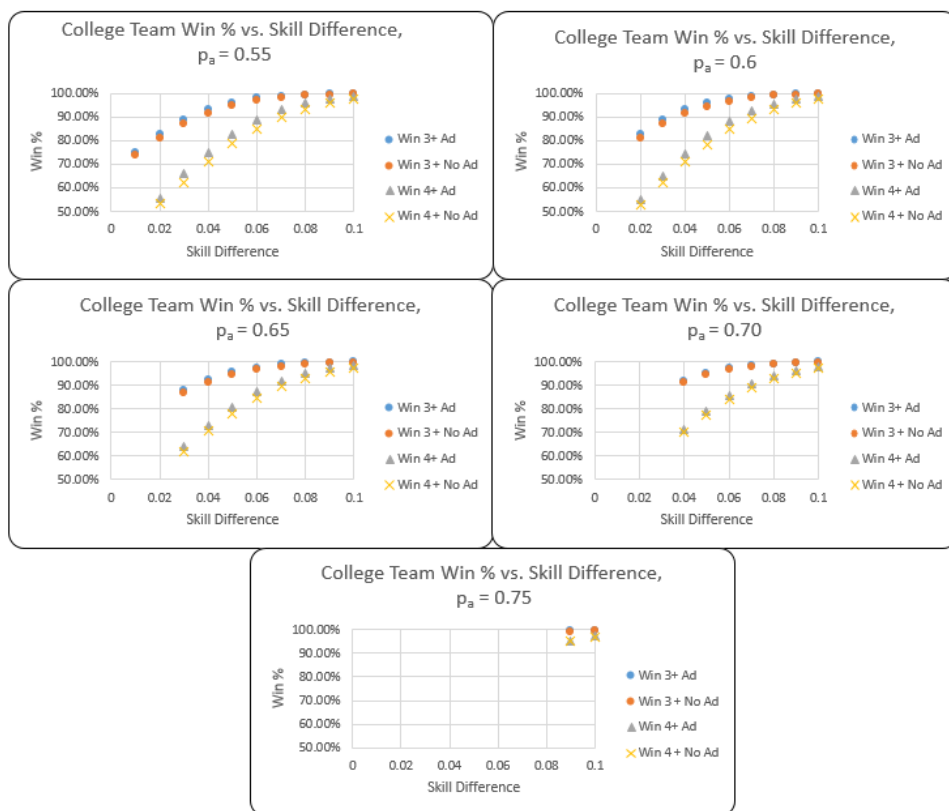


Figure-9. College Team A Win % vs. Skill Difference

While this paper provided a more quantitative analysis of different scoring systems in tennis, there are a few shortcomings. First, as was previously mentioned, a comprehensive data collection on point and break times for matches was not conducted, which means the match length in actual time metric results are not particularly useful at this time. However, the groundwork logic for the simulation model has been laid and future research could lie in conducting the data collection and a more thorough analysis of how scoring systems affect the expected match time. Expressing this in terms of actual time instead of total points played could be of great benefit to tournament organizers and tennis governing bodies. This would be especially interesting in the case of no ad vs. 10-point, where the scoring system that has the higher total points played depends on the skill difference. The structure of the scoring system suggests that 10-point may always be lower in terms of match time because the 10-point tiebreaker is played straight through with no changeover breaks, whereas a no ad third set will still contain many 90 second changeover breaks.

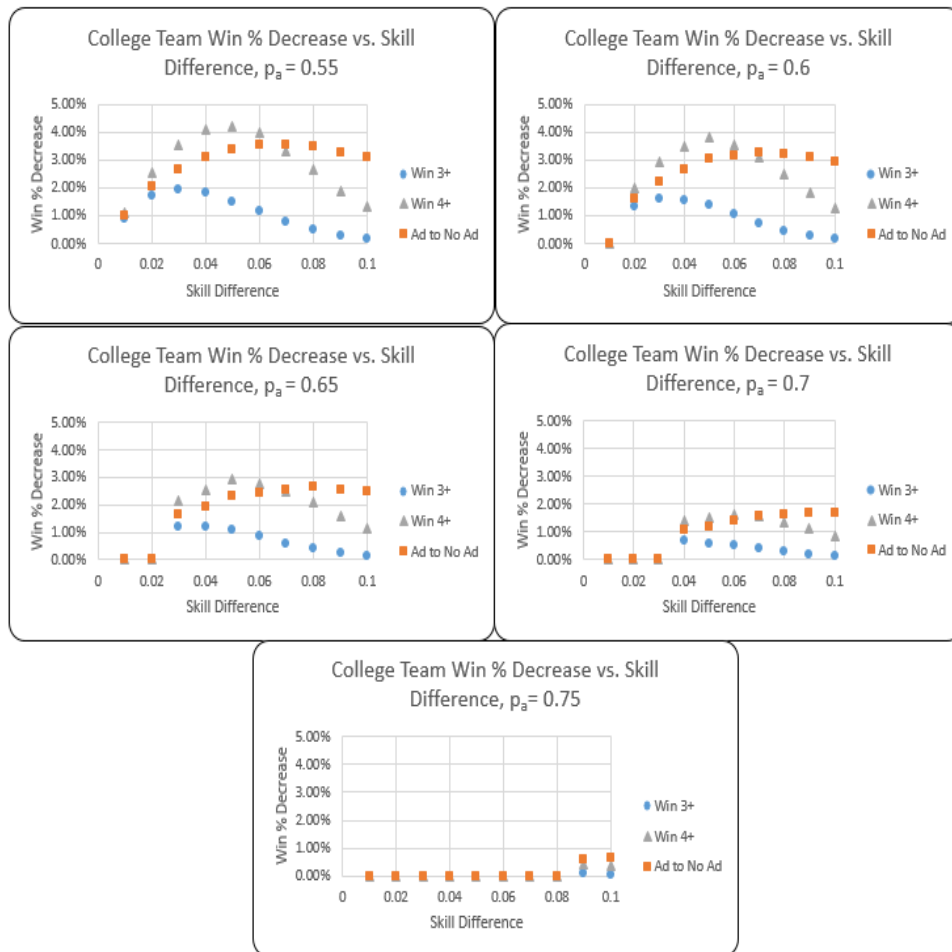


Figure-10. College Team A Relative Win % Decrease

A second shortcoming is the lack of consideration of the receiver getting to choose what side to return from in no ad scoring. The simulation model followed previous research in assuming a constant p_a and p_b . But as Pollard and Pollard (2009) note, it may be possible for players to have a different service point win percentage depending on what side they are serving from, the deuce or ad courts. In this scenario, no ad scoring may present an advantage to the returner, who gets to select what side to return from on the sudden death point at 3-3. Future research could lie in examining this dynamic or conducting data collection on NCAA Division I tennis matches to see if this dynamic occurs frequently.

5. CONCLUSION

This paper used discrete event simulation to examine how the three most common tennis scoring systems in the United States affect the stronger player’s match win percentage and match time in both total points played and actual time elapsed. Results confirmed previous (and intuitive) findings that shorter match formats give the stronger player a lower probability of winning. The main contribution of this paper was to formally quantify this dynamic is a systematic design of experiment. While these shorter formats can result in a greater than 5% drop in the stronger player’s chances of winning, they also may be more practical for tournament organizers and TV scheduling. Governing bodies should weigh all of these factors when considering what format to use.

Future research lies in collecting match data to produce more accurate results for the total time elapsed in matches now that the simulation logic has been laid, and also in examining how service point win percentages may differ in the deuce and ad courts and how this can affect no ad scoring.

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